Workshop and Meeting
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Tie-breaking approaches for collective decision making

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Tie-breaking in collective DM

Overview

1. The problem & framework:reaching collective supported decisions
2. Model-based fusion approach
3. Ties in model-based fusion results
4. Resolving ties: role of context in decision making
5. Characterization: Sensitive/robust decisions
6. Characterization:Skeptical and credulous decisions
7. Conclusions & future work

The problem

- how can a panel reach a supported decision whether to hire an employee?
- rule:

  -a candidate is hired (d) if and only if

  -the candidate has an adequate CV (p)

  and

  -the candidate made a good interview (q)

- method: each member submits his decision and the justifications for it according to the rule.

Trivial ?

$$p \wedge q \leftrightarrow d$$

	p	q	d
$member_1$	1	1	1
$member_2$	1	0	0
$member_3$	0	1	0
panel	1	1	0

Formalization

- a panel of n members (designated by i in [1,n])

Tie-breaking in collective DM

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- all sets submitted by an agent (profile) $\mathcal{P} = \{\mathcal{S}_1, \ldots \mathcal{S}_n\}$
- set of all complete alternatives $\Omega = \{\omega_1, \ldots \omega_l\}$

$$\{\omega_i \cup \mathcal{R}\} \nvDash \perp \qquad \mathcal{P} \subseteq \Omega$$

Model-based fusion

- a method to merge sets of consistent beliefs ($\mathcal{P}$) under integrity constraints ($\mathcal{R}$) .
- distance function d between alternatives (ω_i)

$$\mathrm{d}(\omega_1, \omega_1) = 0 \qquad d(\omega_1, \omega_2) = d(\omega_2, \omega_1)$$

- intuition: how close is each alternative in Ω to $\mathcal{P}$

$$D(\omega_i, \mathcal{P}) = f(d(\omega_i, \mathcal{S}_1), \dots, d(\omega_i, \mathcal{S}_n))$$

- "winner" is the alternative ω closest to $\mathcal{P}$

$$\boxed{D(\omega, \mathcal{P}) = \min\{D(\omega_i, \mathcal{P})\}}$$

Fusion example

- distance function d - Hamming distance
- the judgment sets $\mathcal{P} = \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3\}$:

$\mathcal{S}_1 = \{\neg\varphi_1, \varphi_2, \neg\delta\}$ $\mathcal{S}_2 = \{\varphi_1, \varphi_2, \delta\}$ $\mathcal{S}_3 = \{\neg\varphi_1, \varphi_2, \neg\delta\}$

- the integrity constraint $\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$
- other possible models $\omega_4 = \{\varphi_1, \neg\varphi_2, \neg\delta\}$
  $\omega_5 = \{\neg\varphi_1, \neg\varphi_2, \neg\delta\}$
- distance to the profile $D(\omega_i, \mathcal{P}) = \sum_{j=1}^{n} d(\omega_i, \mathcal{S}_j)$

Fusion example

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

Hamming distance

$\mathcal{P} = \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3\}$

	φ_1	φ_2	δ	$d(\omega_i, \mathcal{S}_1)$	$d(\omega_i, \mathcal{S}_2)$	$d(\omega_i, \mathcal{S}_3)$	$D(\omega_i, \mathcal{P})$
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$\omega_2 = \mathcal{S}_2$	1	1	1				
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ω_4	1	0	0				
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Thursday, October 30, 2008

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Tie-breaking in collective DM

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Tie-breaking in collective DM

Fusion & ties

- in the general case the is no single "winner":

set of tied alternatives: $\mathcal{T} = \{\tau_i, |D(\tau_i, \mathcal{P}) = \min\{D(\omega_i, \mathcal{P})\}$

	φ_1	φ_2	δ	$d(\omega_i, \mathcal{S}_1)$	$d(\omega_i, \mathcal{S}_2)$	$d(\omega_i, \mathcal{S}_3)$	$D(\omega_i, \mathcal{P})$	
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Tie-breaking in collective DM

What ties mean?

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Tie-breaking in collective DM

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- choose randomly from the tied alternatives?
- revise: all tied alternatives are equally good to be the collective decision under the information considered in the fusion!

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More information

- elicit preferential information.

Tie-breaking in collective DM

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- idea: consider the context to resolve the ties!

More information

- elicit preferential information. Which?
- what information do we already have?
- idea: consider the context to resolve the ties!
- our proposal: to break ties by taking into consideration the type of decision more desirable in a given context

Example 1-2

Case1:

δ_1: *enforce tax on cigarettes*

φ_1: *smoking should be reduced among population*

φ_2: *higher cost of cigarettes reduces number of smokers*

The rule:

$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$

Case2:

δ_1: *enforce death penalty for drug trafficking*

φ_1: *less drugs available accounts for less drug abusers*

φ_2: *threat of death penalty reduces the number of drug dealers*

The rule:

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Tie-breaking in collective DM

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Irreversible decision

Tie-breaking in collective DM

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The rule:

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Case2: *All things being equal, be skeptical*

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φ_1: *less drugs available accounts for less drug abusers*

φ_2: *threat of death penalty reduces the number of drug dealers*

The rule:

$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$

Irreversible decision

Example 3-4

Case 3:

δ_1: *hire contractor*

φ_1: *there is enough money in the budget*

φ_2: *contractor appears reliable*

The rule:

$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$

Case 4:

δ_1: *loan to friendly bank*

φ_1: *we possess enough liquid asset*

φ_2: *shares of own bank are expected to maintain market value*

The rule:

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Tie-breaking in collective DM

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Panel will be held responsible

uni.lu

Tie-breaking in collective DM

Example 3-4

Case 3: *All things being equal, go for sensitive alternative*

δ_1: *hire contractor*

φ_1: *there is enough money in the budget*

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Revisable decision

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Case 3:

δ_1: *hire contractor*

φ_1: *there is enough money in the budget*

φ_2: *contractor appears reliable*

The rule:

$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$

Case 4:

All things being equal, go for robust alternative

δ_1: *loan to friendly bank*

φ_1: *we possess enough liquid asset*

φ_2: *shares of own bank are expected to maintain market value*

The rule:

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Context & propositions

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- how to capture decision characteristics in the formalization?

Sensitive/robust

Tie-breaking in collective DM

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Sensitive/robust

- robust alternative - decision will change with many swaps in supporting reasons
- sensitive alternative - decision will change with few swaps in supporting reasons
- sensitive/robust: measure how changing an opinion on a decision or a justification would affect a judgment set

Impact factors

- impact factor of $\sigma_j \in \omega_i$ shows how much ω_i will have to change if σ_j is replaced by $\neg\sigma_j$

$$u^{\Omega}(\sigma_j, \omega_i) = min\{d(\omega_i, \omega_k)\}$$

where $\omega_i, \omega_k \in \Omega$; $\sigma_j \in \omega_i$ and $\neg\sigma_j \in \omega_k$

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$\omega_1 = \mathcal{S}_1$	1(1)	0(2)	0(2)	4
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$\omega_3 = \mathcal{S}_3$	1 (2)	1 (2)	1 (2)	4
ω_4	0 (1)	0 (1)	0 (3)	5

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

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Sensitive to robust

- from most sensitive to most robust:

$$\mathcal{S}_3 \lesssim_u \mathcal{S}_1 \sim_u \mathcal{S}_2 \lesssim_u \omega_4$$

	φ_1	φ_2	δ	$D(\omega_i, \mathcal{P})$	$\lesssim_u$
$\omega_1 = \mathcal{S}_1$	1(1)	0(2)	0(2)	4	2-1=1
$\omega_2 = \mathcal{S}_2$	0(2)	1 (1)	0(2)	4	2-1=1
$\omega_3 = \mathcal{S}_3$	1 (2)	1 (2)	1 (2)	4	2-2=0
ω_4	0 (1)	0 (1)	0 (3)	5	3-1=2

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

Credulous, skeptical

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- credulous alternative - maximizes input from the group on supporting reasons
- skeptical alternative - also takes into account how often a decision appears in the set of all possible alternatives

Frequency factors

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$$v^{\mathcal{G}}(\sigma_j) = |G|$$

$$G = \{\omega_i | \omega_i \subseteq \mathcal{G}; \sigma_j \in \omega_i\}.$$

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$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

over $\mathcal{P}$

over Ω

Credulous & skeptical

- more credulous alternative - the sum of the frequency factors over the reasons in the profile is higher .

$$\omega_i \lesssim_c \omega_j \Leftrightarrow \sum_{k=1}^{m-1} \upsilon^{\mathcal{P}}(\sigma_k) \leqslant \sum_{t=1}^{m-1} \upsilon^{\mathcal{P}}(\sigma_t)$$

$$\sigma_k \in \omega_i \text{ and } \sigma_t \in \omega_j$$

Tie-breaking in collective DM

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$$\sigma_k \in \omega_i \text{ and } \sigma_t \in \omega_j$$

- more skeptical alternative - the sum of the f.factors of the reasons (over $\mathcal{P}$) added to the f.factor of the decision (over Ω) is higher.

$$\omega_i \lesssim_c \omega_j \Leftrightarrow v^{\Omega}(\delta_i) + \sum_{k=1}^{m-1} v^{\mathcal{P}}(\sigma_k) \leqslant v^{\Omega}(\delta_j) + \sum_{t=1}^{m-1} v^{\mathcal{P}}(\sigma_t)$$

$$\sigma_k \in \omega_i \text{ and } \sigma_t \in \omega_j$$

Ordering

- from least to most credulous:
  - $\omega_4 \lesssim_c \mathcal{S}_1 \sim_c \mathcal{S}_2 \lesssim_c \mathcal{S}_3$
- from least to most skeptical:
  - $\omega_4 \sim_s \mathcal{S}_3 \lesssim_s \mathcal{S}_1 \sim_s \mathcal{S}_2$

	φ_1	φ_2	δ	$D(\omega_i, \mathcal{P})$
$\omega_1 = \mathcal{S}_1$	1(2)	0(1)	0 (3)	4(3,6)
$\omega_2 = \mathcal{S}_2$	0(1)	1(2)	0 (3)	4 (3,6)
$\omega_3 = \mathcal{S}_3$	1(2)	1(2)	1 (1)	4 (4,5)
ω_4	0(1)	0(1)	0 (3)	5 (2,5)

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

Conclusions

- context helps to resolve between tied conflicting decisions

Tie-breaking in collective DM

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- but context does not always give an ordering:

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- contexts helps resolve between tied contradicting decisions
- but context does not always give an ordering:

	φ_1	φ_2	δ	$D(\omega_i, \mathcal{P})$
$\omega_1 = \mathcal{S}_1$	1(2/2)	0(2/1)	0 (2/2)	4(0,3,5)
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$\omega_3 = \mathcal{S}_3$	1(2/2)	1(2/2)	1 (2/2)	4 (0,4,6)
ω_4	0(2/1)	0(2/1)	1 (2/2)	5 (0,2,4)

R=$\{\varphi_1 \leftrightarrow \varphi_2 \leftrightarrow \delta\}$

Open questions

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Open questions

- how much can be resolved with context ?
- which other contexts-characterizations for decisions can we distinguish?
- how to resolve what context-characterizations can not ?
  - idea: preferences over justifications based on context

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