

WORKSHOP AND MEETING  
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# Tie-breaking approaches for collective decision making

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# Overview

1. The problem & framework:reaching collective supported decisions
2. Model-based fusion approach
3. Ties in model-based fusion results
4. Resolving ties: role of context in decision making
5. Characterization: Sensitive/robust decisions
6. Characterization:Skeptical and credulous decisions
7. Conclusions & future work

● The problem

2. Model-based  
fusion approach

3. Ties in model-  
based results

4. Resolving ties:  
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decision making

5. Sensitive/robust  
decisions

6. Skeptical and  
credulous  
decisions

7. Conclusions

# The problem

● how can a panel reach a supported decision  
whether to hire an employee?

● rule:

d

-a candidate is hired if and only if

-the candidate has an adequate CV p

and

-the candidate made a good interview q

● method: each member submits his decision and  
the justifications for it according to the rule.



**The problem**

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# Trivial ?

*member<sub>1</sub>*

*member<sub>2</sub>*

*member<sub>3</sub>*

*panel*

$p \wedge q \leftrightarrow d$		
$p$	$q$	$d$
1	1	1
1	0	0
0	1	0
1	1	0



Tie-breaking in  
collective DM

# Formalization

## ● The problem

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● a panel of  $n$  members (designated by  $i$  in  $[1, n]$ )

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- a panel of  $n$  members (designated by  $i$  from  $[1, n]$ )
- propositional language  $\mathcal{L}$

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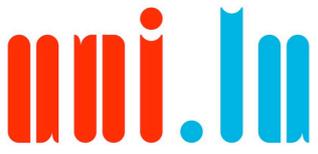
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# Formalization

- a panel of  $n$  members (designated by  $i$  from  $[1, n]$ )
- propositional language  $\mathcal{L}$
- a set of rules (propositional formulas)  $\mathcal{R}$

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- a set of judgments  $\mathcal{S}_i$  submitted by  $i$  (complete and consistent with  $\mathcal{R}$ ):  
$$\mathcal{S}_i = \{\sigma_{i,1}, \dots, \sigma_{i,m-1}, \delta_i\}$$



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  - for  $\sigma_1$  to  $\sigma_{m-1}$ :  $\sigma_4 = \varphi_5$  or  $\sigma_6 = \varphi_5 \rightarrow \varphi_7$
- all sets submitted by the members (profile)  
 $\mathcal{P} = \{\mathcal{S}_1, \dots, \mathcal{S}_n\}$



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- all sets submitted by an agent (profile)  
$$\mathcal{P} = \{\mathcal{S}_1, \dots, \mathcal{S}_n\}$$
- set of all complete alternatives  $\Omega = \{\omega_1, \dots, \omega_l\}$   
$$\{\omega_i \cup \mathcal{R}\} \neq \perp \quad \mathcal{P} \subseteq \Omega$$

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# Model-based fusion

- a method to merge sets of **consistent** beliefs ( $\mathcal{P}$ ) under integrity constraints ( $\mathcal{R}$ ).

- distance function  $d$  between alternatives ( $\omega_i$ )

$$d(\omega_1, \omega_1) = 0 \quad d(\omega_1, \omega_2) = d(\omega_2, \omega_1)$$

- intuition: how close is each alternative in  $\Omega$  to  $\mathcal{P}$

$$D(\omega_i, \mathcal{P}) = f(d(\omega_i, \mathcal{S}_1), \dots, d(\omega_i, \mathcal{S}_n))$$

- “winner” is the alternative  $\omega$  closest to  $\mathcal{P}$

$$D(\omega, \mathcal{P}) = \min\{D(\omega_i, \mathcal{P})\}$$

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# Fusion example

- distance function  $d$  - Hamming distance

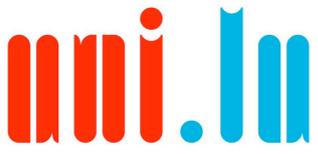
- the judgment sets  $\mathcal{P} = \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3\}$  :

$$\mathcal{S}_1 = \{\neg\varphi_1, \varphi_2, \neg\delta\} \quad \mathcal{S}_2 = \{\varphi_1, \varphi_2, \delta\} \quad \mathcal{S}_3 = \{\neg\varphi_1, \varphi_2, \neg\delta\}$$

- the integrity constraint  $\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$

- other possible models  $\omega_4 = \{\varphi_1, \neg\varphi_2, \neg\delta\}$   
 $\omega_5 = \{\neg\varphi_1, \neg\varphi_2, \neg\delta\}$

- distance to the profile  $D(\omega_i, \mathcal{P}) = \sum_{j=1}^n d(\omega_i, \mathcal{S}_j)$



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Hamming distance

$\mathcal{P} = \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3\}$

	$\varphi_1$	$\varphi_2$	$\delta$	$d(\omega_i, \mathcal{S}_1)$	$d(\omega_i, \mathcal{S}_2)$	$d(\omega_i, \mathcal{S}_3)$	$D(\omega_i, \mathcal{P})$
$\omega_1 = \mathcal{S}_1$	0	1	0				
$\omega_2 = \mathcal{S}_2$	1	1	1				
$\omega_3 = \mathcal{S}_3$	0	1	0				
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$$D(\omega_i, \mathcal{P}) = \sum_{j=1}^n d(\omega_i, \mathcal{S}_j)$$



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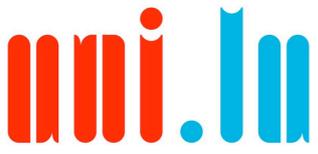
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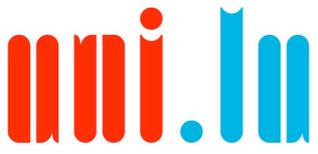
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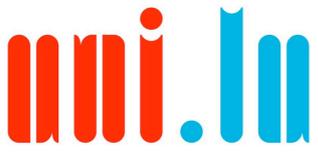
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$\omega_5$	0	0	0	1	3	1	5

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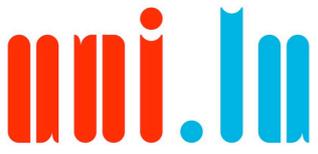
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$$D(\omega_i, \mathcal{P}) = \sum_{j=1}^n d(\omega_i, \mathcal{S}_j)$$



# Fusion & ties

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• in the general case there is no single “winner”:

set of tied alternatives

$$\mathcal{T} = \{\tau_i, | D(\tau_i, \mathcal{P}) = \min\{D(\omega_i, \mathcal{P})\}\}$$

	$\varphi_1$	$\varphi_2$	$\delta$	$d(\omega_i, \mathcal{S}_1)$	$d(\omega_i, \mathcal{S}_2)$	$d(\omega_i, \mathcal{S}_3)$	$D(\omega_i, \mathcal{P})$	
$\omega_1 = \mathcal{S}_1$	1	0	0	0	2	2	4	$\mathcal{T}_1$
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$\omega_3 = \mathcal{S}_3$	1	1	1	2	2	0	4	$\mathcal{T}_3$
$\omega_4$	0	0	0	1	1	3	5	

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$



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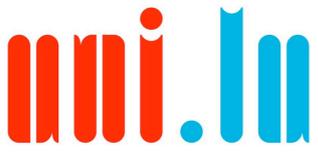
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# What ties mean?

- all elements in  $\mathcal{T} = \{\tau_i, | D(\tau_i, \mathcal{P}) = \min\{D(\omega_i, \mathcal{P})\}$   
are equally good to be the collective alternative



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# What ties mean?

- all elements in  $\mathcal{T} = \{\tau_i, | D(\tau_i, \mathcal{P}) = \min\{D(\omega_i, \mathcal{P})\}$   
are equally good to be the collective alternative
- choose randomly from the tied alternatives?

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- choose randomly from the tied alternatives?

	$\varphi_1$	$\varphi_2$	$\delta$	$d(\omega_i, \mathcal{S}_1)$	$d(\omega_i, \mathcal{S}_2)$	$d(\omega_i, \mathcal{S}_3)$	$D(\omega_i, \mathcal{P})$
$\omega_1 = \mathcal{S}_1$	1	0	0	0	2	2	4
$\omega_2 = \mathcal{S}_2$	0	1	0	2	0	2	4
$\omega_3 = \mathcal{S}_3$	1	1	1	2	2	0	4
$\omega_4$	0	0	0	1	1	3	5

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

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# What ties mean?

- all elements in  $\mathcal{T} = \{\tau_i, | D(\tau_i, \mathcal{P}) = \min\{D(\omega_i, \mathcal{P})\}$  are equally good to be the collective alternative
- choose randomly from the tied alternatives?
- revise: all tied alternatives are equally good to be the collective decision **under the information considered in the fusion!**

	$\varphi_1$	$\varphi_2$	$\delta$	$d(\omega_i, \mathcal{S}_1)$	$d(\omega_i, \mathcal{S}_2)$	$d(\omega_i, \mathcal{S}_3)$	$D(\omega_i, \mathcal{P})$
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$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$



Tie-breaking in  
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# More information

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● elicit preferential information.

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● elicit preferential information. Which?

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# More information

● elicit preferential information. Which?

● what information do we already have?

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# More information

- elicit preferential information. Which?
- what information do we already have?
- idea: consider the context to resolve the ties!



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# More information

- elicit preferential information. Which?
- what information do we already have?
- idea: consider the context to resolve the ties!
- our proposal: to break ties by taking into consideration the type of decision more desirable in a given context



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# Example 1-2

Case1:

$\delta_1$ : *enforce tax on cigarettes*

$\varphi_1$ : *smoking should be reduced among population*

$\varphi_2$ : *higher cost of cigarettes reduces  
number of smokers*

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$

Case2:

$\delta_1$ : *enforce death penalty for drug trafficking*

$\varphi_1$ : *less drugs available*

*accounts for less drug abusers*

$\varphi_2$ : *threat of death penalty reduces the  
number of drug dealers*

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$



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Reversible decision



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Irreversible decision



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# Example 1-2

Case1: *All things being equal, be credulous*

$\delta_1$ : *enforce tax on cigarettes*

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$\varphi_2$ : *higher cost of cigarettes reduces  
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The rule:

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Reversible Decision



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# Example 1-2

Case1:

$\delta_1$ : enforce tax on cigarettes

$\varphi_1$ : smoking should be reduced among population

$\varphi_2$ : higher cost of cigarettes reduces  
number of smokers

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$

Case2: *All things being equal, be skeptical*

$\delta_1$ : enforce death penalty for drug trafficking

$\varphi_1$ : less drugs available

accounts for less drug abusers

$\varphi_2$ : threat of death penalty reduces the  
number of drug dealers

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$

Irreversible Decision



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# Example 3-4

Case 3:

$\delta_1$ : *hire contractor*

$\varphi_1$ : *there is enough money in the budget*

$\varphi_2$ : *contractor appears reliable*

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$

Case 4:

$\delta_1$ : *loan to friendly bank*

$\varphi_1$ : *we possess enough liquid asset*

$\varphi_2$ : *shares of own bank are expected to  
maintain market value*

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$



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Revisable decision

Case 4:

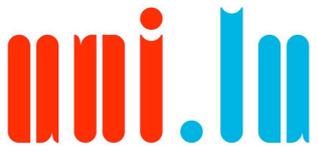
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$\varphi_2$ : shares of own bank are expected to  
maintain market value

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$

Panel will be held responsible



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# Example 3-4

Case 3: *All things being equal, go for*

$\delta_1$ : *hire contractor sensitive alternative*

$\varphi_1$ : *there is enough money in the budget*

$\varphi_2$ : *contractor appears reliable*

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$

Case 4:

$\delta_1$ : *loan to friendly bank*

$\varphi_1$ : *we possess enough liquid asset*

$\varphi_2$ : *shares of own bank are expected to  
maintain market value*

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$



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# Example 3-4

Case 3:

$\delta_1$ : hire contractor

$\varphi_1$ : there is enough money in the budget

$\varphi_2$ : contractor appears reliable

The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$

Case 4:

$\delta_1$ : loan to friendly bank *robust alternative*

$\varphi_1$ : we possess enough liquid asset

$\varphi_2$ : shares of own bank are expected to  
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The rule:

$$R : \delta_1 \leftrightarrow \varphi_1 \wedge \varphi_2$$

Panel will be held responsible



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# Context & propositions

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● in all examples, the logic formalization of the problem is the same

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# Context & propositions

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- in all examples, the logic formalization of the problem is the same
- how to capture decision characteristics in the formalization?

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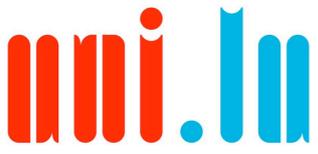


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# Sensitive/robust

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# Sensitive/robust

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● **robust alternative** - decision will change with many swaps in supporting reasons

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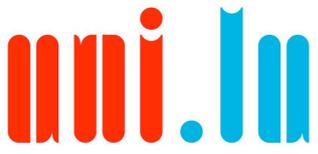
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# Sensitive/robust

● **robust alternative** - decision will change with many swaps in supporting reasons

● **sensitive alternative** - decision will change with few swaps in supporting reasons



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# Sensitive/robust

- **robust alternative** - decision will change with many swaps in supporting reasons
- **sensitive alternative** - decision will change with few swaps in supporting reasons
- **sensitive/robust**: measure how changing an opinion on a decision or a justification would affect a judgment set

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# Impact factors

● **impact factor** of  $\sigma_j \in \omega_i$  shows how much  $\omega_i$  will have to change if  $\sigma_j$  is replaced by  $\neg\sigma_j$

$$u^\Omega(\sigma_j, \omega_i) = \min\{d(\omega_i, \omega_k)\}$$

where  $\omega_i, \omega_k \in \Omega$  ;  $\sigma_j \in \omega_i$  and  $\neg\sigma_j \in \omega_k$

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$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

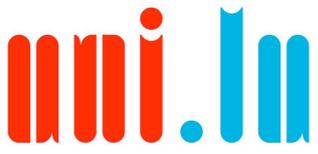


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# Sensitive to robust

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# Sensitive to robust

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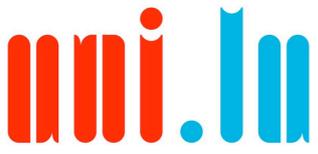
● **more robust alternative** - its decision has higher  
impact factor than its supporting reasons.



# Sensitive to robust

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- **more robust alternative** - its decision has higher impact factor than its supporting reasons.
- **more sensitive alternative** - its decision has lower impact factors than its supporting reasons.



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# Sensitive to robust

- **more robust alternative** - its decision has higher impact factor than its supporting reasons.
- **more sensitive alternative** - its decision has lower impact factors than its supporting reasons.
- define  $M(\omega_i) = \min\{u^{\Omega}(\sigma_j, \omega_i)\}, j \in [1, m - 1]$

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# Sensitive to robust

- **more robust alternative** - its decision has higher impact factor than its supporting reasons.
- **more sensitive alternative** - its decision has lower impact factors than its supporting reasons.

● define  $M(\omega_i) = \min\{u^\Omega(\sigma_j, \omega_i)\}, j \in [1, m - 1]$

$$\mathcal{M}(\omega_i) = \{\sigma_j | u^\Omega(\sigma_j, \omega_i) = M(\omega_i)\}$$

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# Sensitive to robust

- **more robust alternative** - its decision has higher impact factor than its supporting reasons.
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- define  $M(\omega_i) = \min\{u^\Omega(\sigma_j, \omega_i)\}, j \in [1, m - 1]$   
 $\mathcal{M}(\omega_i) = \{\sigma_j | u^\Omega(\sigma_j, \omega_i) = M(\omega_i)\}$

$$\omega_i \lesssim_u \omega_j \Rightarrow (u^\Omega(\delta, \omega_i) - M(\omega_i)) \leq (u^\Omega(\delta, \omega_j) - M(\omega_j))$$

$$\omega_i \lesssim_u \omega_j \Rightarrow u^\Omega(\delta, \omega_i) - M(\omega_i) = u^\Omega(\delta, \omega_j) - M(\omega_j) \text{ and } |\mathcal{M}(\omega_i)| \geq |\mathcal{M}(\omega_j)|$$

$$\omega_i \sim_u \omega_j \Leftrightarrow u^\Omega(\delta, \omega_i) - M(\omega_i) = u^\Omega(\delta, \omega_j) - M(\omega_j) \text{ and } |\mathcal{M}(\omega_i)| = |\mathcal{M}(\omega_j)|$$

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# Sensitive to robust

- **more robust alternative** - its decision has higher impact factor than its supporting reasons.
- **more sensitive alternative** - its decision has lower impact factors than its supporting reasons.

● define  $M(\omega_i) = \min\{u^\Omega(\sigma_j, \omega_i)\}, j \in [1, m - 1]$

$$\mathcal{M}(\omega_i) = \{\sigma_j | u^\Omega(\sigma_j, \omega_i) = M(\omega_i)\}$$

$$\omega_i \lesssim_u \omega_j \Rightarrow (u^\Omega(\delta, \omega_i) - M(\omega_i)) \leq (u^\Omega(\delta, \omega_j) - M(\omega_j))$$

$$\omega_i \lesssim_u \omega_j \Rightarrow u^\Omega(\delta, \omega_i) - M(\omega_i) = u^\Omega(\delta, \omega_j) - M(\omega_j) \text{ and } |\mathcal{M}(\omega_i)| \geq |\mathcal{M}(\omega_j)|$$

$$\omega_i \sim_u \omega_j \Leftrightarrow u^\Omega(\delta, \omega_i) - M(\omega_i) = u^\Omega(\delta, \omega_j) - M(\omega_j) \text{ and } |\mathcal{M}(\omega_i)| = |\mathcal{M}(\omega_j)|$$

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● Sensitive/robust decisions

5. Skeptical and credulous decisions

6. Conclusions

# Sensitive to robust

- **more robust alternative** - its decision has higher impact factor than its supporting reasons.
- **more sensitive alternative** - its decision has lower impact factors than its supporting reasons.

● define  $M(\omega_i) = \min\{u^\Omega(\sigma_j, \omega_i)\}, j \in [1, m - 1]$

$$\mathcal{M}(\omega_i) = \{\sigma_j | u^\Omega(\sigma_j, \omega_i) = M(\omega_i)\}$$

$$\omega_i \lesssim_u \omega_j \Rightarrow (u^\Omega(\delta, \omega_i) - M(\omega_i)) \leq (u^\Omega(\delta, \omega_j) - M(\omega_j))$$

$$\omega_i \lesssim_u \omega_j \Rightarrow u^\Omega(\delta, \omega_i) - M(\omega_i) = u^\Omega(\delta, \omega_j) - M(\omega_j) \text{ and } |\mathcal{M}(\omega_i)| \geq |\mathcal{M}(\omega_j)|$$

$$\omega_i \sim_u \omega_j \Leftrightarrow u^\Omega(\delta, \omega_i) - M(\omega_i) = u^\Omega(\delta, \omega_j) - M(\omega_j) \text{ and } |\mathcal{M}(\omega_i)| = |\mathcal{M}(\omega_j)|$$



# Sensitive to robust

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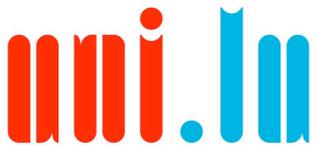
6. Conclusions

● from most sensitive to most robust:

$$\mathcal{S}_3 \lesssim_u \mathcal{S}_1 \sim_u \mathcal{S}_2 \lesssim_u \omega_4$$

	$\varphi_1$	$\varphi_2$	$\delta$	$D(\omega_i, \mathcal{P})$	$\lesssim_u$
$\omega_1 = \mathcal{S}_1$	1(1)	0(2)	0(2)	4	2-1=1
$\omega_2 = \mathcal{S}_2$	0(2)	1(1)	0(2)	4	2-1=1
$\omega_3 = \mathcal{S}_3$	1(2)	1(2)	1(2)	4	2-2=0
$\omega_4$	0(1)	0(1)	0(3)	5	3-1=2

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$



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# Credulous, skeptical

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6. Conclusions

● **credulous alternative** - maximizes input from the group on supporting reasons

● **Skeptical and credulous decisions**

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# Credulous, skeptical

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3. Ties in model-  
based results

4. Resolving ties:  
role of context in  
decision making

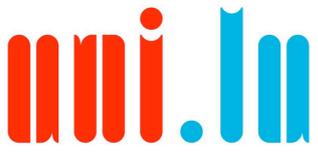
5. Sensitive/robust  
decisions

● Skeptical and  
credulous  
decisions

6. Conclusions

● **credulous alternative** - maximizes input from the group on supporting reasons

● **skeptical alternative** - also takes into account how often a decision appears in the set of all possible alternatives



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● **Skeptical and credulous decisions**

# Frequency factors

- **frequency factor** of  $\sigma_j \in \omega_i$  counts in how many  $\omega_i$  in a given set of alternatives  $\mathcal{G}$ ,  $\sigma_j$  appear.

$$v^{\mathcal{G}}(\sigma_j) = |\mathcal{G}|$$

$$\mathcal{G} = \{\omega_i \mid \omega_i \subseteq \mathcal{G}; \sigma_j \in \omega_i\}.$$

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# Frequency factors

- ⦿ **frequency factor** of  $\sigma_j \in \omega_i$  counts in how many  $\omega_i$  in a given set of alternatives  $\mathcal{G}$ ,  $\sigma_j$  appear.

$$v^{\mathcal{G}}(\sigma_j) = |G|$$

$$G = \{\omega_i | \omega_i \subseteq \mathcal{G}; \sigma_j \in \omega_i\}.$$

	$\varphi_1$	$\varphi_2$	$\delta$	$D(\omega_i, \mathcal{P})$
$\omega_1 = \mathcal{S}_1$	1(2)	0(1)	0(3)	4
$\omega_2 = \mathcal{S}_2$	0(1)	1(2)	0(3)	4
$\omega_3 = \mathcal{S}_3$	1(2)	1(2)	1(1)	4
$\omega_4$	0(1)	0(1)	0(3)	5

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

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# Frequency factors

-  **frequency factor** of  $\sigma_j \in \omega_i$  counts in how many  $\omega_i$  in a given set of alternatives  $\mathcal{G}$ ,  $\sigma_j$  appear.

$$v^{\mathcal{G}}(\sigma_j) = |\mathcal{G}|$$

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	$\varphi_1$	$\varphi_2$	$\delta$	$D(\omega_i, \mathcal{P})$
$\omega_1 = \mathcal{S}_1$	1(2)	0(1)	0(3)	4
$\omega_2 = \mathcal{S}_2$	0(1)	1(2)	0(3)	4
$\omega_3 = \mathcal{S}_3$	1(2)	1(2)	1(1)	4
$\omega_4$	0(1)	0(1)	0(3)	5

over  $\mathcal{P}$

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$

over  $\Omega$

# Credulous & skeptical

- **more credulous alternative** - the sum of the frequency factors over the reasons in the profile is higher .

$$\omega_i \lesssim_c \omega_j \Leftrightarrow \sum_{k=1}^{m-1} v^{\mathcal{P}}(\sigma_k) \leq \sum_{t=1}^{m-1} v^{\mathcal{P}}(\sigma_t)$$

$$\sigma_k \in \omega_i \text{ and } \sigma_t \in \omega_j$$

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# Credulous & skeptical

• **more credulous alternative** - the sum of the frequency factors over the reasons in the profile is higher .

$$\omega_i \lesssim_c \omega_j \Leftrightarrow \sum_{k=1}^{m-1} v^{\mathcal{P}}(\sigma_k) \leq \sum_{t=1}^{m-1} v^{\mathcal{P}}(\sigma_t)$$

$$\sigma_k \in \omega_i \text{ and } \sigma_t \in \omega_j$$

• **more skeptical alternative** - the sum of the f.factors of the reasons (over  $\mathcal{P}$ ) added to the f.factor of the decision (over  $\Omega$ ) is higher.

$$\omega_i \lesssim_c \omega_j \Leftrightarrow v^{\Omega}(\delta_i) + \sum_{k=1}^{m-1} v^{\mathcal{P}}(\sigma_k) \leq v^{\Omega}(\delta_j) + \sum_{t=1}^{m-1} v^{\mathcal{P}}(\sigma_t)$$

$$\sigma_k \in \omega_i \text{ and } \sigma_t \in \omega_j$$

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# Ordering

from least to most credulous:

$$\omega_4 \lesssim_c \mathcal{S}_1 \sim_c \mathcal{S}_2 \lesssim_c \mathcal{S}_3$$

from least to most skeptical:

$$\omega_4 \sim_s \mathcal{S}_3 \lesssim_s \mathcal{S}_1 \sim_s \mathcal{S}_2$$

	$\varphi_1$	$\varphi_2$	$\delta$	$D(\omega_i, \mathcal{P})$
$\omega_1 = \mathcal{S}_1$	1(2)	0(1)	0(3)	4(3,6)
$\omega_2 = \mathcal{S}_2$	0(1)	1(2)	0(3)	4(3,6)
$\omega_3 = \mathcal{S}_3$	1(2)	1(2)	1(1)	4(4,5)
$\omega_4$	0(1)	0(1)	0(3)	5(2,5)

$$\mathcal{R} = \{\varphi_1 \wedge \varphi_2 \leftrightarrow \delta\}$$



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# Conclusions

● context helps to resolve between tied conflicting decisions



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# Conclusions

- context helps to resolve between tied conflicting decisions
- but context does not always give an ordering:



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# Conclusions

- contexts helps resolve between tied contradicting decisions
- but context does not always give an ordering:

	$\varphi_1$	$\varphi_2$	$\delta$	$D(\omega_i, \mathcal{P})$
$\omega_1 = \mathcal{S}_1$	1(2/2)	0(2/1)	0(2/2)	4(0,3,5)
$\omega_2 = \mathcal{S}_2$	0(2/1)	1(2/2)	0(2/2)	4(0,3,5)
$\omega_3 = \mathcal{S}_3$	1(2/2)	1(2/2)	1(2/2)	4(0,4,6)
$\omega_4$	0(2/1)	0(2/1)	1(2/2)	5(0,2,4)

$$R = \{\varphi_1 \leftrightarrow \varphi_2 \leftrightarrow \delta\}$$



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# Open questions

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● **Conclusions**

● how much can be resolved with context ?

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● **Conclusions**

● how much can be resolved with context ?

● which other contexts can we distinguish?

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# Open questions

-  how much can be resolved with context ?
-  which other contexts can we distinguish?
-  how to resolve what context can not ?



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● **Conclusions**

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# Open questions

- how much can be resolved with context ?
- which other contexts-characterizations for decisions can we distinguish?
- how to resolve what context-characterizations can not ?
  - idea: preferences over justifications based on context



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