Multi-Agent Systems and Social Influence Part 2

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(Only apparently overwhelming) plan

- Fenrong Liu, Jeremy Seligman, Patrick Girard: Logical dynamics of belief change in the community
- 2 Zoé Christoff, Jens Ulrik Hansen:

A Two-Tiered Formalization of Social Influence

- Alexandru Baltag, Zoé Christoff, Rasmus Rendsvig, Sonja Smets Dynamic Epistemic Logic of Diffusion and Prediction in Threshold Models
- Truls Pedersen, Marija Slavkovik Formal Models of Conflicting Social Influence
- Sonja Smets, Fernando Velázquez-Quesada How to Make Friends: A Logical Approach to Social Group Creation

Logical dynamics of belief change in the community

• Explores formalization of the relationship between

- norms of belief revision, and
- properties of networks.

Modal Logic in two minutes

• Simple language

$$\phi \quad ::= \quad p \mid \neg \phi \mid \phi \land \phi \mid \Box \phi$$

• Usually extended

$$\phi \lor \psi := \neg (\neg \phi \land \neg \psi)$$
$$\Diamond \phi := \neg \Box \neg \phi$$

Modal Logic in two minutes

- Models (W, R, V)
 - ► *W* non-empty set of worlds,
 - $R \subseteq W \times W$, and
 - $V: W \to \wp(P)$.

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 - ► *W* non-empty set of worlds,
 - $R \subseteq W \times W$, and
 - $\blacktriangleright V: W \to \wp(P).$
- Models, in some world, may satisfy a formula

$$M, w \models p \Leftrightarrow p \in V(w)$$

$$M, w \models \neg \phi \Leftrightarrow \text{ it is not the case that } M, w \models \phi$$

$$\vdots$$

$$M, w \models \Box \phi \Leftrightarrow \text{ for every } v \in W \text{ s.t. } wRv : M, v \models \phi$$

Hybrid Logic in two minutes

• In addition to regular propositional symbols, NOM is a set of names

- *name* : $NOM \rightarrow W$ names the worlds
- Behave similar to propositions, but

$$M, w \models p \Leftrightarrow p \in V(w)$$
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• Usually add *global modality* G

$$M, w \models G\phi \Leftrightarrow$$
 for every v $M, v \models \phi$

Doxastic Influence

"To be influenced by my friends is to change my beliefs so that they correspond better to theirs. To begin with, we will consider influence regarding a single proposition p. If I do not believe p and some significant number or proportion of my friends do believe it, there are several ways I could respond. I could, of course, ignore their opinions and remain doxastically unperturbed. But if I am influenced to change my beliefs there are at least two ways of doing so: I may revise so that I too believe p or (more cautiously) merely contract, removing my belief in its negation $\neg p$."



Doxastic Influence

Will I change my belief?

- my own attitude regarding p,
- the cohesiveness of my *friends*' beliefs concerning p, and
- So the extent to which I regard any particular friend as an authority on *p*.



Two kinds of influence

We consider two kinds of influence:

Strong If we are "*strongly influenced to believe p*", then we *revise* in favour of *p*.

We denote this S(p), or simply Sp.

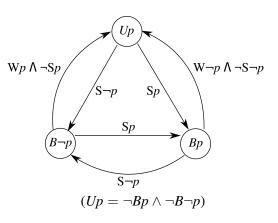
Weak If we are "weakly influenced to believe p", then we contract our (possible) belief in $\neg p$.

We denote this W(p), or simply Wp.

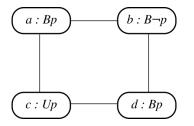
Consequence of influence

if S(p) then R(p)else if W(p) then $C(\neg p)$ end if end if Consequence of influence

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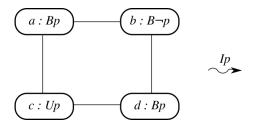


Influence as an action/event



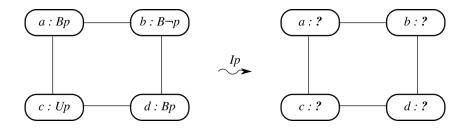
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Influence as an action/event



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- an event is an operation which *updates* the model,

Influence as an action/event



- We are considering how a given network evolves,
- an event is an operation which *updates* the model,
- want to describe the *rational outcome*.

A first instantiation

Threshold influence (conservative)

• Recall threshold models: if the fraction of your friends that believe p exceeds some threeshold θ , you are influenced to believe p.

Conservative threshold model

Sp If all your friends believe p, and you have at least one friend. Wp If at least one of your friends believe p, and none believe $\neg p$.

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- "Social hermits" are not affected by social influence.
- "Conservative" in the sense that our conditions are not "risky". If belief spreads across network edges and *every friend* believes *p*, we must accept it. If there is *no support* for ¬*p*, but at least one friend believes *p*, we should not contradict her.

Logical language

Language

The language describing influence is given by

$$\phi \quad ::= \quad \underbrace{Bp \mid B \neg p \mid \mathsf{S}p \mid \mathsf{S} \neg p \mid \mathsf{W}p \mid \mathsf{W} \neg p}_{\text{atoms}} \mid \underbrace{\neg \phi \mid \phi \land \phi}_{\text{boolean}} \mid \underbrace{F\phi \mid \langle F \rangle \phi}_{\text{new}}$$

 $F\phi$ For every friend, ϕ .

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Conservative threshold model formalized

 $Sp \Leftrightarrow FBp \land \langle F \rangle Bp$ $Wp \Leftrightarrow F \neg B \neg p \land \langle F \rangle Bp$

Reading formulas

Assume we are discussing a particular agent, a, with "friends" N(a)

 $\mathsf{S}p \Leftrightarrow FBp \land \langle F \rangle Bp$ \Leftrightarrow every $b \in N(a)$ believes p, and there is at least one $b \in N(a)$ that believes p

Reading formulas

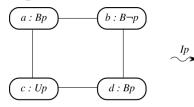
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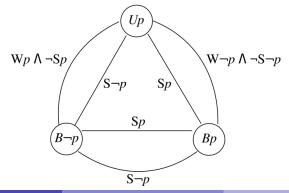
 $\begin{aligned} \mathsf{S}p \Leftrightarrow FBp \land \langle F \rangle Bp \\ \Leftrightarrow \text{ every } b \in N(a) \text{ believes } p, and \\ \text{ there is at least one } b \in N(a) \text{ that believes } p \\ \Rightarrow \text{ no } b \in N(a) \text{ believes } \neg p, and \\ \text{ there is at least one } b \in N(a) \text{ that believes } p \end{aligned}$

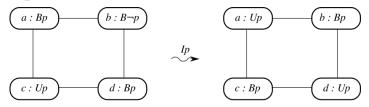
Reading formulas

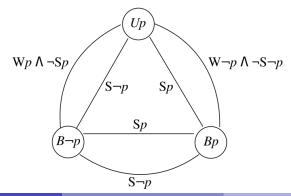
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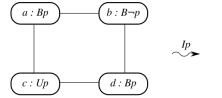
 $Sp \Leftrightarrow FBp \land \langle F \rangle Bp$ $\Leftrightarrow every \ b \in N(a) \text{ believes } p, and$ there is at least one $b \in N(a)$ that believes p $\Rightarrow \text{ no } b \in N(a) \text{ believes } \neg p, and$ there is at least one $b \in N(a)$ that believes p $\Leftrightarrow F \neg B \neg p \land \langle F \rangle Bp$ $\Leftrightarrow Wp$

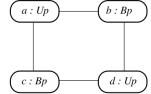




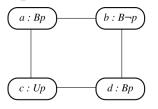


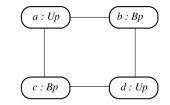






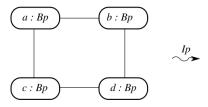


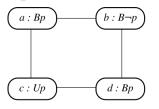


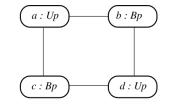




 $\stackrel{Ip}{\checkmark}$

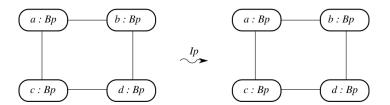








 $\stackrel{Ip}{\checkmark}$



Stability and flux

Stable Models that are "unaffected" by the update operation, are *stable* They are *fixed points* of the update operation.

 $\mathcal{M} = I_p(\mathcal{M})$

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Becoming stable Models that, after a number of applications of the update operator, become such a fixed point, are *becoming stable*.

$$I_p^n(\mathcal{M}) = I_p(I_p^n(\mathcal{M}))$$

Stability and flux

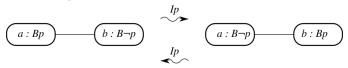
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In flux The remaining models are in flux.

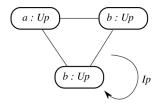


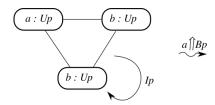
Local conditions of stability

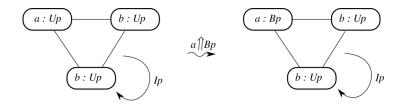
• If no agent changes, the network is *stable*.

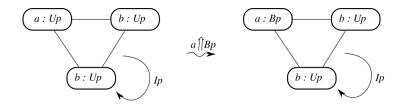
$$\neg (B \neg p \land \mathsf{W}p) \land \neg (Up \land \mathsf{S}p) \land \neg (Up \land \mathsf{S}\neg p) \land \neg (Bp \land \mathsf{W}\neg p)$$

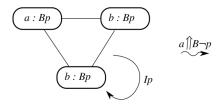
- Network is stable if, and only if, every agent satisfies the above.
 - (We already saw $Sp \Rightarrow Wp$.)



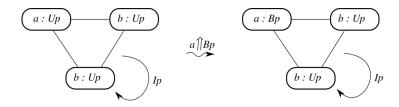


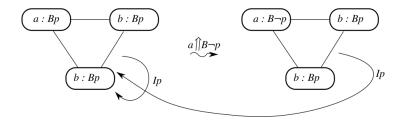


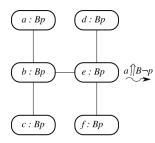


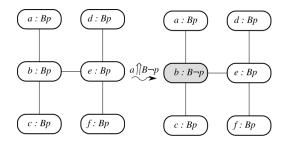


Unilateral Update

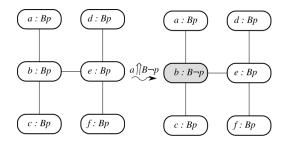


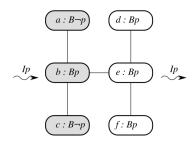


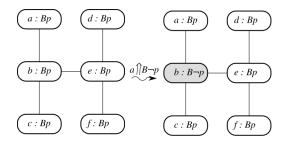


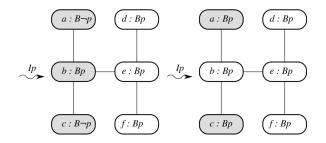


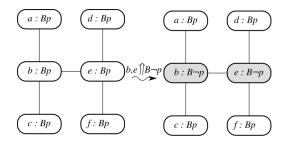




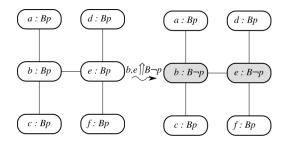












$$\begin{array}{c} a: B\neg p \\ \hline \\ b: B\neg p \\ \hline \\ c: B\neg p \\ \hline \\ c: B\neg p \\ \hline \\ f: B\neg p \\ \hline \end{array}$$

Logical dynamics of belief change in the community Liu, Seligman & Girard

• Establishes general terminology and preliminary issues

- strong and weak influence,
- position in network matters,
- ▶ *n*-resistance,
- timing (consecutive \neq simultaneous),
- conditions for stability.
- Goes on to discuss
 - Ranking friends' reliability (degree of influence),
 - possibility of *chaning* the network
 - ★ adding/removing edges
 - Appendix discusses similar investigations in other fields

A Two-Tiered Formalization of Social Influence

Zoé Christoff and Jens Ulrik Hansen

"The [framework of Liu, Seligman, and Girard] makes it unproblematic to identify the stability and stablization conditions of social-doxastic configurations, both of which can be characterized directly in the language of friendship and belief. However, this simplicity is pricey: [...] it relies on an extremely strong assumption: agents' belief states are influenced directly by their friends' belief states.

Thus, either all agents have direct access to their friends' beliefs (as mind-readers would), or their observed behavior always reflects their private beliefs, i.e., there is no difference between what they <u>seem to believe</u> and what they actually believe."

Pluralistic Ignorance

Pluralistic Ignorance

- Agents believe that their private attitudes differ from the others', but
- agents' private attitudes actually coincide.

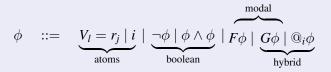
- Need a *two-tiered* model:
 - private (true) belief (dislikes hats), and
 - expressed/observable behavior (wears a hat).
- We expand the framework, but keep
 - three states of belief $(Bp, B \neg p \text{ and } Up)$,
 - conservative threshold model for strong and weak influence.

Hybrid Network Logic

Language

Language

The language of Hybrid Network Logic is given by



where

- V_l is the name of the *j*th *characteristic*,
- r_j is a possible value for the *j*th characteristic ($r_j \in R_j$), and
- $i \in \text{NOM}$ is a nominal.

Characteristic

There are a finite set of *characteristics*: V_1, V_2, \ldots, V_n , each with an associated finite domain R_1, R_2, \ldots, R_n (e.g., $R_1 = \{r_{1,1}, r_{1,2}, \ldots, r_{1,m}\}$).

Hybrid Network Logic

Hybrid Network Logic Model

A *Hybrid Netwrok Logic Model* is a tuple $\mathcal{M} = (A, \sim, g, v)$ where

- A is a non-emtpy set of agents,
- $\sim \subseteq A \times A$ is a symmetric and irreflexive network relation,
- $g : \text{NOM} \rightarrow A$ names agents, and
- for every *a* ∈ *A*, *v*(*a*) maps each characteristic to a possible value:
 v(*a*)(*V*_l) ∈ *R*_l.

Hybrid Network Logic

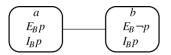
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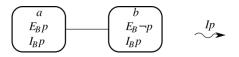
- A is a non-emtpy set of agents,
- $\sim \subseteq A \times A$ is a symmetric and irreflexive network relation,
- $g : \text{NOM} \rightarrow A$ names agents, and
- for every a ∈ A, v(a) maps each characteristic to a possible value:
 v(a)(V_l) ∈ R_l.

Suppose there is a characteristic V_I , with possible values $\{B_p, B_{\neg p}, U_p\}$, corresponding to the *internal belief state*. If agent *a* believes *p*, *b* believes $\neg p$, and agent *c* is undecided, we get:

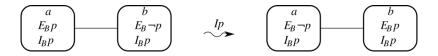
- $v(a)(V_I) = B_p$,
- $v(b)(V_I) = B_{\neg p}$, and
- $v(c)(V_I) = U_p$.



- Both agents actually agree (internal belief),
- each agent *apparently contradicts* the other.



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Now we need to decide how the *internal* beliefs affects the agents' belief revision.

	Inner state	$\langle F \rangle E_B \varphi$	$\langle F \rangle E_B \neg \varphi$	$\langle F \rangle E_U \varphi$	Type 1	Type 2	Type 3
1	$I_B \varphi$				$\sim E_B \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \varphi$
2	$I_B \neg \varphi$	1	1	1	$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
3	$I_U \varphi$				$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
4	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$	$\sim E_B \varphi$
5	$I_B \neg \varphi$	1	1	0	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
6	$I_U \varphi$				$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
7	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
8	$I_B \neg \varphi$	1	0	1	$ ightarrow E_U arphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
9	$I_U \varphi$				$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
10	$I_B \varphi$						
11	$I_B \neg \varphi$	1	0	0	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightarrow E_B \varphi$
12	$I_U \varphi$						
13	$I_B \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
14	$I_B \neg \varphi$	0	1	1	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
15	$I_U \varphi$				$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\sim E_B \neg \varphi$
16	$I_B \varphi$					_	
17	$I_B \neg \varphi$	0	1	0	$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\sim E_B \neg \varphi$
18	$I_U \varphi$						
19	$I_B \varphi$	0	0	1	$\sim E_B \varphi$	$\sim E_B \varphi$	$\sim E_U \varphi$
20	$I_B \neg \varphi$	0	0	1	$\rightarrow E_B \neg \varphi$	$\sim E_B \neg \varphi$	$\sim E_U \varphi$
21	$I_U \varphi$				$\rightarrow E_U \varphi$	$\rightarrow E_U \varphi$	$\rightarrow E_U \varphi$
22	$I_B \varphi$		0		$\rightarrow E_B \varphi$	$\sim E_B \varphi$	$\sim E_B \varphi$
23	$I_B \neg \varphi$	0	0	0	$\rightarrow E_B \neg \varphi$	$\sim E_B \neg \varphi$	$\sim E_B \neg \varphi$
24	$I_U \varphi$				$\rightarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$

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11	$I_B \neg \varphi$	1	0	0	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
12	$I_U \varphi$						
13	$I_B \varphi$				$\sim E_U \varphi$	$\rightarrow E_U \varphi$	$ ightarrow E_U \varphi$
14	$I_B \neg \varphi$	0	1	1	$\rightarrow E_B \neg \varphi$		$\rightsquigarrow E_B \neg \varphi$
15	$I_U \varphi$				$\rightarrow E_B \neg \varphi$	$\rightarrow E_B \neg \varphi$	$\rightarrow E_B \neg \varphi$
16	$I_B \varphi$						
17	$I_B \neg \varphi$	0	1	0	$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
18	$I_U \varphi$						
19	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$
20	$I_B \neg \varphi$	0	0	1	$\sim E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$
21	$I_U \varphi$				$\rightarrow E_U \varphi$	$ ightarrow E_U arphi$	$\rightsquigarrow E_U \varphi$
22	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
23	$I_B \neg \varphi$	0	0	0	$\sim E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
24	$I_U \varphi$				$\rightarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$

	Inner state	$\langle F \rangle E_B \varphi$	$\langle F \rangle E_B \neg \varphi$	$\langle F \rangle E_U \varphi$	Type 1	Type 2	Type 3
1	$I_B \varphi$				$\rightarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \varphi$
2	$I_B \neg \varphi$	1	1	1	$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
3	$I_U \varphi$				$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
4	$I_B \varphi$				$\rightarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \varphi$
5	$I_B \neg \varphi$	1	1	0	$\sim E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
6	$I_U \varphi$				$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
7	$I_B \varphi$				$\sim E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
8	$I_B \neg \varphi$	1	0	1	$\rightarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
9	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
10	$I_B \varphi$						
11	$I_B \neg \varphi$	1	0	0	$ ightarrow E_B arphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
12	$I_U \varphi$						
13	$I_B \varphi$				$\rightarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
14	$I_B \neg \varphi$	0	1	1	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
15	$I_U \varphi$				$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
16	$I_B \varphi$						
17	$I_B \neg \varphi$	0	1	0	$\sim E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
18	$I_U \varphi$						
19	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$
20	$I_B \neg \varphi$	0	0	1	$\sim E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$
21	$I_U \varphi$				$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
22	$I_B \varphi$				$ ightarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightarrow E_B \varphi$
23	$I_B \neg \varphi$	0	0	0	$\sim E_B \neg \varphi$	$\sim E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
24	$I_U \varphi$				$ ightarrow E_U arphi$	$\rightsquigarrow E_U \varphi$	$\rightarrow E_U \varphi$

	Inner state	$\langle F \rangle E_B \varphi$	$\langle F \rangle E_B \neg \varphi$	$\langle F \rangle E_U \varphi$	Type 1	Type 2	Type 3
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2	$I_B \neg \varphi$	1	1	1	$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
3	$I_U \varphi$				$\sim E_U \varphi$	$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$
4	$I_B \varphi$				$\sim E_B \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \varphi$
5	$I_B \neg \varphi$	1	1	0	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
6	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
7	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
8	$I_B \neg \varphi$	1	0	1	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
9	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
10	$I_B \varphi$						
11	$I_B \neg \varphi$	1	0	0	$\sim E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
12	$I_U \varphi$						
13	$I_B \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
14	$I_B \neg \varphi$	0	1	1	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
15	$I_U \varphi$				$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
16	$I_B \varphi$						
17	$I_B \neg \varphi$	0	1	0	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
18	$I_U \varphi$						
19	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\sim E_U \varphi$
20	$I_B \neg \varphi$	0	0	1	$\sim E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$
21	$I_U \varphi$				$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$	$ ightarrow E_U arphi$
22	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
23	$I_B \neg \varphi$	0	0	0	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
24	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$

	Inner state	$\langle F \rangle E_B \varphi$	$\langle F \rangle E_B \neg \varphi$	$\langle F \rangle E_U \varphi$	Type 1	Type 2	Type 3
1	$I_B \varphi$				$\rightarrow E_B \varphi$	$\rightarrow E_U \varphi$	$\sim E_B \varphi$
2	$I_B \neg \varphi$	1	1	1	$\sim E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
3	$I_U \varphi$				$\sim E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\sim E_U \varphi$
4	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$	$\rightarrow E_B \varphi$
5	$I_B \neg \varphi$	1	1	0	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
6	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
7	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\sim E_{B} arphi$	$\sim E_B \varphi$
8	$I_B \neg \varphi$	1	0	1	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
9	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U arphi$
10	$I_B \varphi$				\frown		
11	$I_B \neg \varphi$	1	0	0		$\rightsquigarrow E_B \varphi$	$\sim E_B \varphi$
12	$I_U \varphi$				\sum_{r}		
13	$I_B \varphi$				$\sim E_V \varphi$	$\rightarrow E_U \varphi$	$\rightarrow E_U \varphi$
14	$I_B \neg \varphi$	0 V	$Vp \Lambda \neg Sp$	/ 1 /	$\sim E_B - \varphi$	$\sim \lambda W$	$p \wedge \neg S \neg p$
15	$I_U \varphi$		· ·/		$\rightarrow E_B \neg \varphi$	$\sim E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
16	$I_B \varphi$			/S¬j	o Sp`		
17	$I_B \neg \varphi$	0	1	0	$\rightsquigarrow E_B \neg \varphi$	$\rightarrow E_B - \varphi$	$\rightsquigarrow E_B \neg \varphi$
18	$I_U \varphi$				C.,		
19	$I_B \varphi$			<	SpErc		$\sim E_U \varphi$
20	$I_B \neg \varphi$	0	$0(B\neg$	p) 1	$\rightsquigarrow E_B \neg \varphi$	$\sim (Bp)$	$\sim E_U \varphi$
21	$I_U \varphi$		\sim				
22	$I_B \varphi$				~ Epp	$\rightsquigarrow E_B \varphi$	$\sim E_B \varphi$
23	$I_B \neg \varphi$	0	0	0	$S \neg p_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
24	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\sim E_U \varphi$

Slavkovik & Pedersen

Social Influence

New Models, New Phenomena

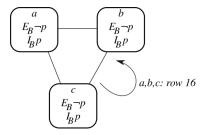
• We are now able to model *Pluralistic Ignorance*

$$PI_{\phi} := G(I_B\phi \wedge E_B \neg \phi)$$

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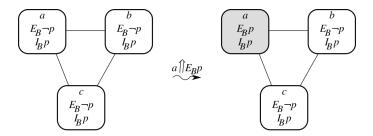


Two-Tired update

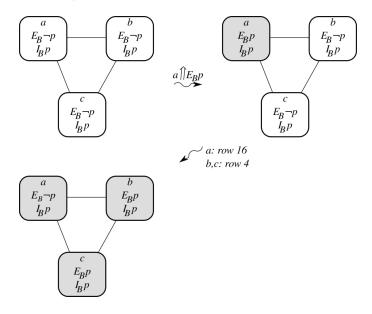
Notice rows: 4, 10, and 16.

	Inner state	$\langle F \rangle E_B \varphi$	$\langle F \rangle E_B \neg \varphi$	$\langle F \rangle E_U \varphi$	Type 1	Type 2	Type 3
1	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \varphi$
2	$I_B \neg \varphi$	1	1	1	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
3	$I_U \varphi$				$\rightarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
4	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$	$\sim E_B \varphi$
5	$I_B \neg \varphi$	1	1	0	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_B \neg \varphi$
6	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
7	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
8	$I_B \neg \varphi$	1	0	1	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
9	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
10	$I_B \varphi$						
11	$I_B \neg \varphi$	1	0	0	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\sim E_B \varphi$
12	$I_U \varphi$						
13	$I_B \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
14	$I_B \neg \varphi$	0	1	1	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
15	$I_U \varphi$				$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
16	$I_B \varphi$						
17	$I_B \neg \varphi$	0	1	0	$\sim E_B \neg \varphi$	$\sim E_B \neg \varphi$	$\sim E_B \neg \varphi$
18	$I_U \varphi$						
19	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_U \varphi$
20	$I_B \neg \varphi$	0	0	1	$\rightarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_U \varphi$
21	$I_U \varphi$				$\rightarrow E_U \varphi$	$\rightarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$
22	$I_B \varphi$				$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$	$\rightsquigarrow E_B \varphi$
23	$I_B \neg \varphi$	0	0	0	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$	$\rightsquigarrow E_B \neg \varphi$
24	$I_U \varphi$				$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$	$\rightsquigarrow E_U \varphi$

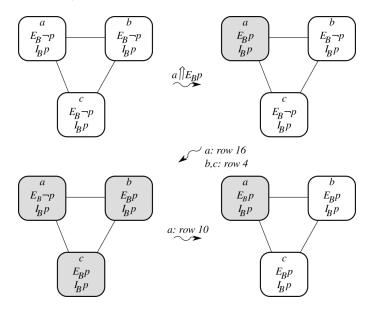
"Fragile sability"



"Fragile sability"



"Fragile sability"



A Two-Tiered Formalization of Social Influence Christoff & Hansen

- Establishes a two-tiered hybrid logic where
 - privately held beliefs affect transition,
- demonstrates conditions for stability and (eventual) dissolution of state of pluralistic ignorance
- various *types* of agents

Dynamic Epistemic Logics of Diffusion and Prediction in Social Networks

Alexandru Baltag, Zoé Christoff, Rasmus K. Rendsvig, and Sonja Smets

"We introduce an <u>epistemic</u> dimension to threshld models, thus taking into account the real-life limitations posed by the agents' limited access to information; for this, we propose an epistemic variant of the [belief/behavior] adoption rule: agents adopt a behavior only if they <u>know</u> that enough of [...] their neighbours have adopted it."

Epistemic Threshold Models

- Seen simple (extreme) threshold models expressed in terms of:
 - *all, none,* or *some* (at least one)
- Fix a single belief/behavior we are interested in, having adopted p, say.
 - Denote the set agents that have adopted p by $B \subseteq A$.

Epistemic Threshold Models

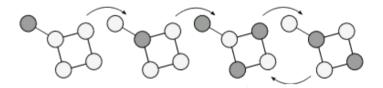
- Seen simple (extreme) threshold models expressed in terms of:
 - *all, none,* or *some* (at least one)
- Fix a single belief/behavior we are interested in, having adopted p, say.
 - Denote the set agents that have adopted p by $B \subseteq A$.
- Let $\theta \in [0, 1]$ be some threshold.

$$B' \;=\; \left\{ a \in \mathcal{A} \; \left| \; rac{|N(a) \cap B|}{|N(a)|} \geq heta
ight\}$$

- ► *B* represents initial belivers,
- B' represents the result of the model update.

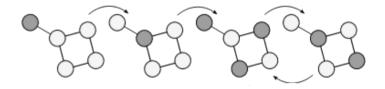
Inflationary adoption

Non-inflationary Let $\theta = 1/4$.

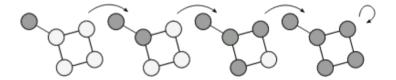


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Inflationary Let $\theta = 1/4$.



Inflationary adoption

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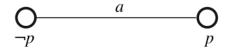
$$B' \;=\; \left\{ a \in \mathcal{A} \; \left| \; rac{|N(a) \cap B|}{|N(a)|} \geq heta
ight\}$$

Inflationary

$$B' \;=\; B \cup \left\{ a \in \mathcal{A} \; \left| \; rac{|N(a) \cap B|}{|N(a)|} \geq heta
ight\}$$

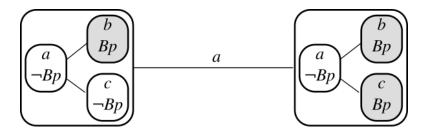
Epistemic dimension

- When should an agent adopt a belief?
 - Proposal: when she *knows* that at least θ of her friends already have adopted.



Epistemic dimension

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 - Proposal: when she *knows* that at least θ of her friends already have adopted.



Epistemic Threshold Model

Epistemic Threshold Model

An ETM is a tuple $\mathcal{M} = (W, \mathcal{A}, N, B, \theta, \{\sim_a\}_{a \in \mathcal{A}})$ where

- W is a finite, non-empty set of possible worlds,
- $\sim_a \subseteq W \times W$ is an *indistinguishably* (equivalence) relation,

•
$$N: W \to (\mathcal{A} \to \wp(\mathcal{A})),$$

- $a \notin N(w)(a)$ (irreflexive),
- $b \in N(w)(a) \Leftrightarrow a \in N(w)(b)$ (symmetric), and

$$N(w)(a) \neq \emptyset$$
 (serial).

• $B: W \to \wp(\mathcal{A})$

n-sight

- These models permit that agents do not "know" their friends.
 - (like the previous example)
- If a model has 1-sight, then every agent *knows* whether every (immediate) friend has adopted *p* (and who they are).
 - ► If a model has 2-sight, then every agent *knows* whether every friend, and friends of her friends, has adopted *p*.
- How much knowledge does the agent need to comply to non-epistemic threshold models?

1-sight is sufficient

• *Informed update* only when the agent *knows* (*de dicto*) that the threshold is exceeded.

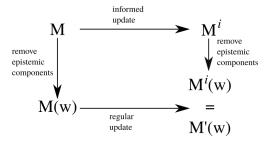
$$B'(w) = B(w) \cup \left\{ a \in \mathcal{A} \mid \forall v \sim_a w \frac{|N(v)(a) \cap B(v)|}{|N(v)(a)|} \ge \theta \right\}$$

1-sight is sufficient

• *Informed update* only when the agent *knows* (*de dicto*) that the threshold is exceeded.

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• When a model has 1-sight, the following holds:



Dynamic Epistemic Logics of Diffusion and Prediction in Social Networks

Alexandru Baltag, Zoé Christoff, Rasmus K. Rendsvig, and Sonja Smets

- Goes on to show:
 - adding epistemic dimension can *slow down* diffusion,
 - this slowdown can be counteracted be a speed up when agents are permitted to predict the development
- provides logics for reasoning about the systems

Formal Models of Conflicting Social Influence

Truls Pedersen and Marija Slavkovik

"Assume that you have a group of friends that are convinced that climate change is a hoax. You also have another group of friends that are climate change researchers devoted to slowing down climate change. You would be under pressure to choose an opinion to support. To avoid the conflict you would necessarily have to stop your relations with at least one of (or parts of one of) the groups."

Social Network

Social Network

A model is a tuple $\mathcal{M} = (\mathcal{A}, N, \mathcal{I}, pro)$ where

• \mathcal{I} is a set of issues (comes in pairs), e.g.,

$$\mathcal{I} = \{p, \neg p\}$$

•
$$pro: \mathcal{I} \to \wp(\mathcal{A}) \text{ s.t., } pro(\phi) \cap pro(\neg \phi) = \emptyset$$

• We keep three doxasitc agent states: can be in either or none.

Strength of social influence

- Keep the normative question from Liu, Seligman & Girard: when does an agent feel compelled to revise her belief?
- Primarily interested in expressing strength/degree: by what unit do we measure this degree?
- We propose a framework for reformulating the well-known models:

$$\Omega^i:\mathcal{I}\to\wp(\wp(\mathcal{A}))$$

Pivotal sets

Pivotal sets

 $A \in \Omega(i, \phi)$ is a set of *i*'s neighbours such that, *cetiris paribus*, after agent *i* drops all ties to the agents in *A*, *i* is no longer under pressure to adopt ϕ .

Pivotal sets

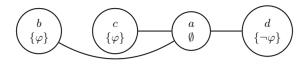
Pivotal sets

 $A \in \Omega(i, \phi)$ is a set of *i*'s neighbours such that, *cetiris paribus*, after agent *i* drops all ties to the agents in *A*, *i* is no longer under pressure to adopt ϕ .

• Formalize (non-inflationary) threshold model:

$$\Omega_t(i,\phi) := \left\{ A \subseteq N(i) \mid \frac{\mid (N(i) \cap pro(\phi)) \setminus A \mid}{\mid N(I) \setminus A \mid} \not\geq \theta \right\}$$

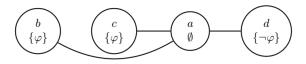
Conflicting Social Influence Threshold $\theta = 1/3$



$$\Omega(a, \neg \phi) = \{\{d\}, \{b, d\}, \{c, d\}, \underbrace{\{b, c, d\}}_{N(a)}\}$$

• Can not avoid influence unless ties to d are cut.

Conflicting Social Influence Threshold $\theta = 1/3$



$$\Omega(a, \neg \phi) = \{\{d\}, \{b, d\}, \{c, d\}, \underbrace{\{b, c, d\}}_{N(a)}\}$$

• Can not avoid influence unless ties to d are cut.

$$\Omega(a,\phi) = \{\{b,c\}, \underbrace{\{b,c,d\}}_{N(a)}\}$$

• Can not avoid influence unless ties to both b and c are cut.

Properties of Influence Models

- In both cases, N(i) were included:
 - Social hermits are immune in the threshold model.
- In neither case was \emptyset included:
 - *a* was influenced to adopt ϕ , and influenced to adopt $\neg \phi$.
 - Provides condition for restoring dichotomous models.

Formal Models of Conflicting Social Influence

Truls Pedersen and Marija Slavkovik

- Goes on to discuss
 - temporal consequences, and
 - consistency of agents' beliefs.
- Under what conditions do elements of $\Omega(i, \phi)$ function as "conflict resolving actions"?

How to Make Friends: A Logical Approach to Social Group Creation Sonja Smets and Fernando R. Velázques-Quesada

"It is commonly accepted that our social contacts affect the way we form our opinions about the world. [...] This paper focuses on the logical structure behind the creation of social networks. Our basic mechanism for group-creation focusses on agents who become socially connected when the number of features in which they differ is small enough In line with this idea we propose several group-creation policies, exploring the properties of the resulting networks."

Making Friends

- "Birds of a feather flock together"
 - Agents that are similar may form new edges in the network.

Mismatch

The feature mismatch between a and b

$$mismatch^{M}(a,b) := (V(a) \setminus V(b)) \cup (V(b) \setminus V(a))$$

The mismatch *distance* between a and b

$$dist^M = |mismatch^M(a, b)|$$

Social Network Models

Social Network Models

- Some finite *P* of *agent features* (propositions)
- Some updates we discuss violate the regular assumptions of symmetry and irreflexivity.

Similarity Update (I)

• We do *not* update agent features, *only* the network.

Similarity Update (I)

Given a social network (\mathcal{A}, S, V)

- *S* is any set of (social) edges,
- $V: \mathcal{A} \to \wp(P)$ maps every agent to the set of her features.

The similarity updated network (\mathcal{A}, S', V) with threshold θ where

$$S' = \{(a,b) \in \mathcal{A} \times \mathcal{A} \mid dist^{M}(a,b) \le \theta\}$$

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 $\leq \theta$

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Similarity Update (I)

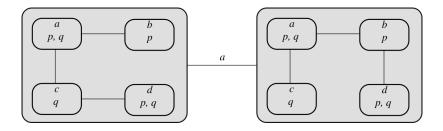
Given a social network (\mathcal{A}, S, V)

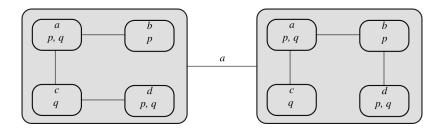
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The similarity updated network (\mathcal{A}, S', V) with threshold θ where

$$S' = \{(a,b) \in \mathcal{A} \times \mathcal{A} \mid dist^{M}(a,b) \leq \theta\}$$

$$\xrightarrow{a}_{p,q} \qquad \xrightarrow{b}_{p} \qquad \xrightarrow{c}_{p,q} \qquad \xrightarrow{b}_{p} \qquad \xrightarrow{c}_{p,q} \qquad \xrightarrow{c}_{p,q}$$

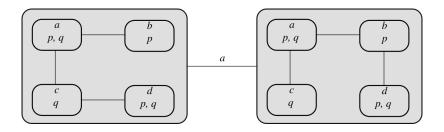




de re (a, b) is added in w if there is a c such that, for every $u \sim_a w$:

•
$$dist_u^M(a,b)$$
, and

•
$$(a,c) \in S, (c,b) \in S$$



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- $dist_u^M(a, b)$, and
- $(a,c) \in S, (c,b) \in S$

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How to Make Friends: A Logical Approach to Social Group Creation Sonia Smets and Fernando R. Velázques-Quesada

- Describes several variations
 - possibility of other distances
 - with/without midleman
 - with/without epistemic dimension
- provides logics describing networks and dynamics



Thank you!

by the way...

FYI: EUMAS2018

https://eumas2018.w.uib.no/

