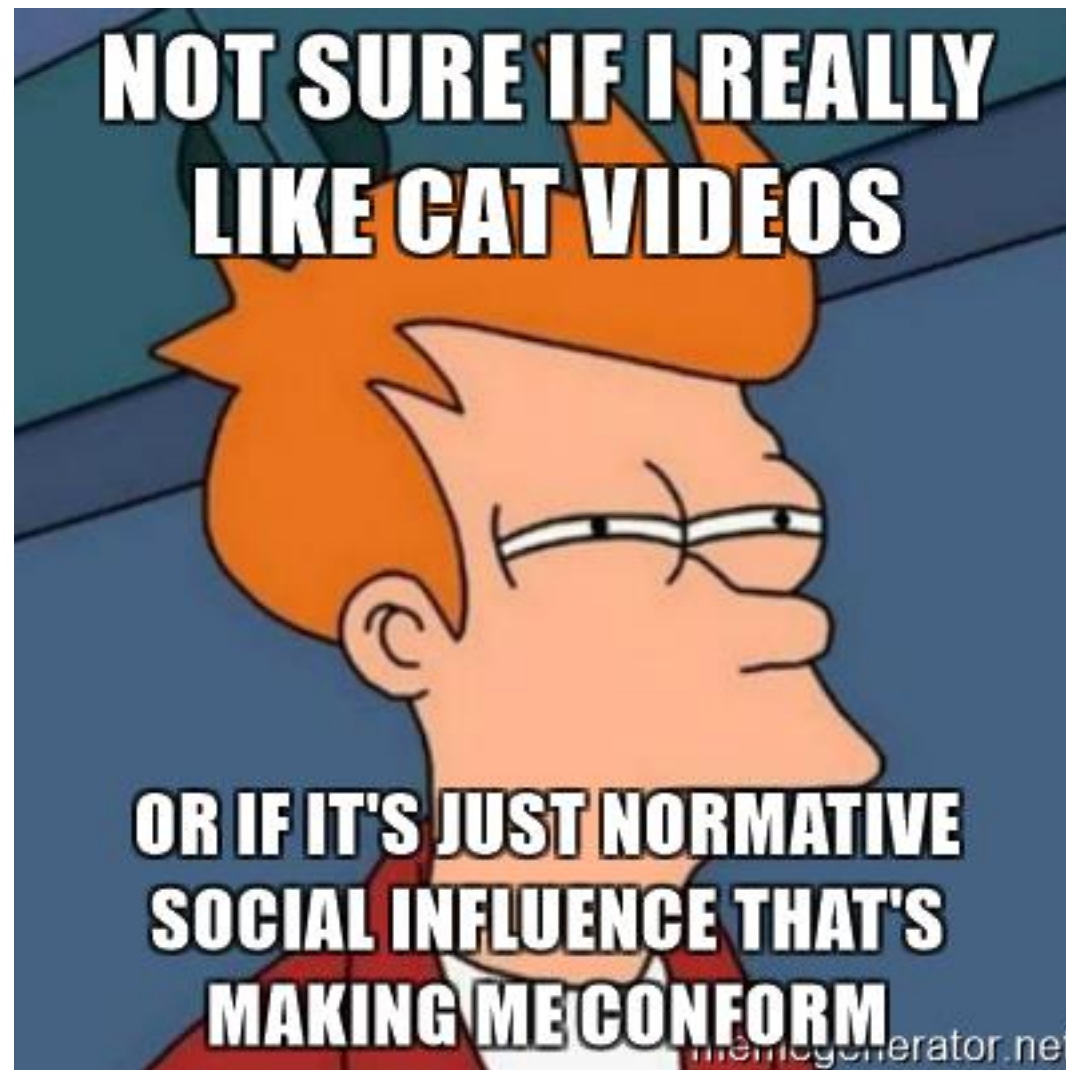


# MAS and Social Influence

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MARIJA SLAVKOVIC & TRULS PEDERSEN  
AAMAS-2018 TUTORIAL

# Why social networks & influence



# Overview

- Social networks models and measures
- Symptoms of social influence: diffusion and convergence
- Macro level effects of micro level actions: cascades, fake majorities and pluralistic ignorance

## Class and Committees in a Norwegian Island Parish

J. A. Barnes

First Published February 1, 1954 | Other

[Article information](#) ▾



# CLASS AND COMMITTEES IN A NORWEGIAN ISLAND PARISH<sup>1</sup>

J. A. BARNES

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# Representing and measuring networks

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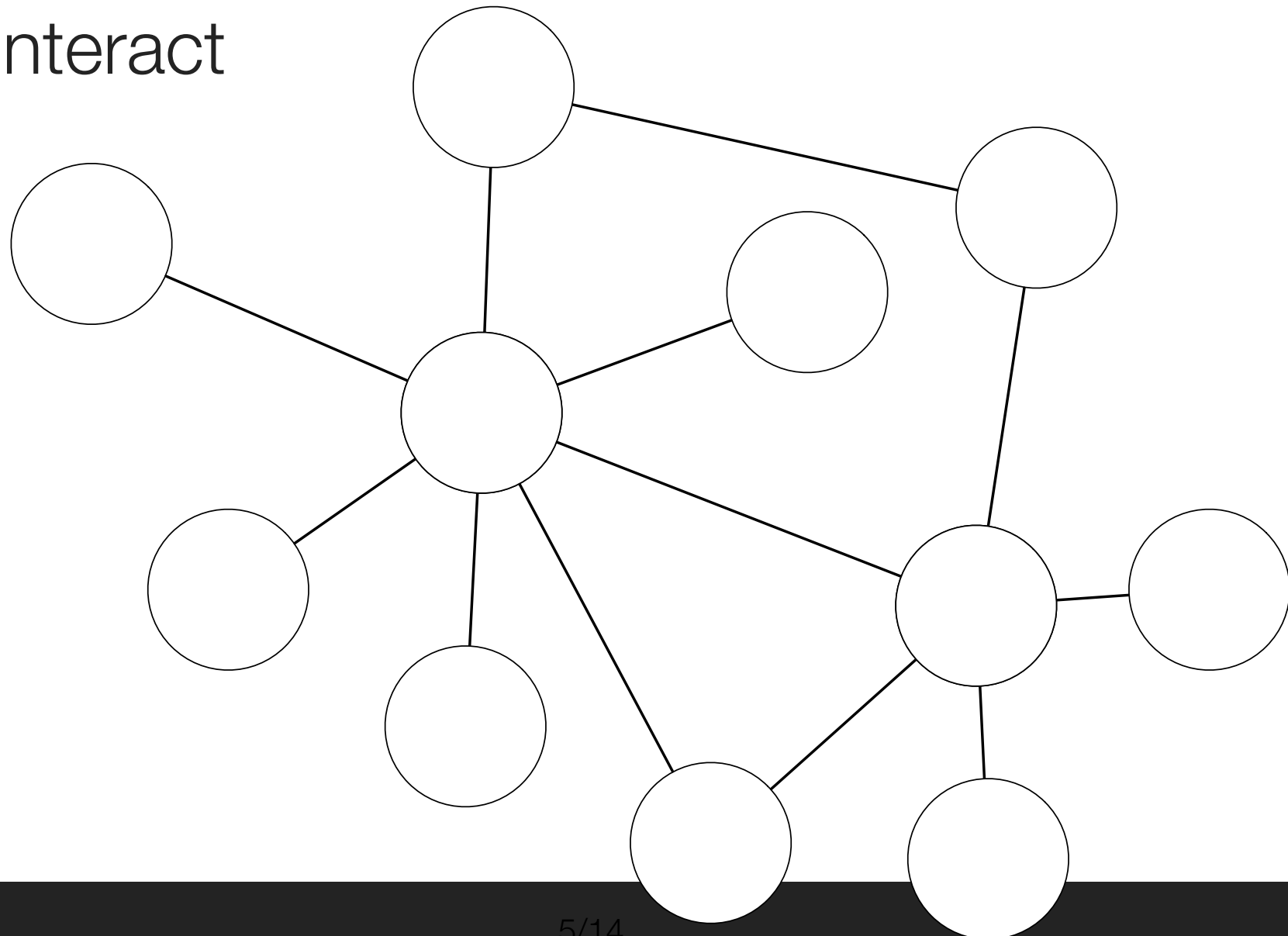
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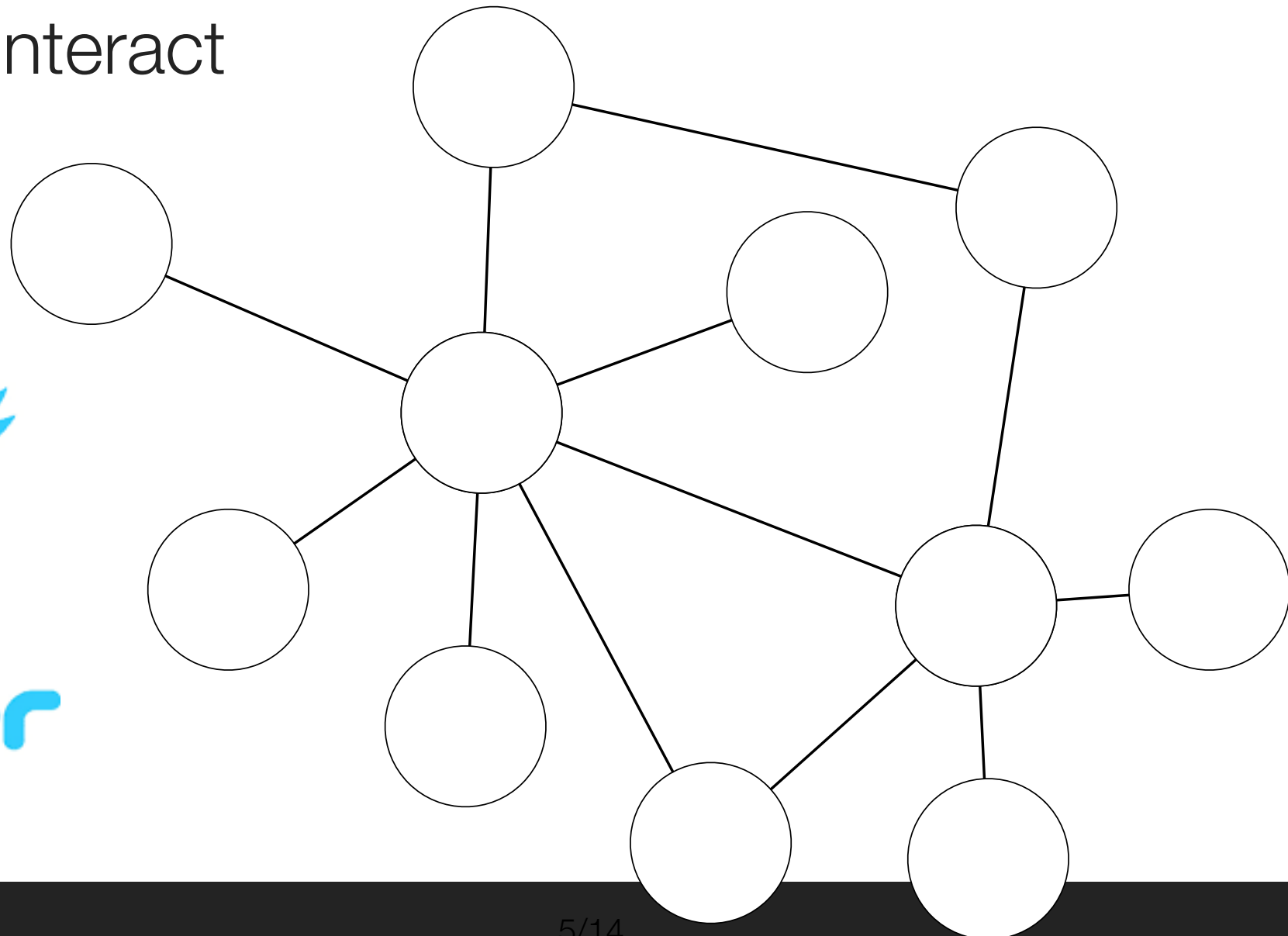
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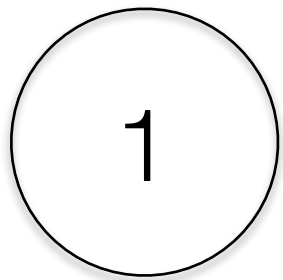


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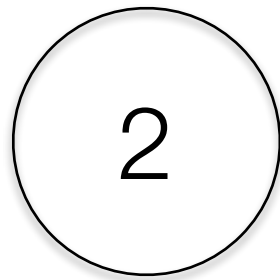
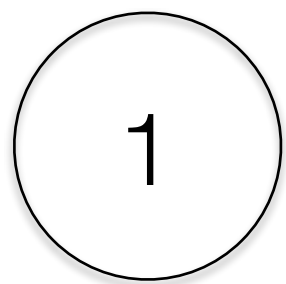
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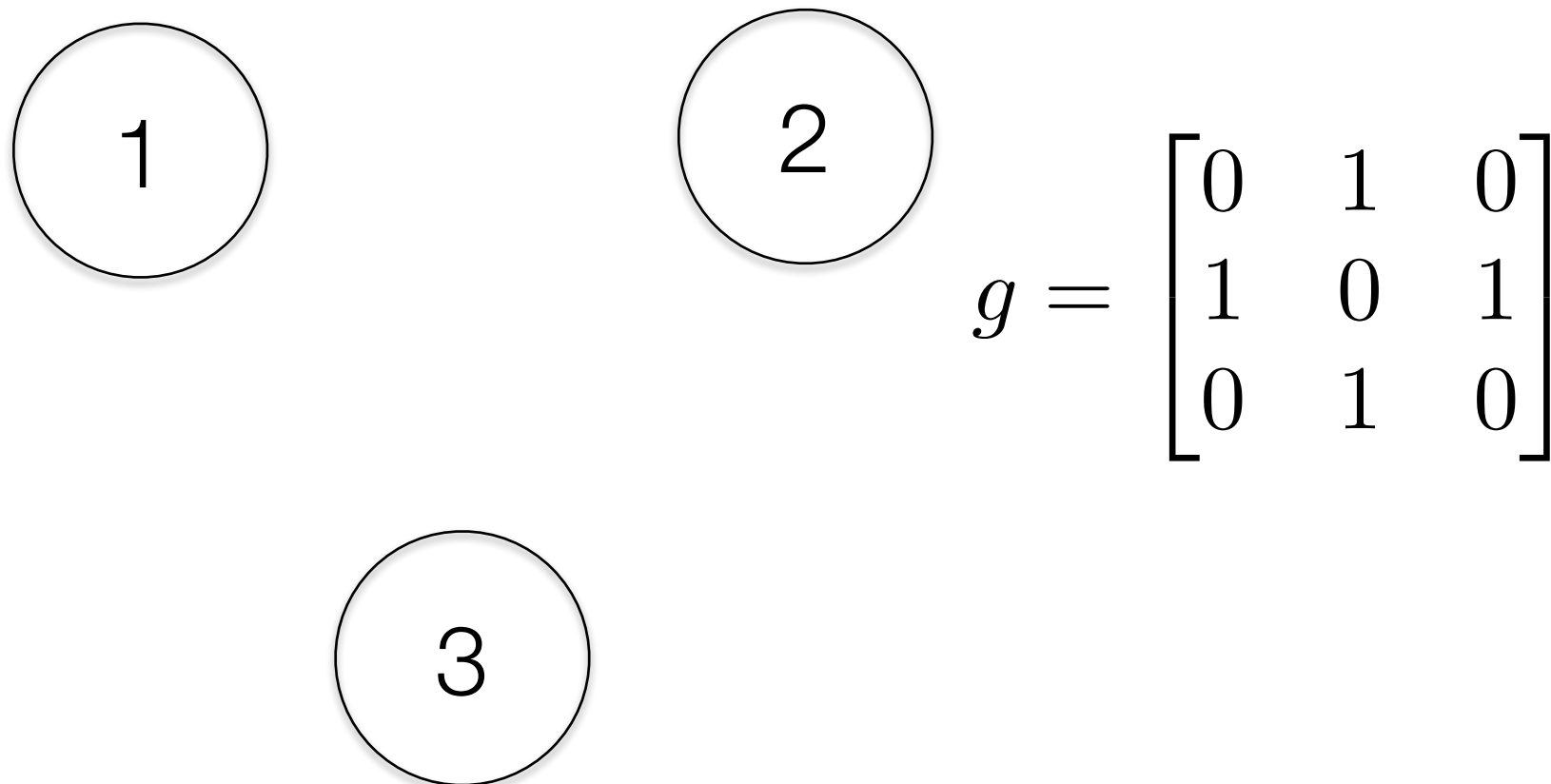
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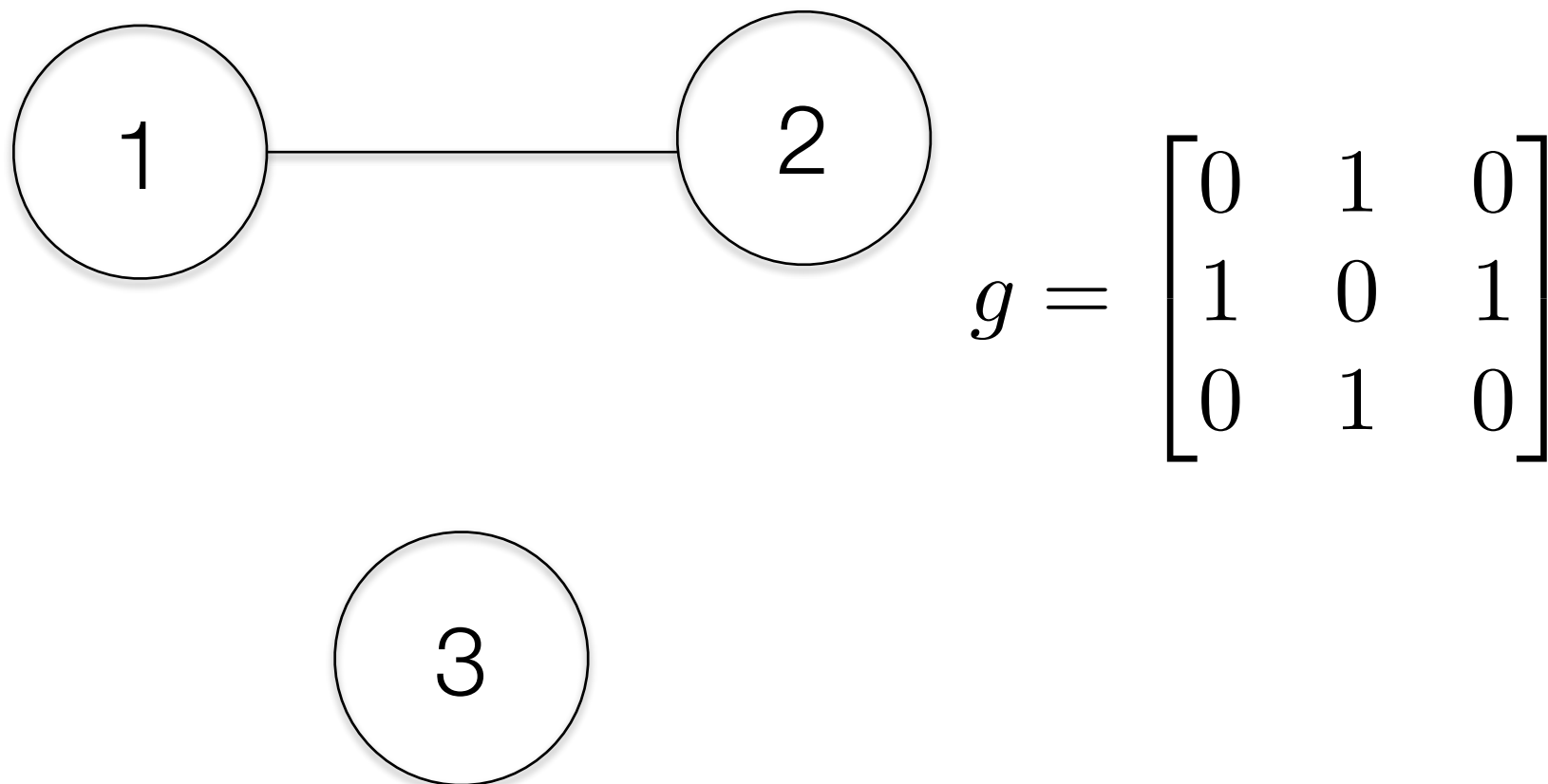


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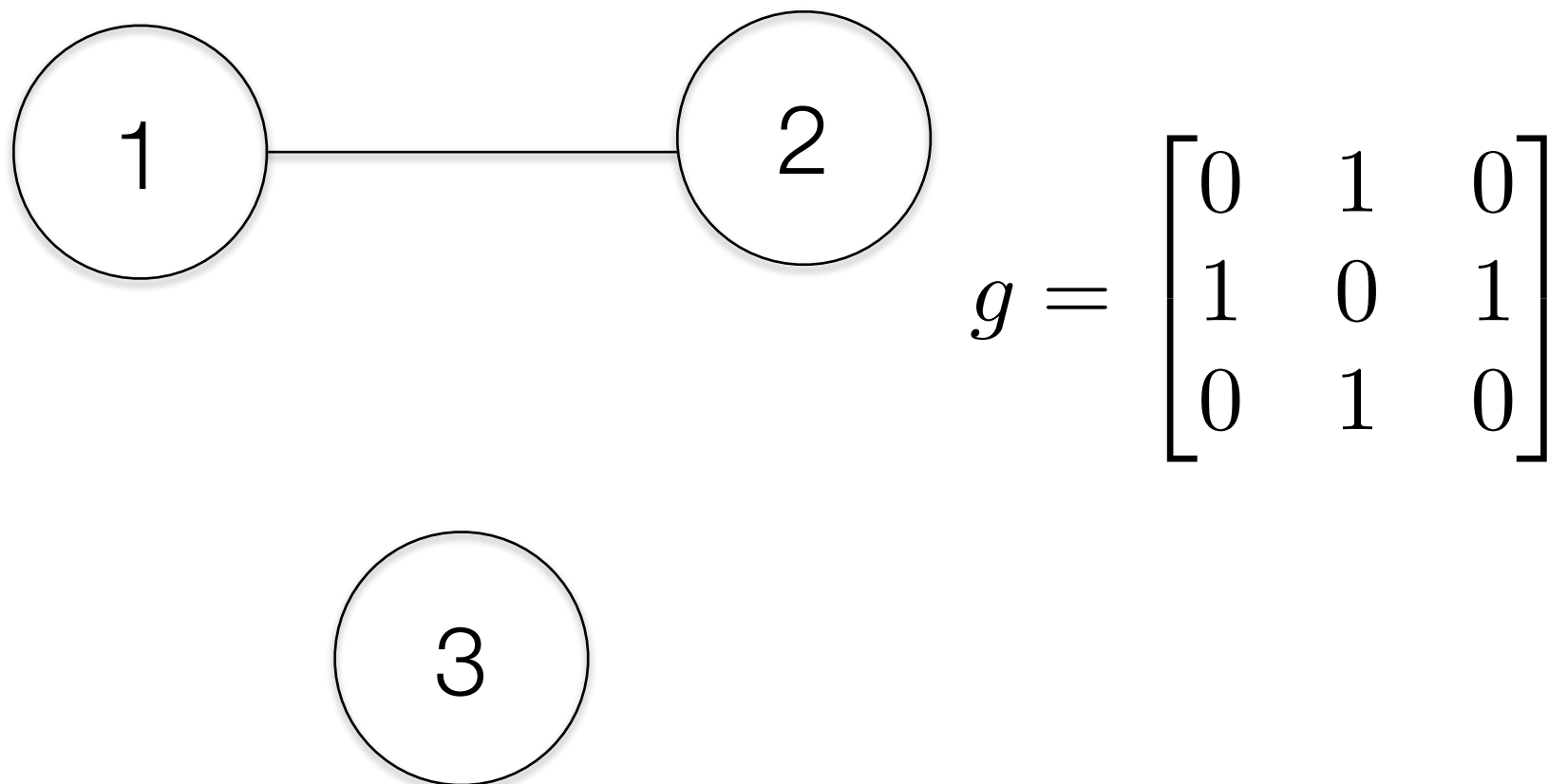


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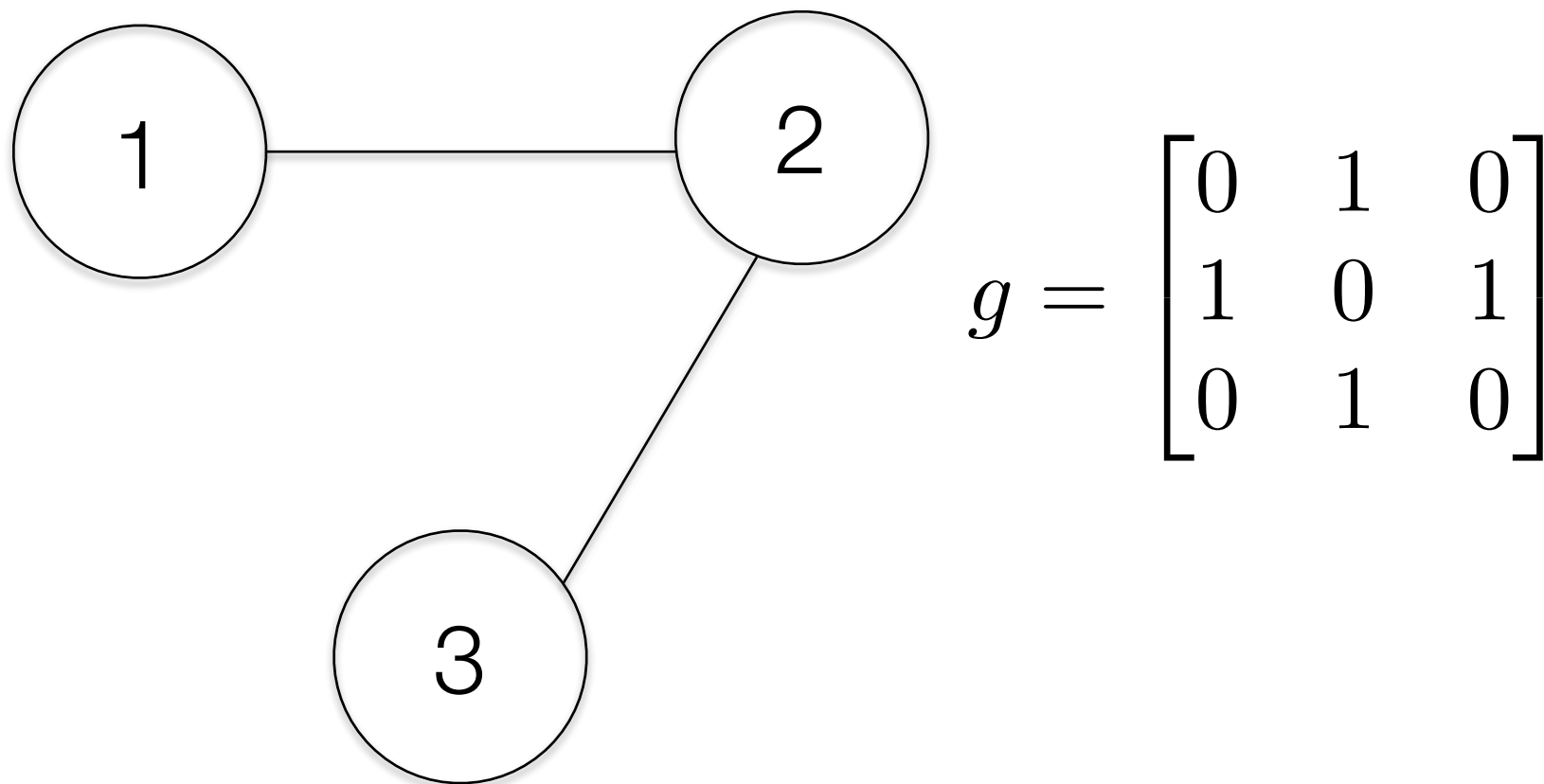




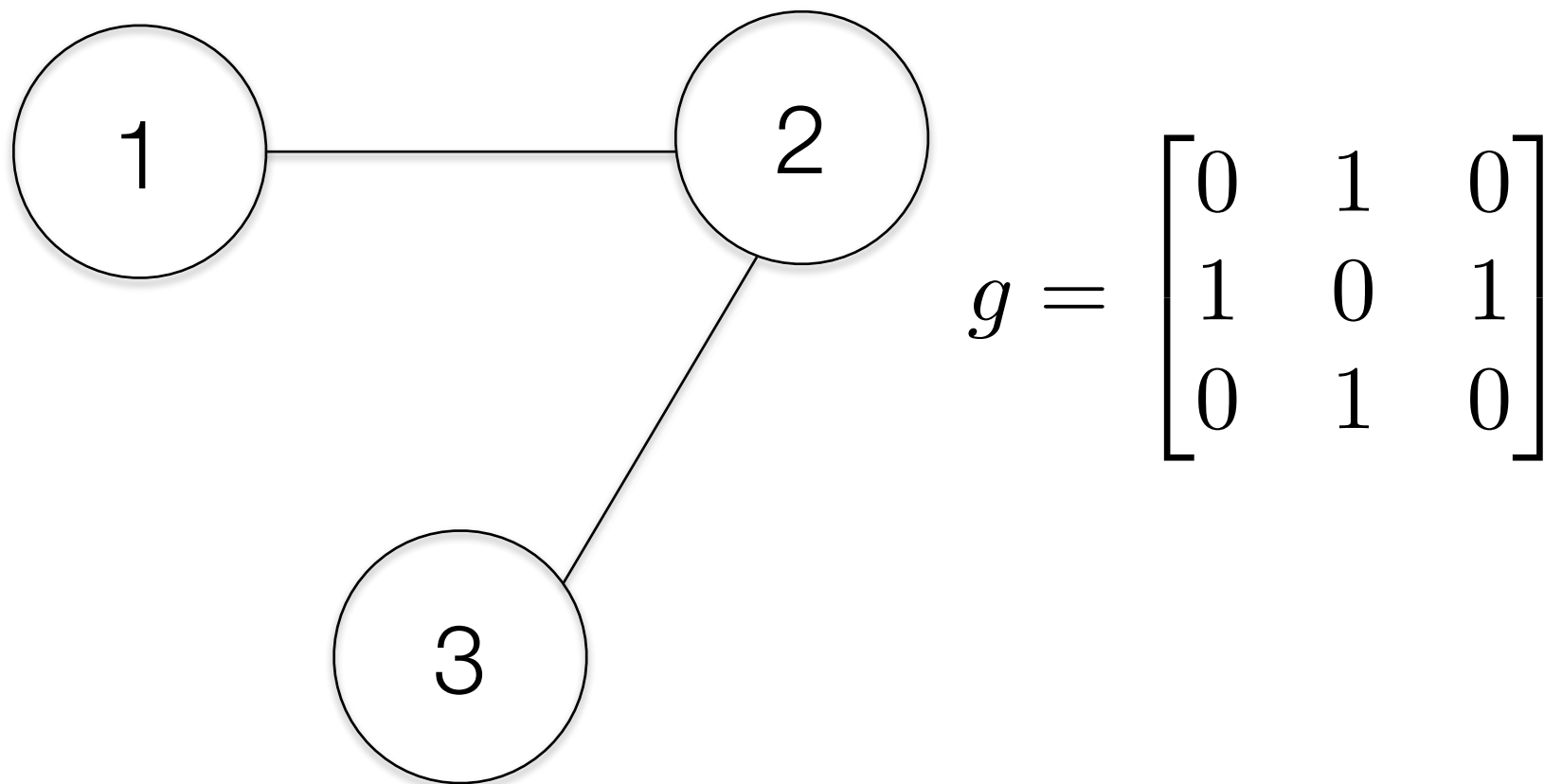
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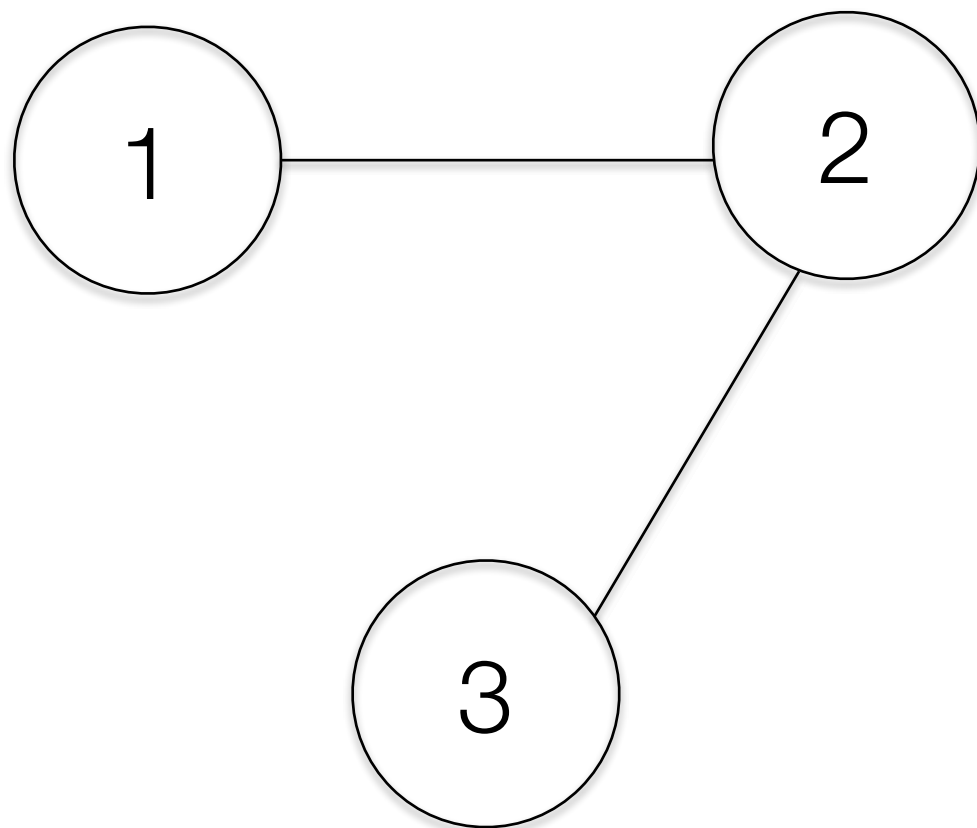
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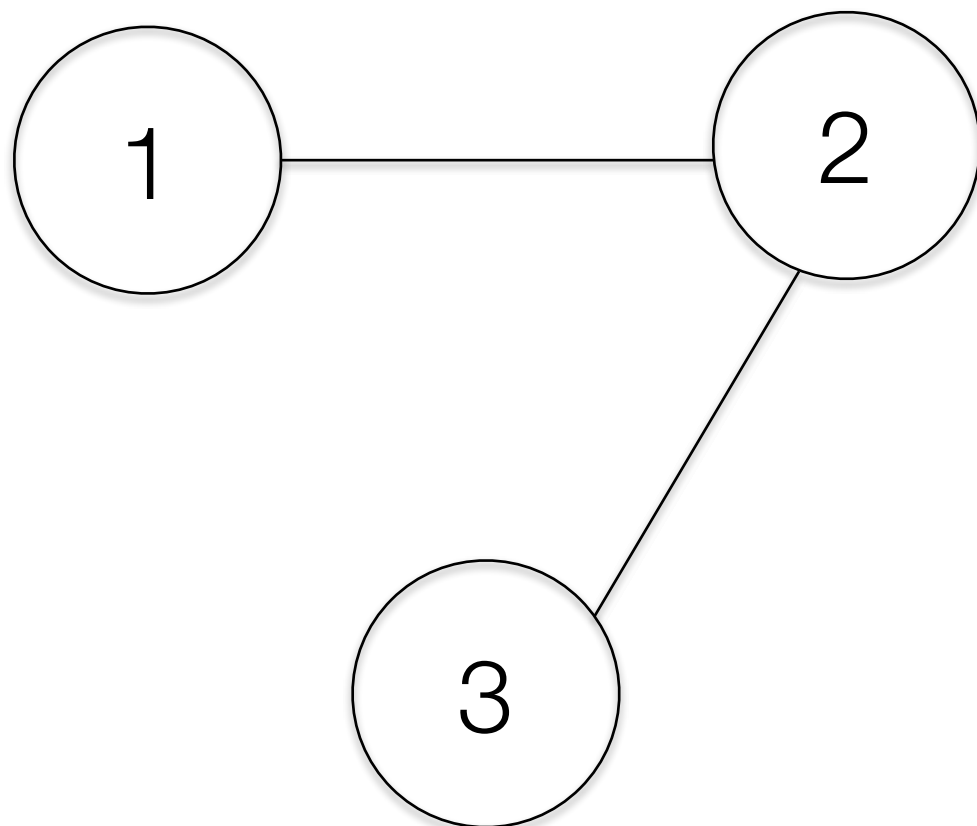
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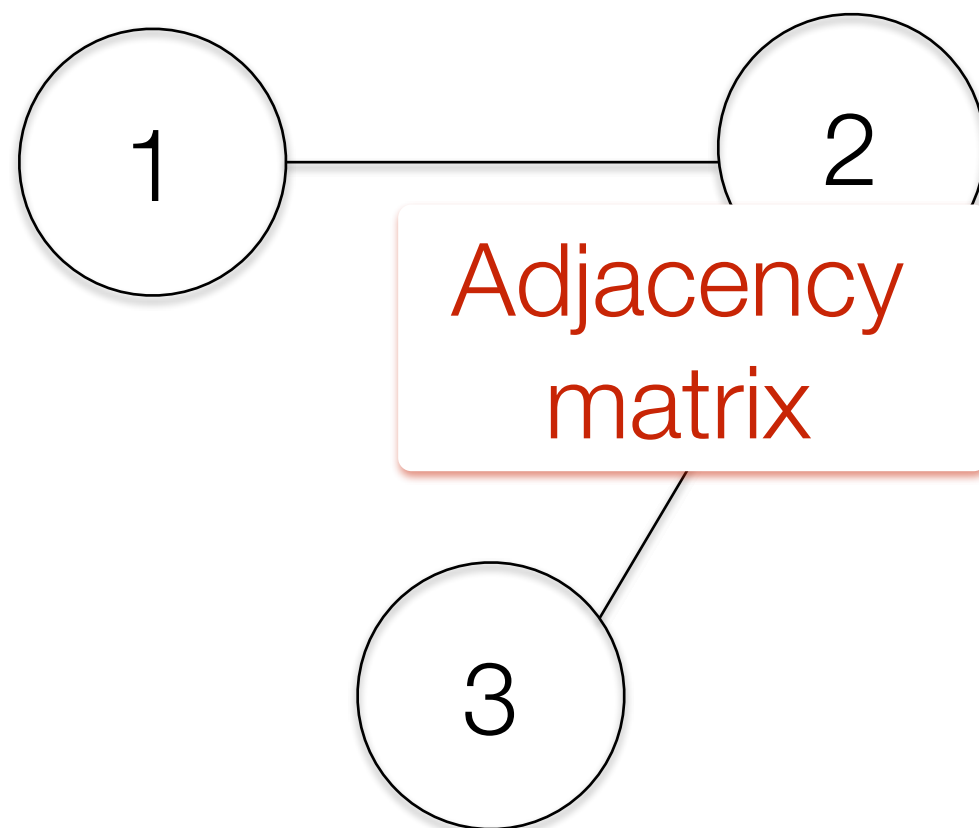


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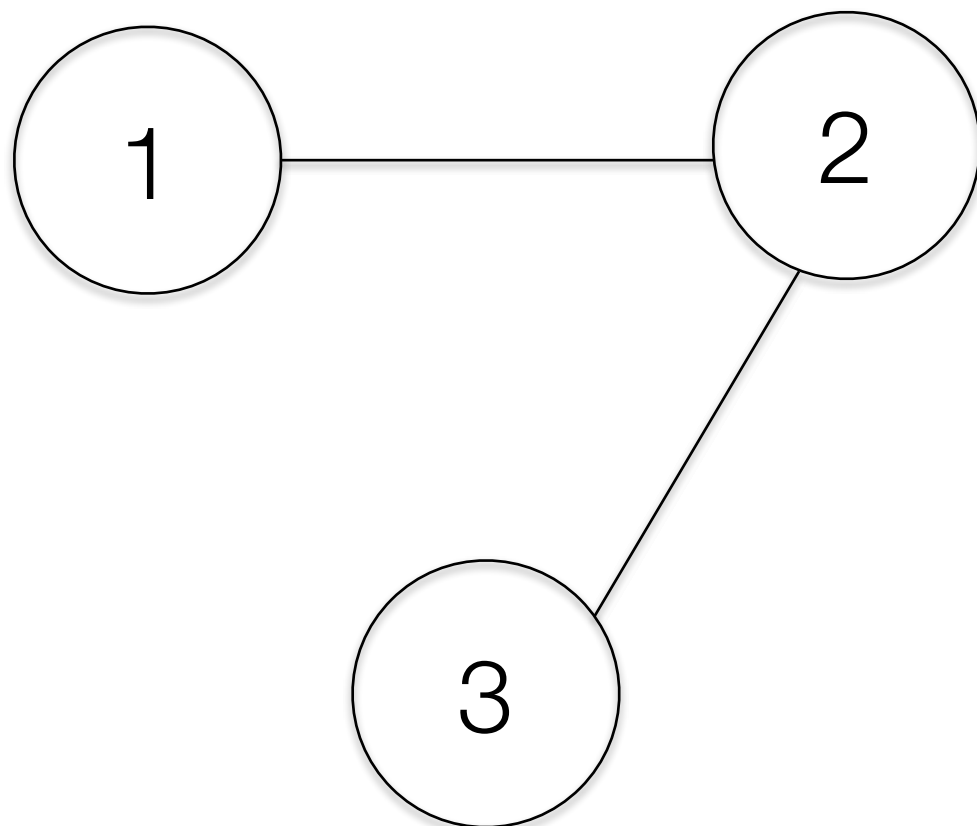


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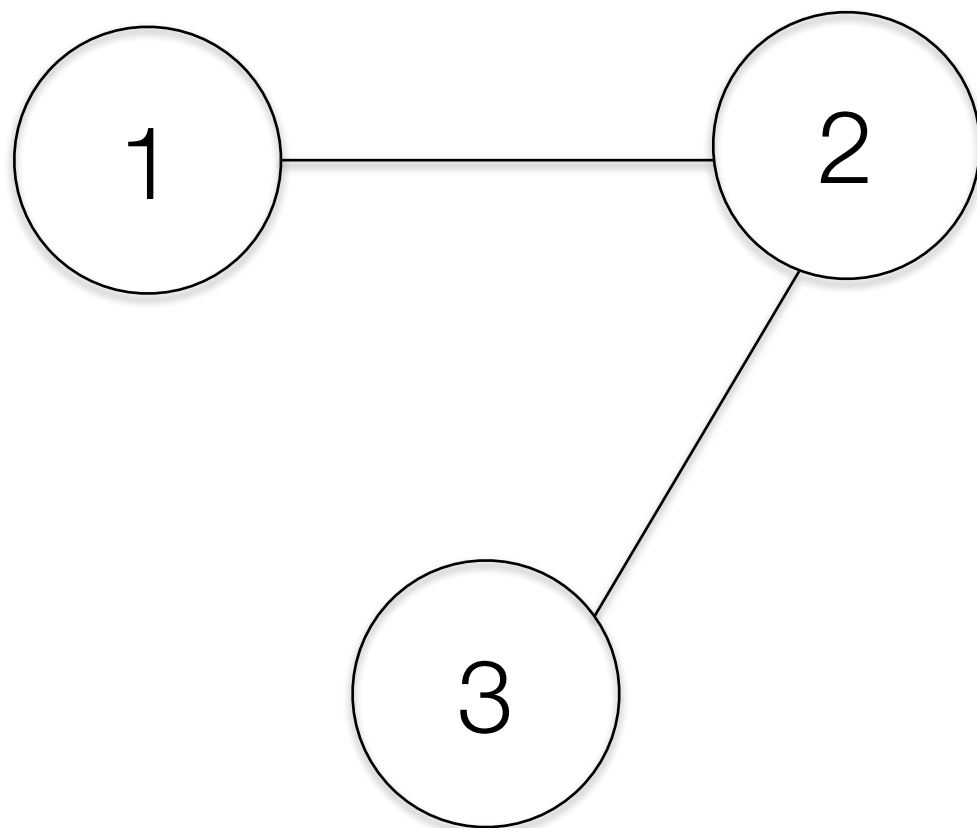


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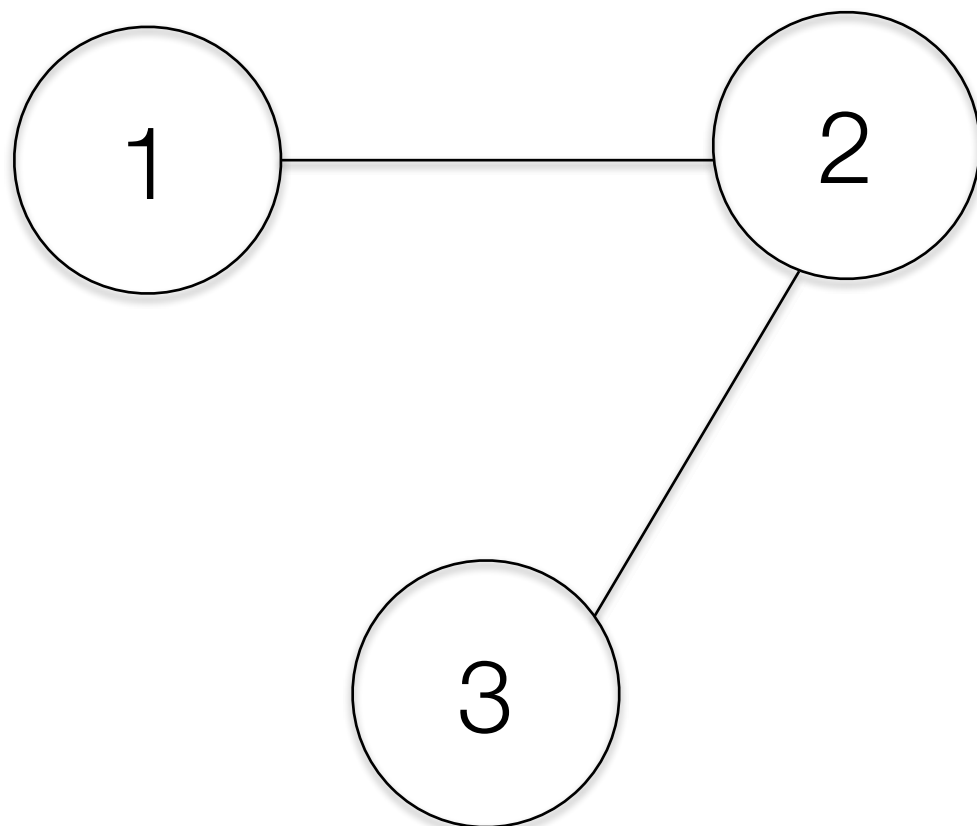
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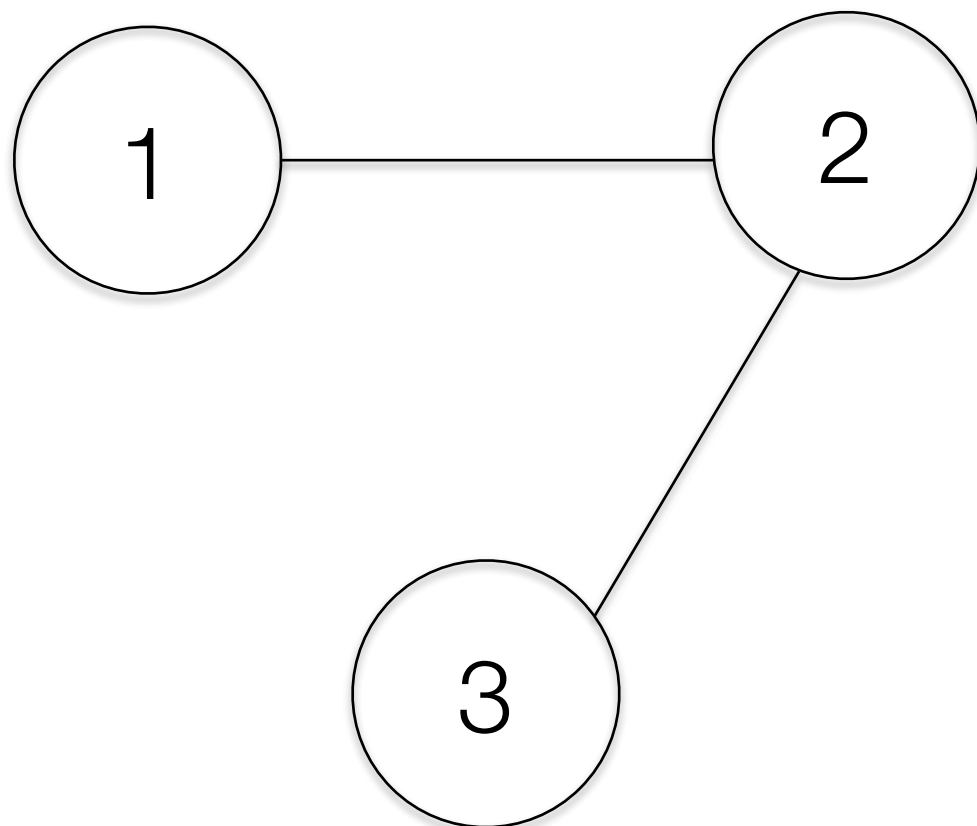


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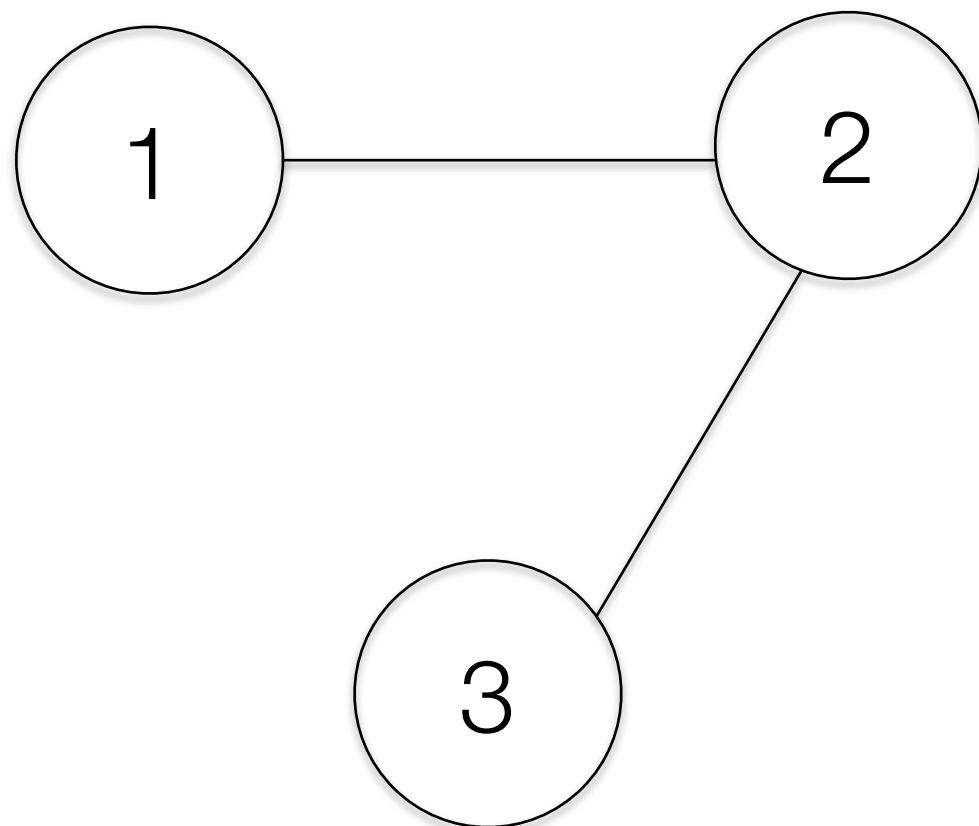


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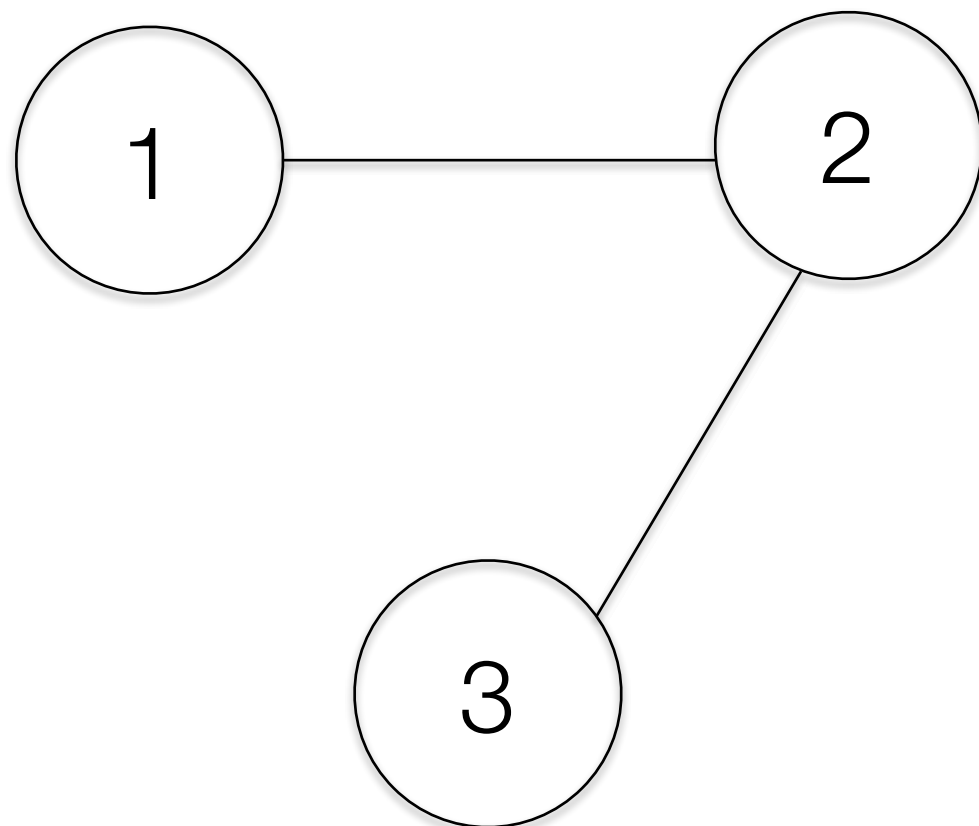
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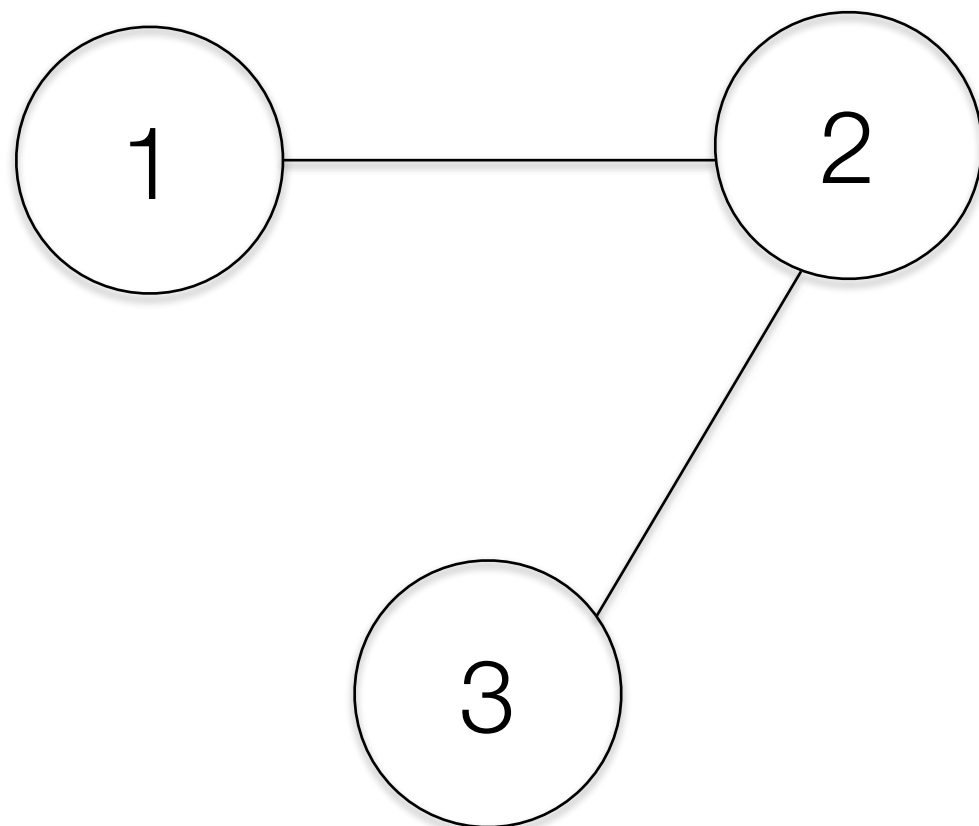
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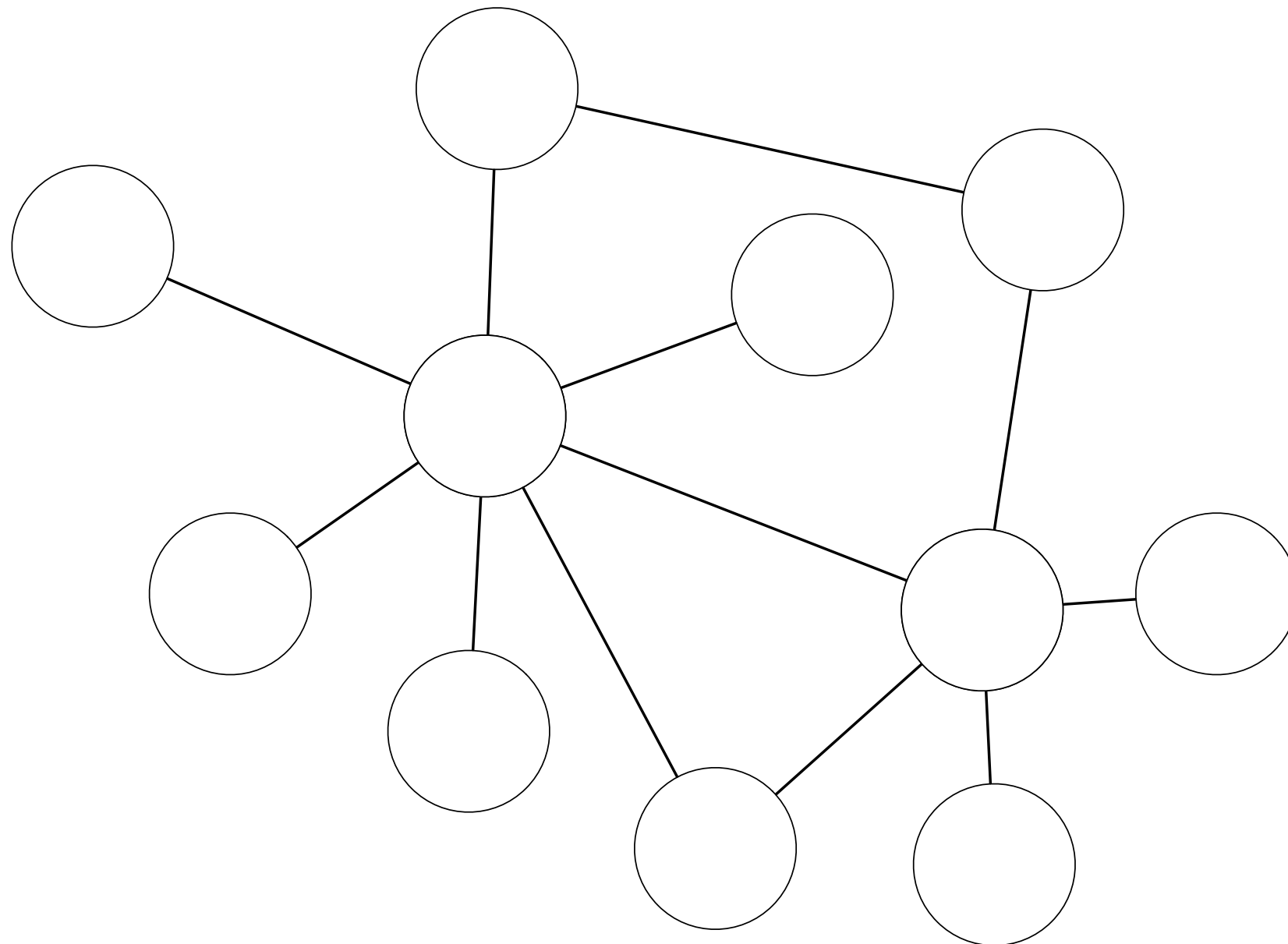
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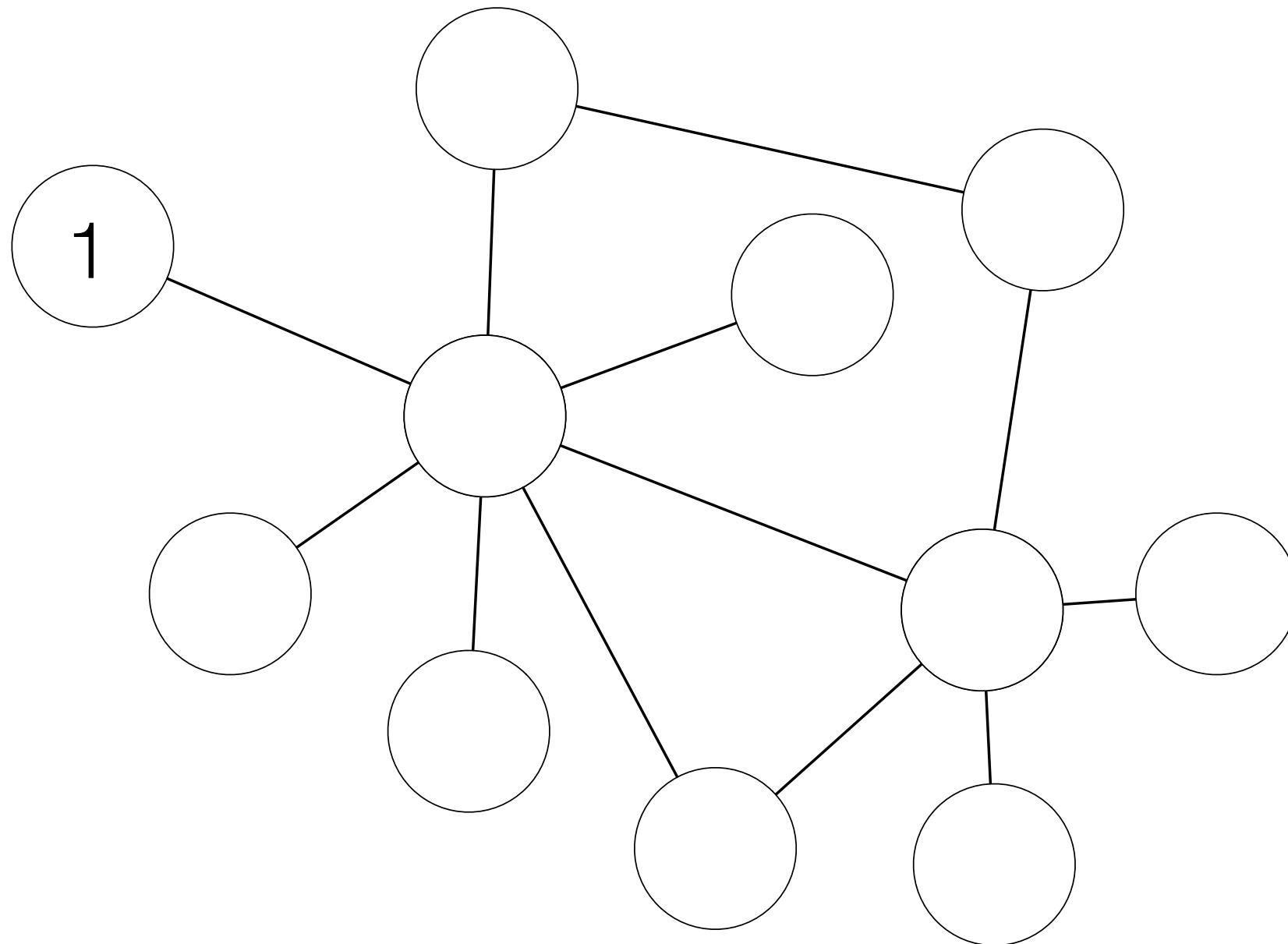
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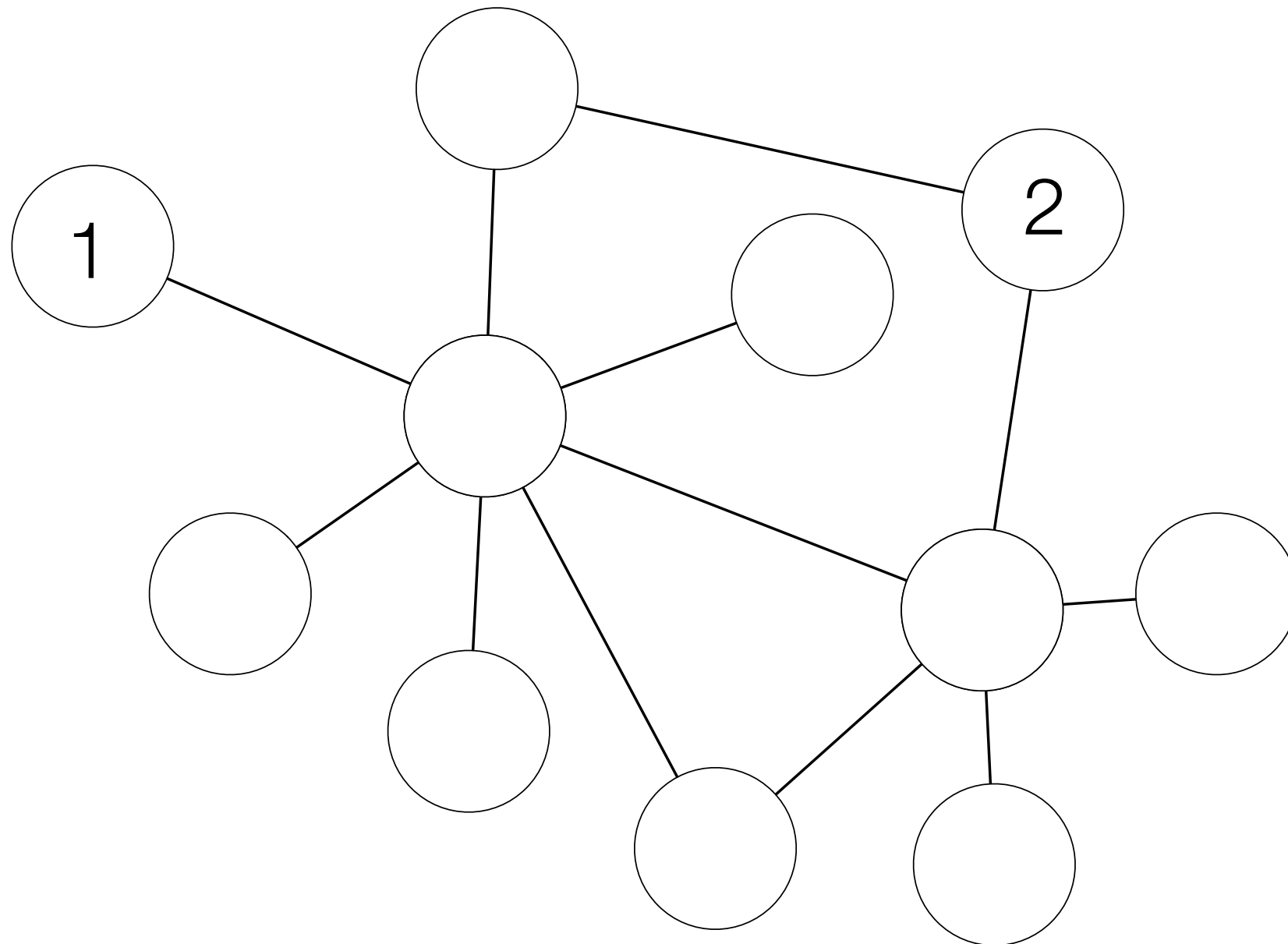
# Path, cycle, distance



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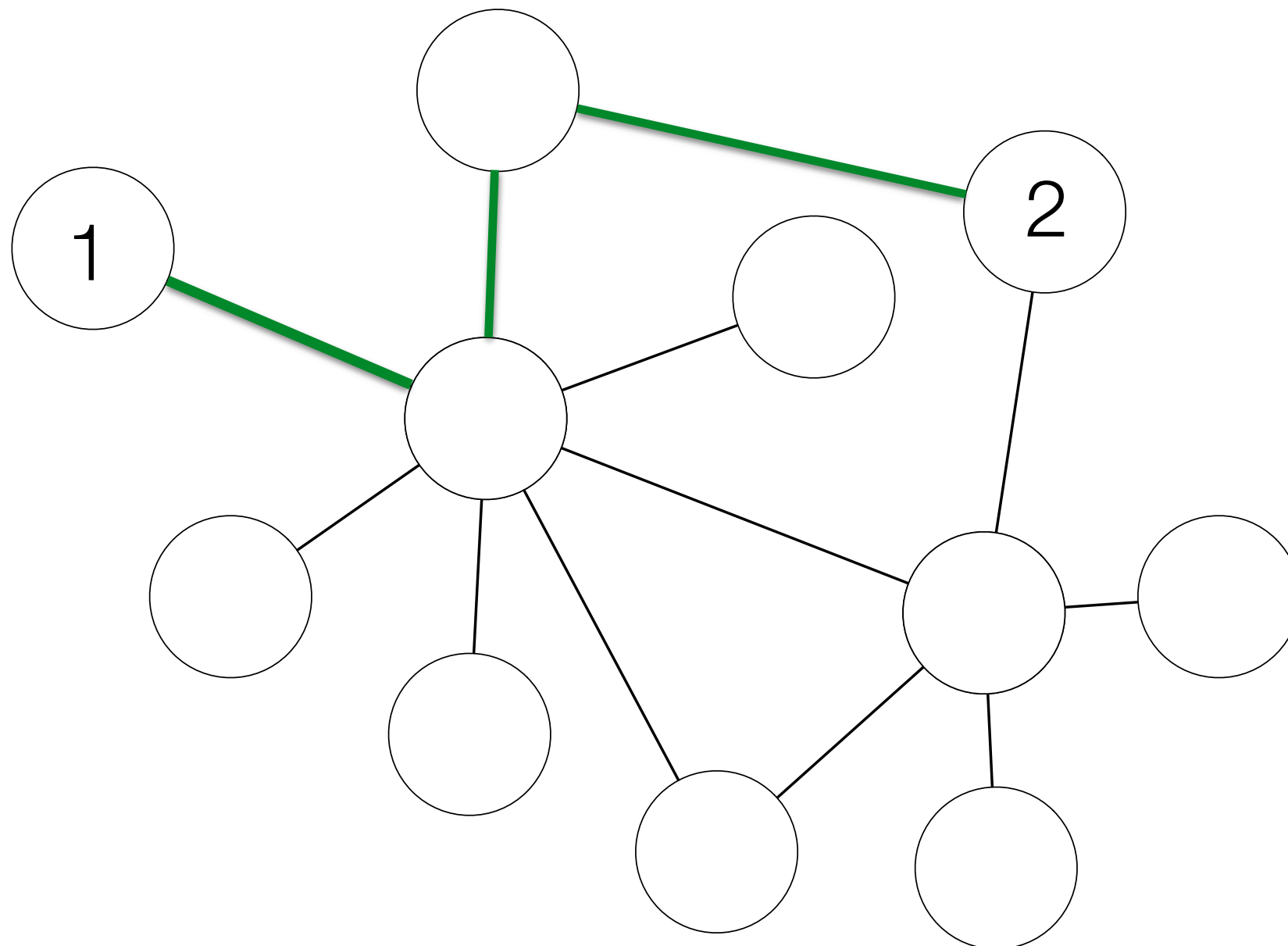


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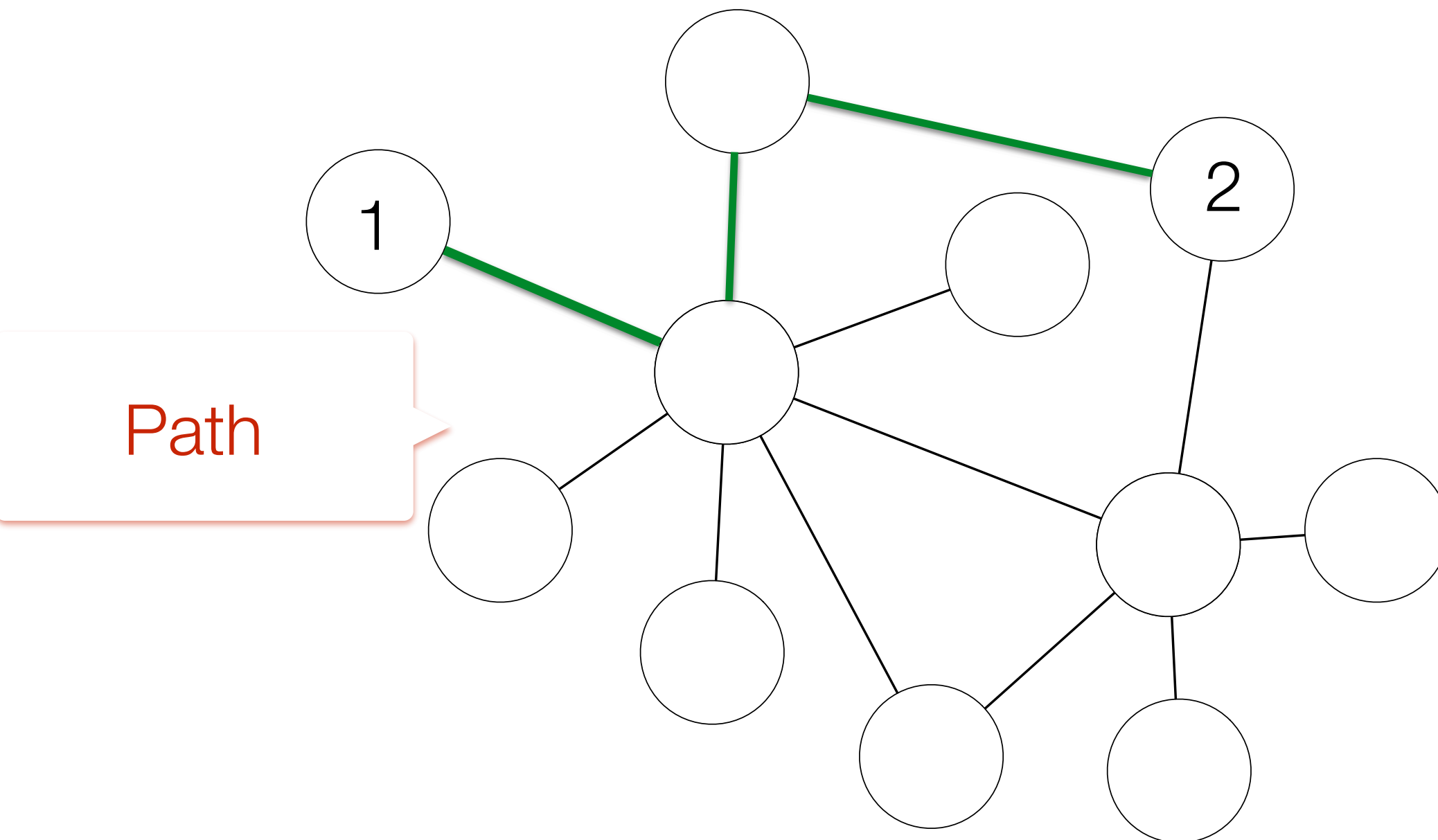




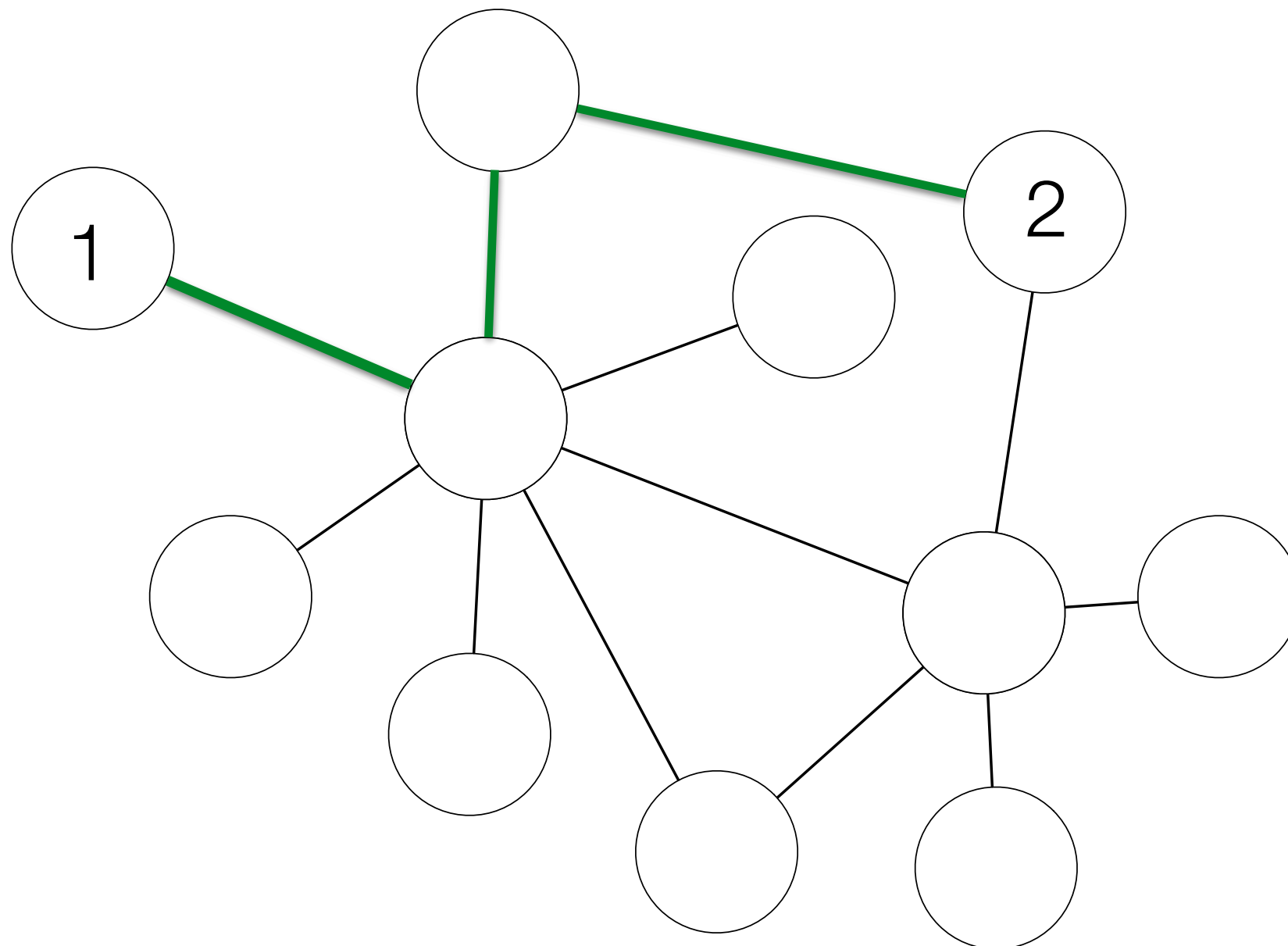
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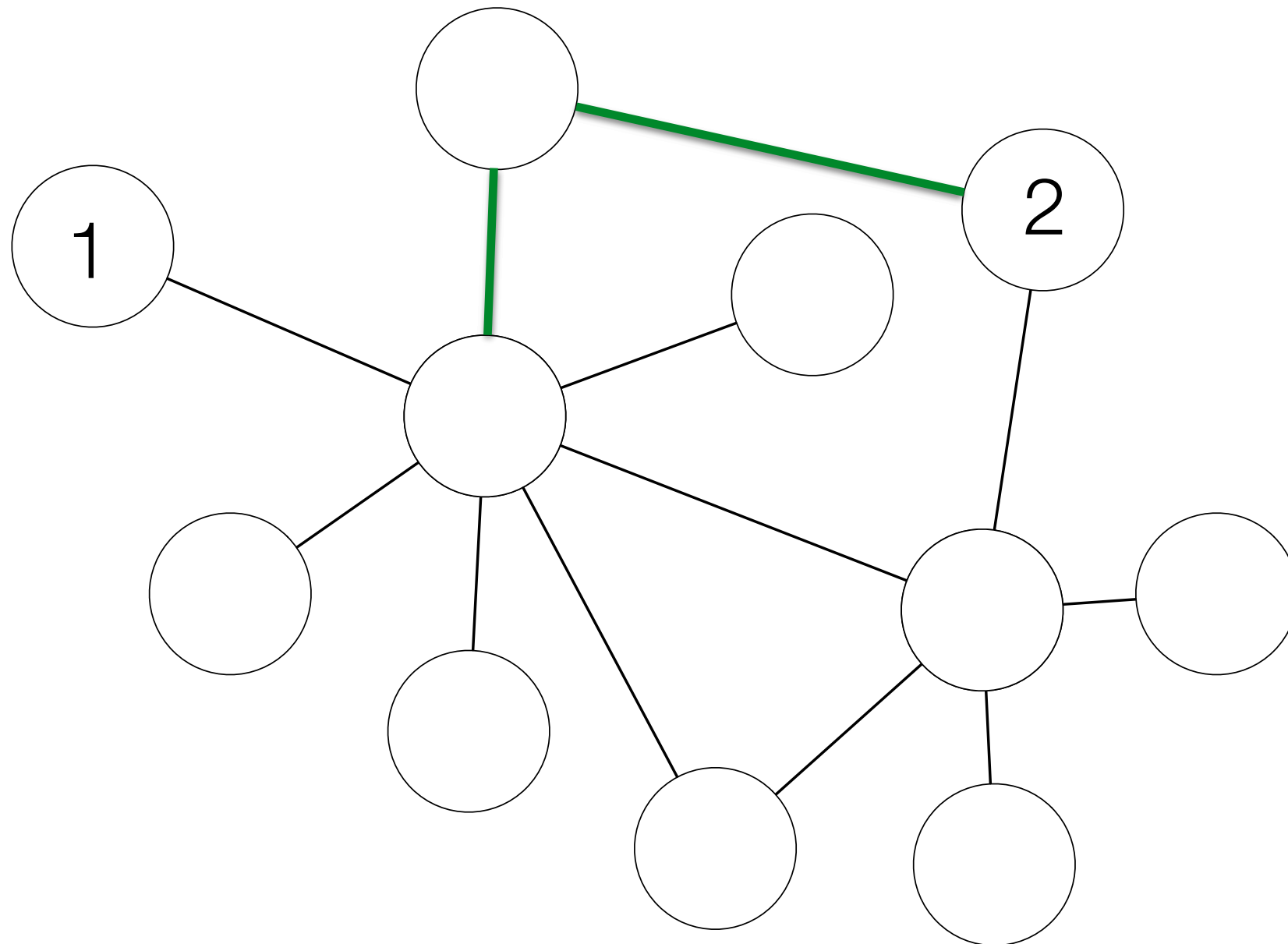
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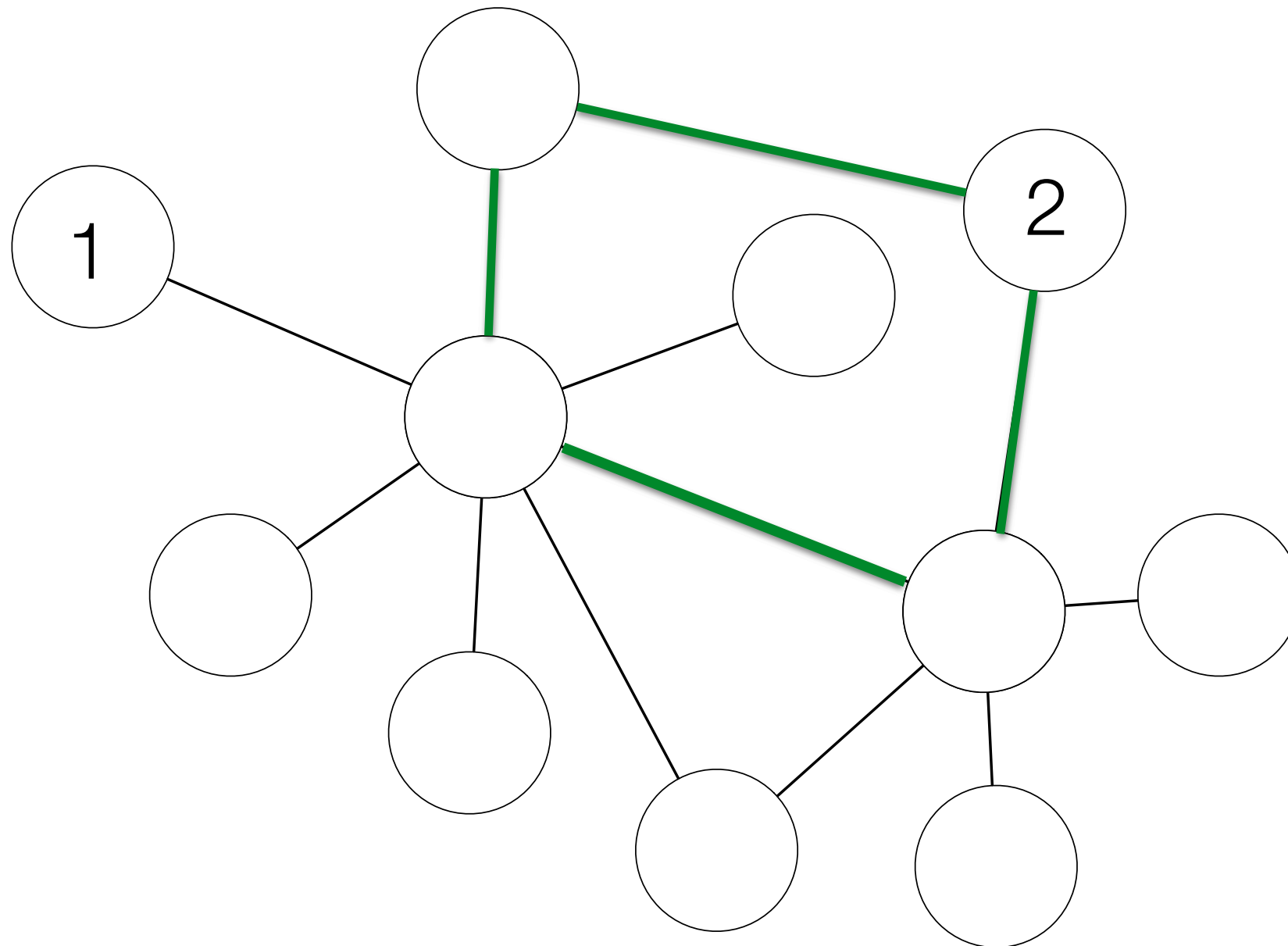
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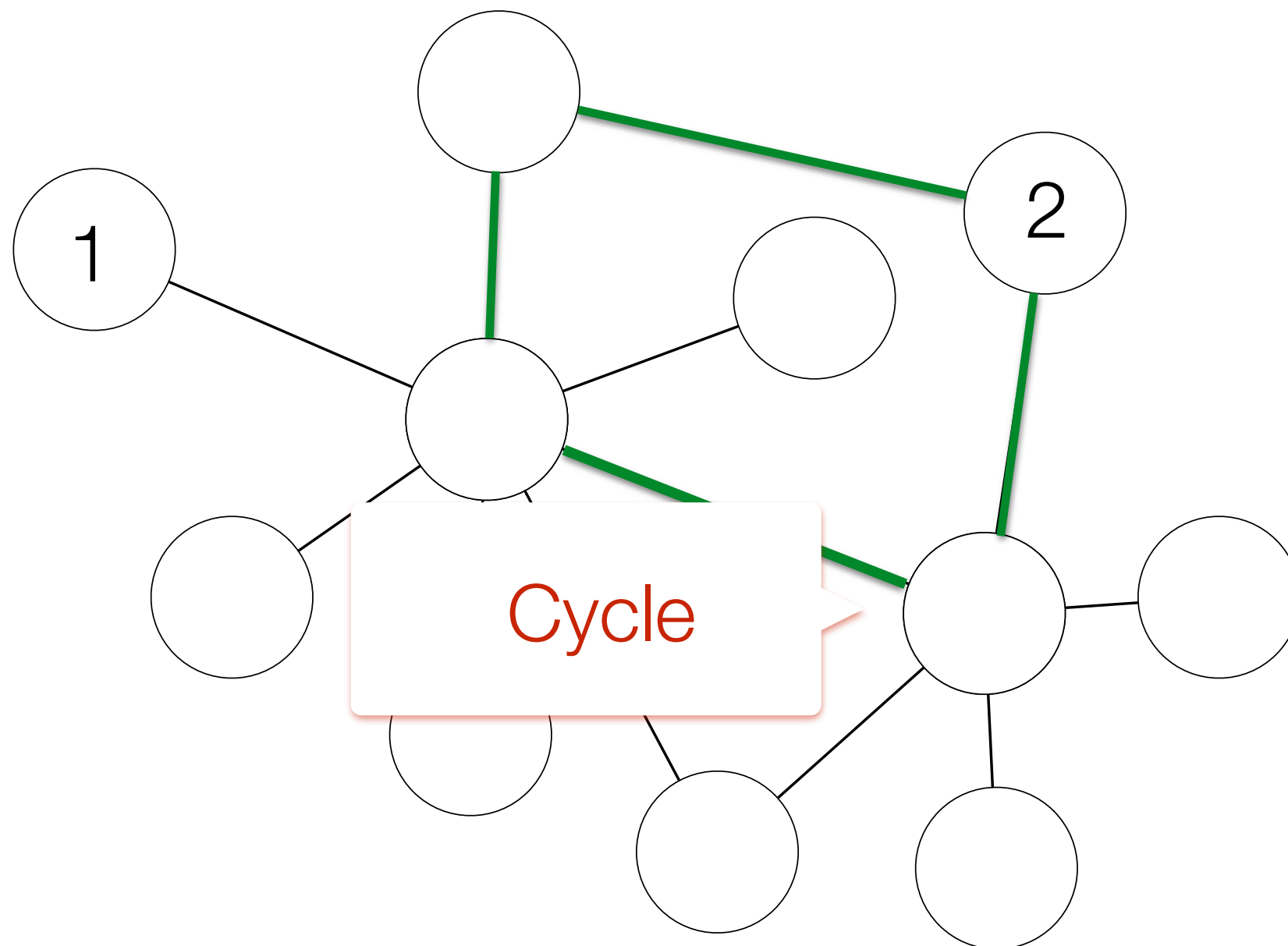
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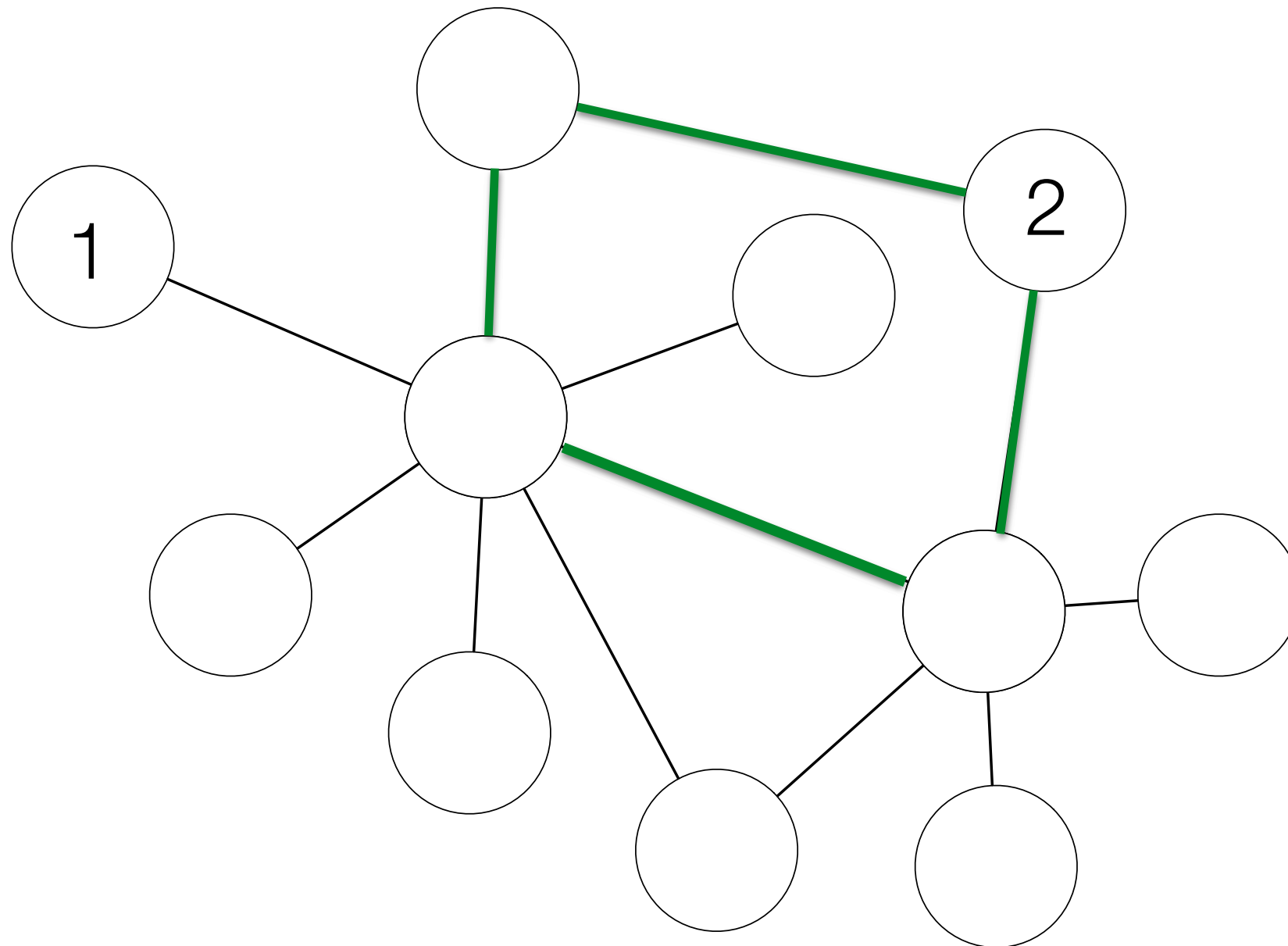
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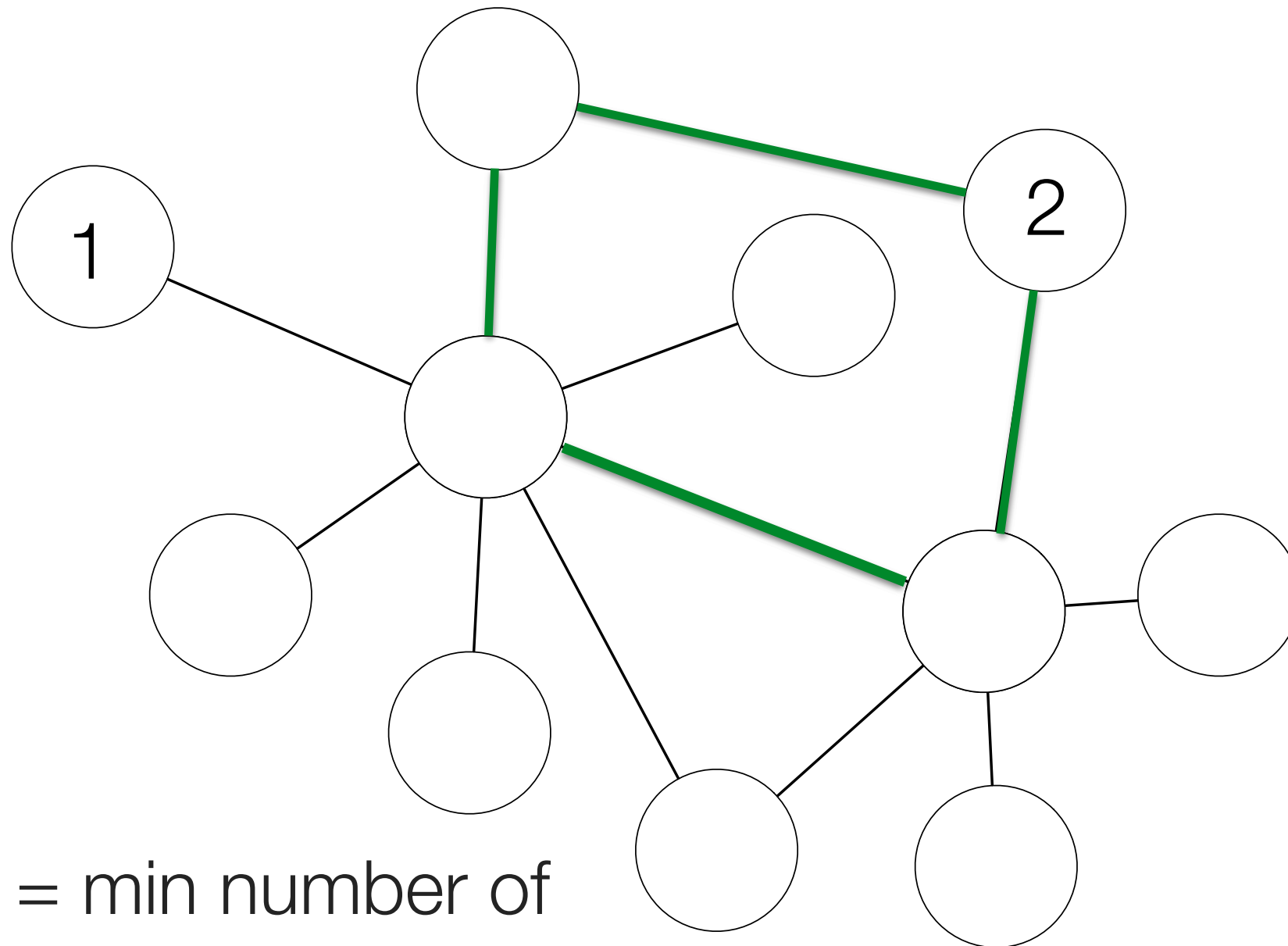
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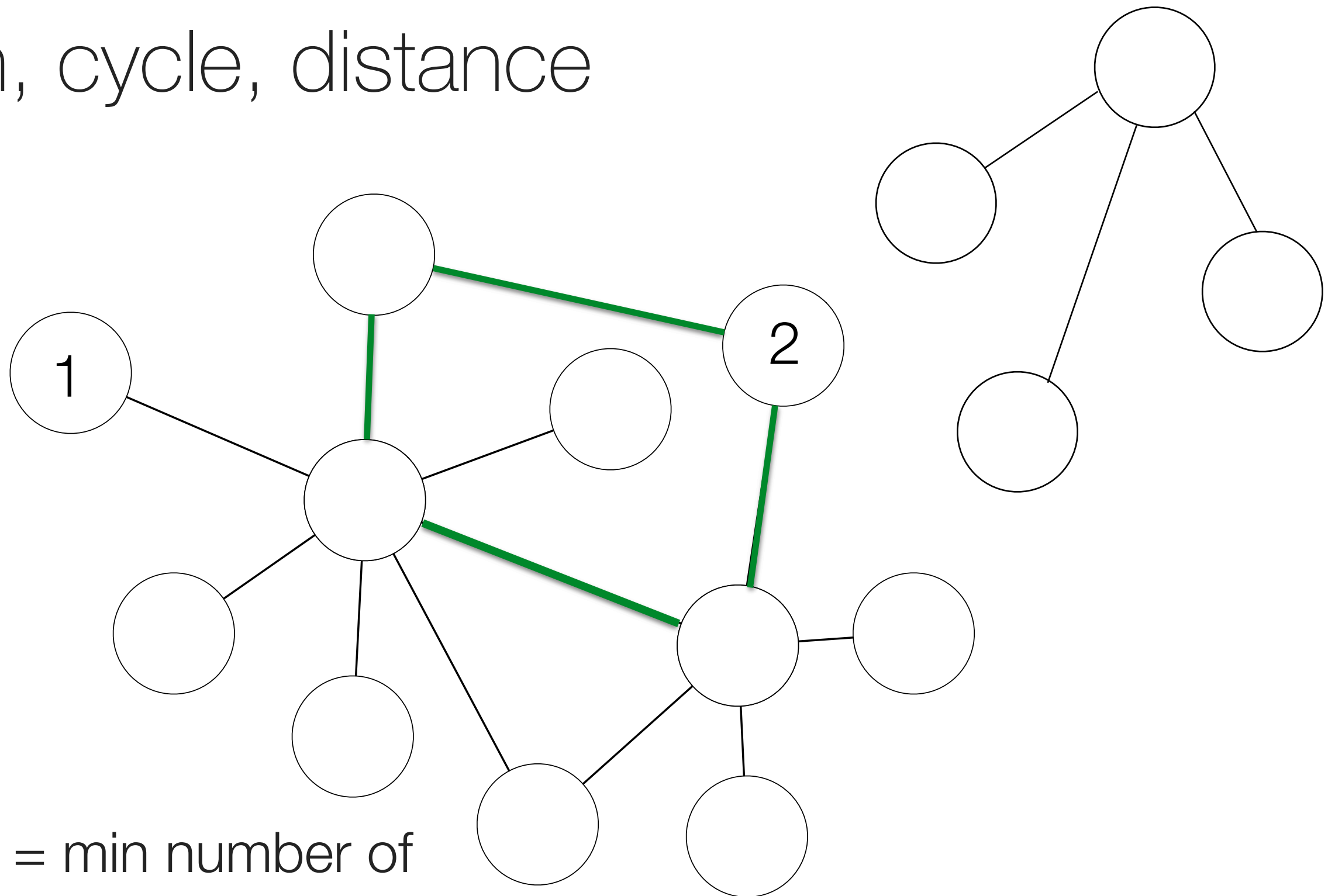
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Distance = min number of edges between two nodes

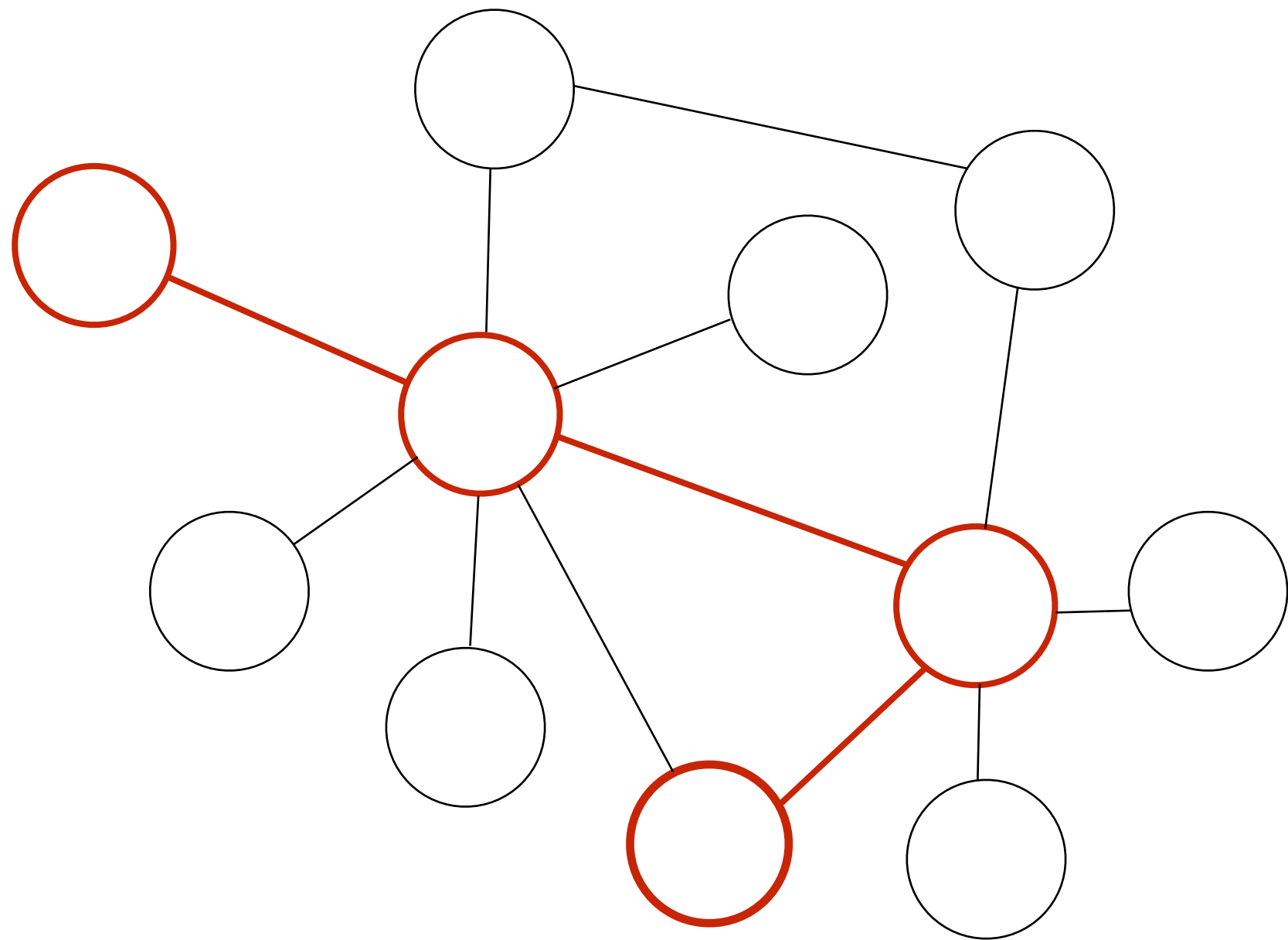


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# Subgraph



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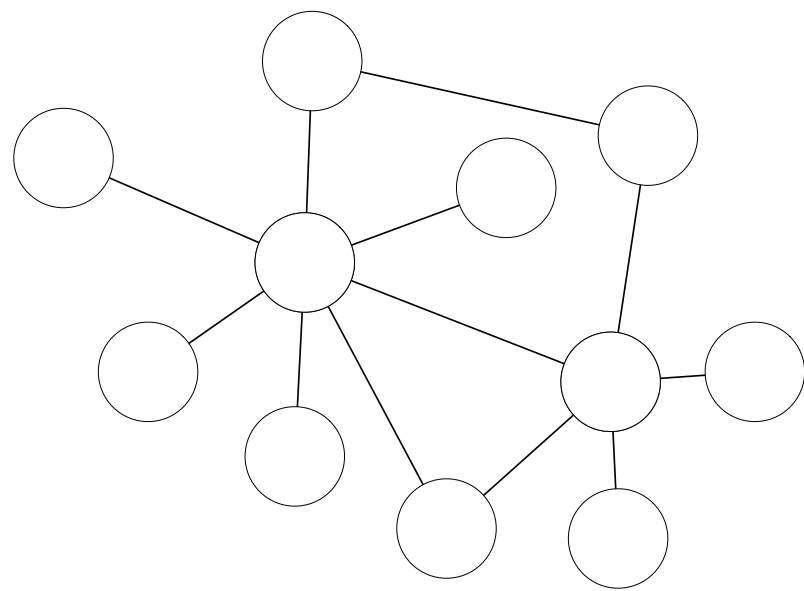
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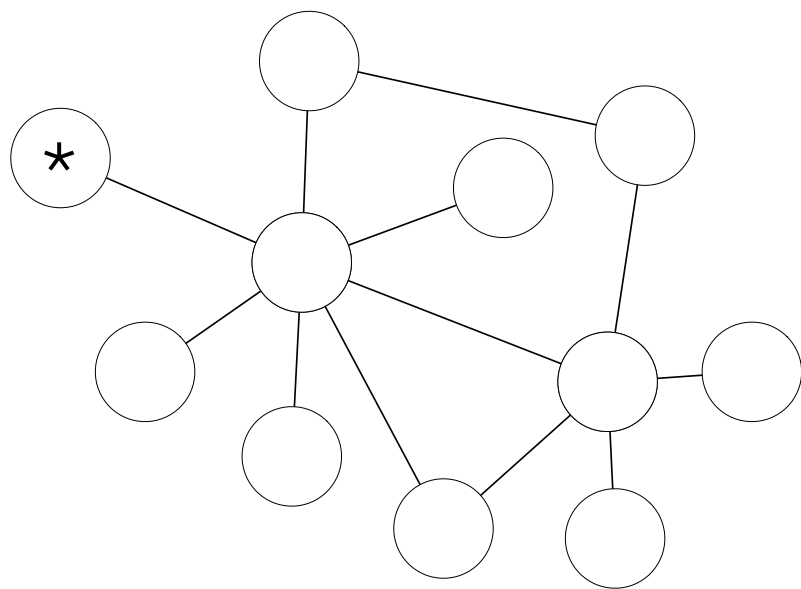


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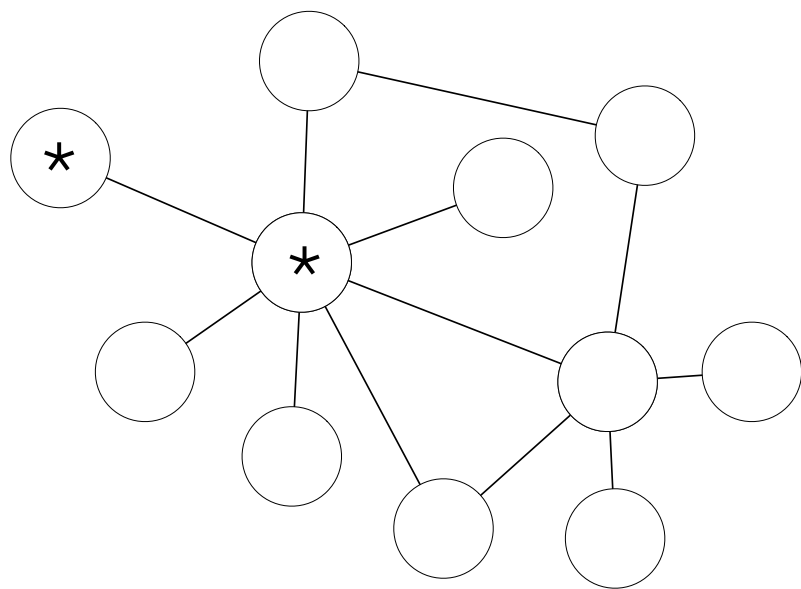


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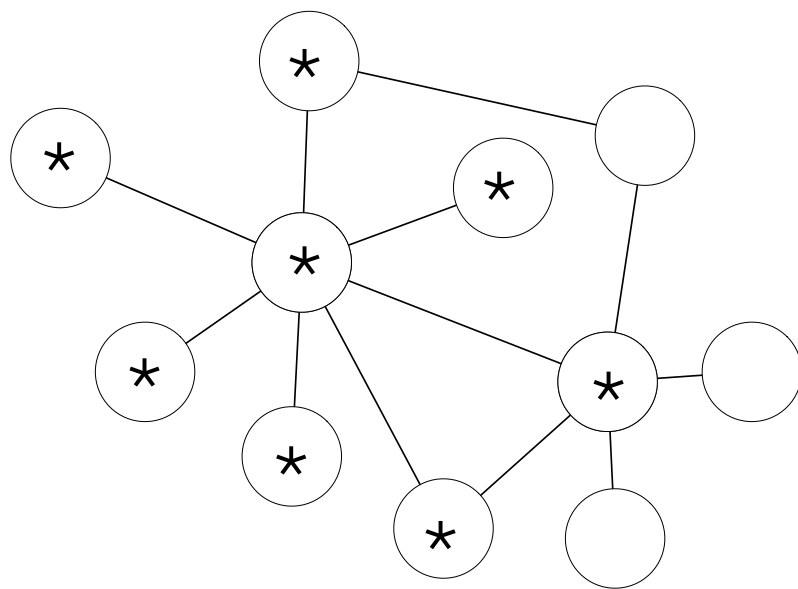


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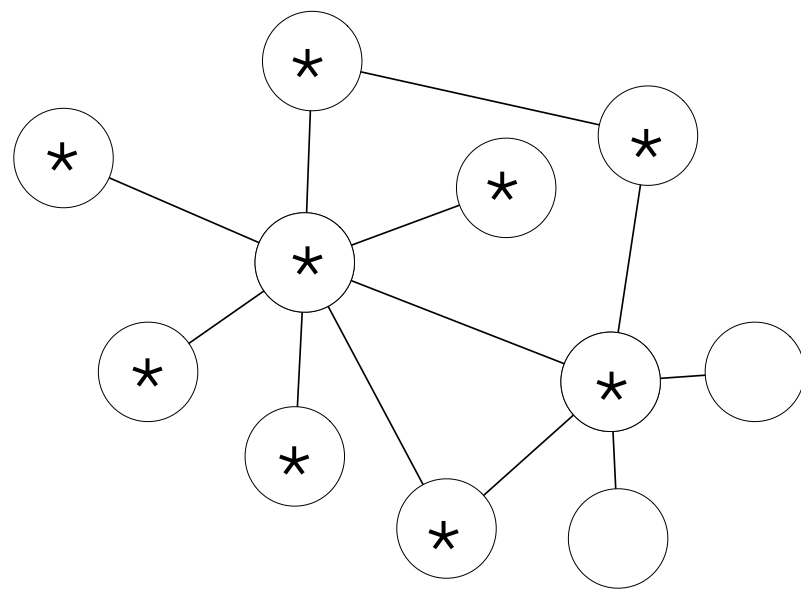


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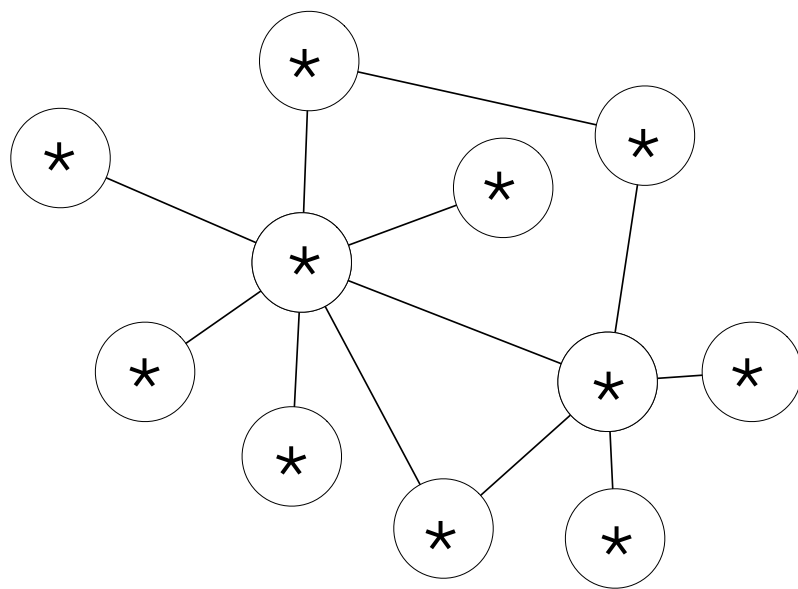


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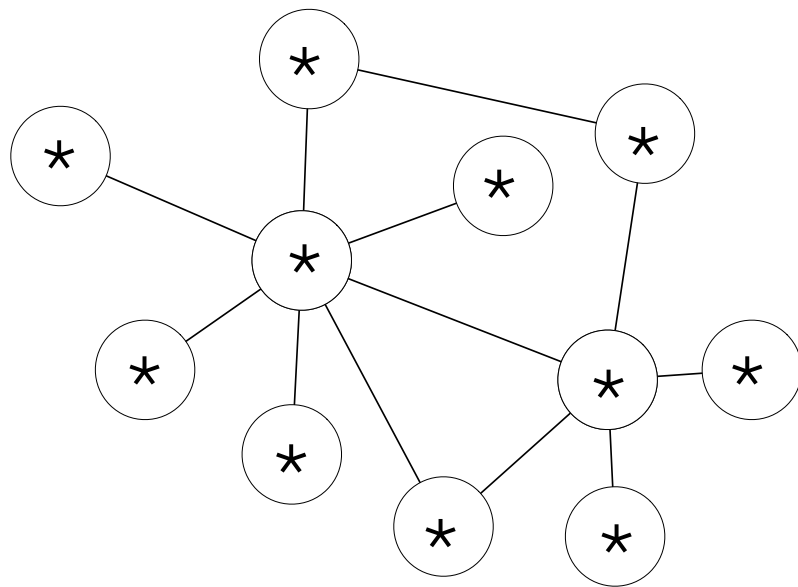


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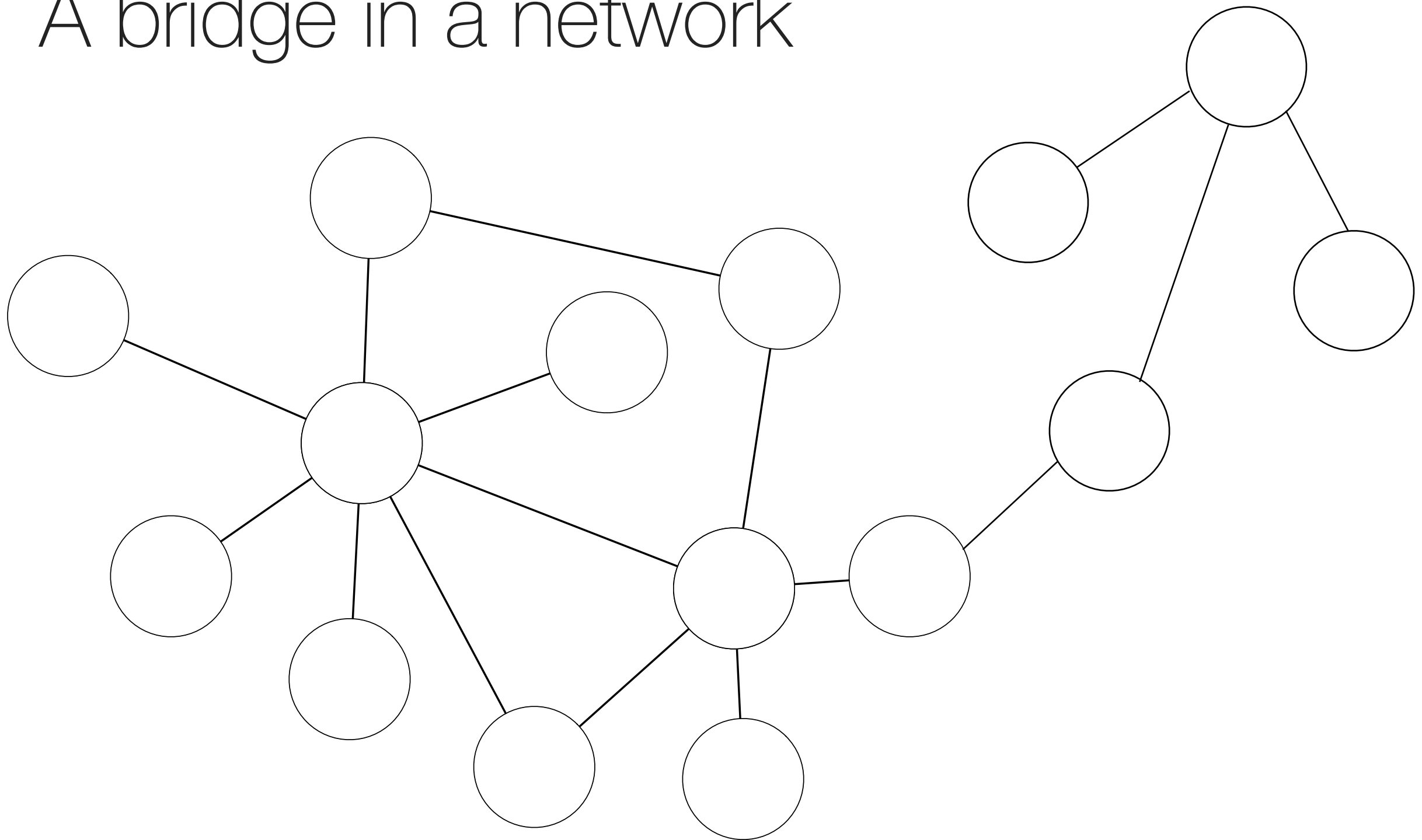
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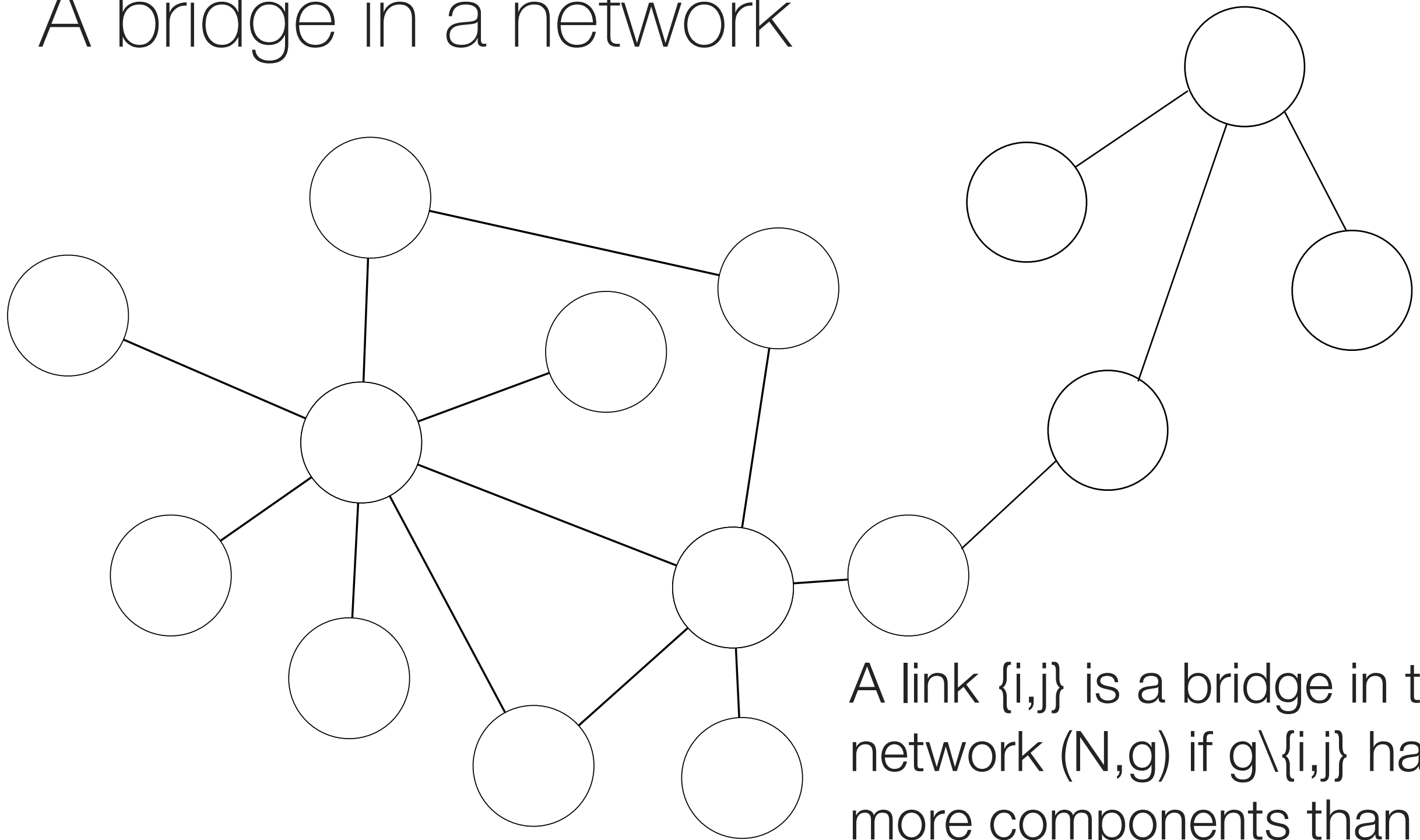


A network is connected iff it has one single component

# A bridge in a network

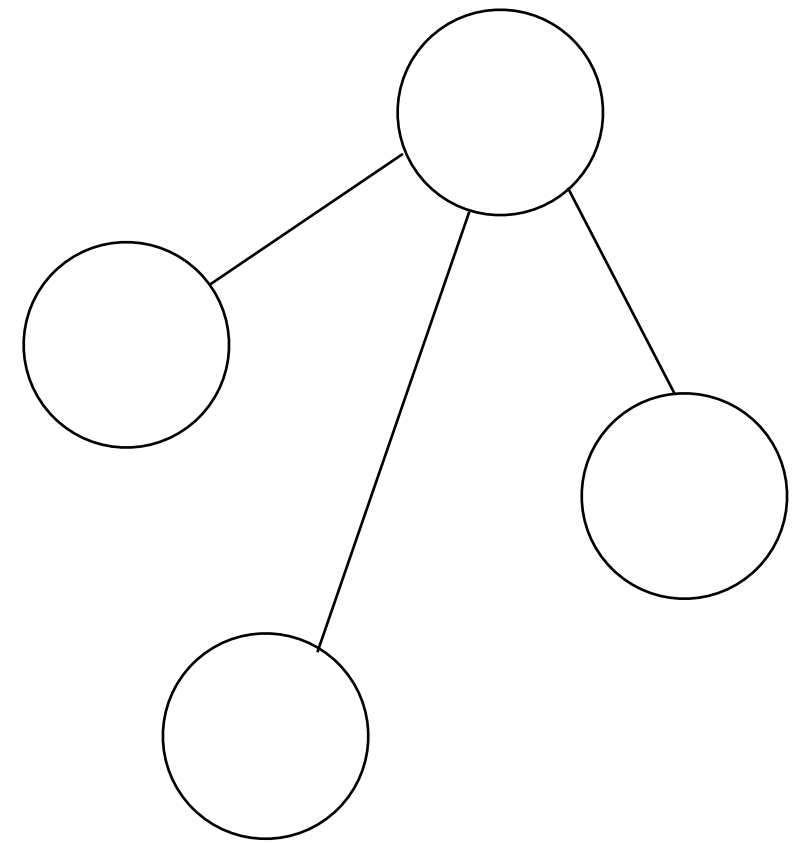
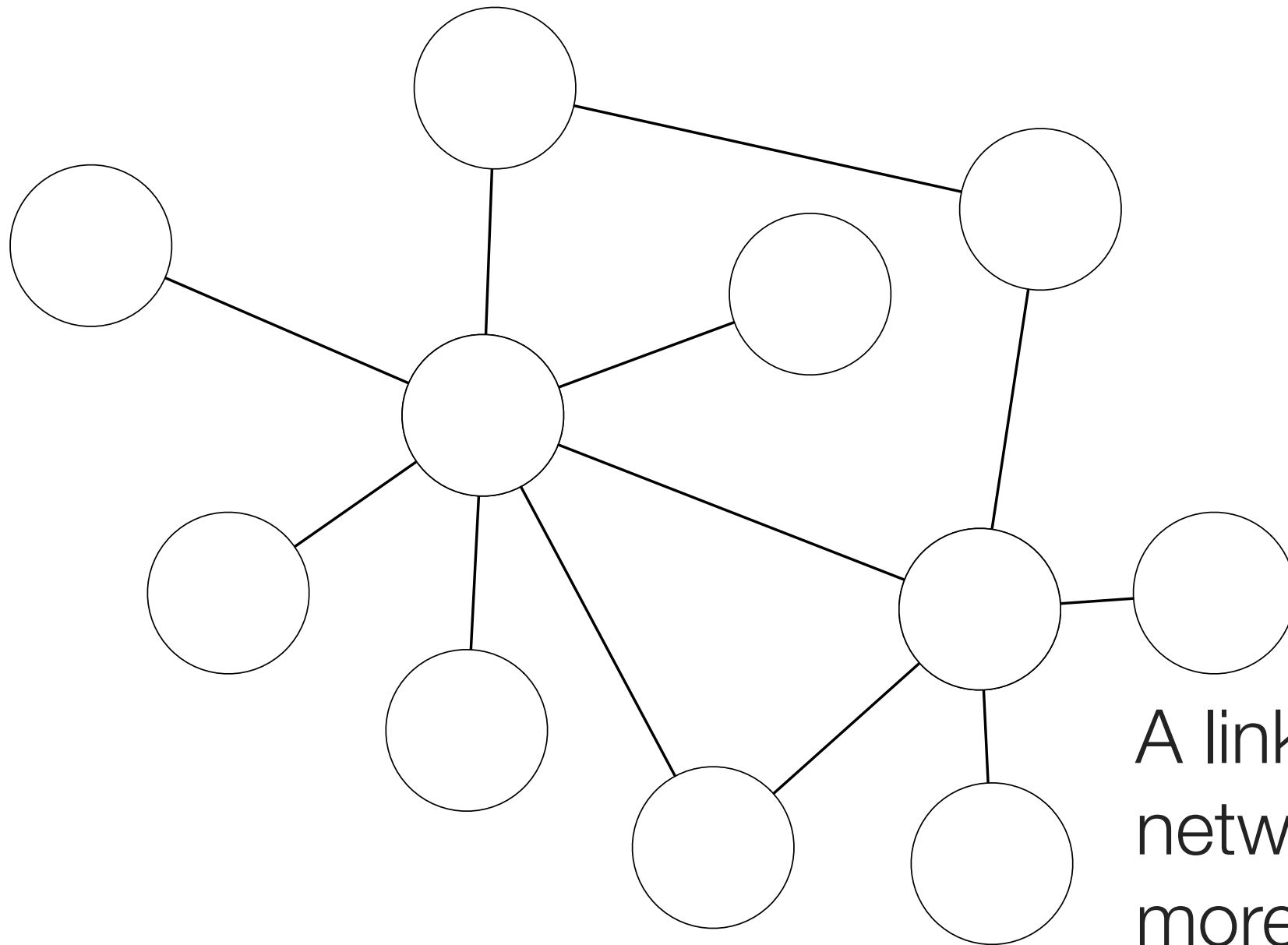


# A bridge in a network



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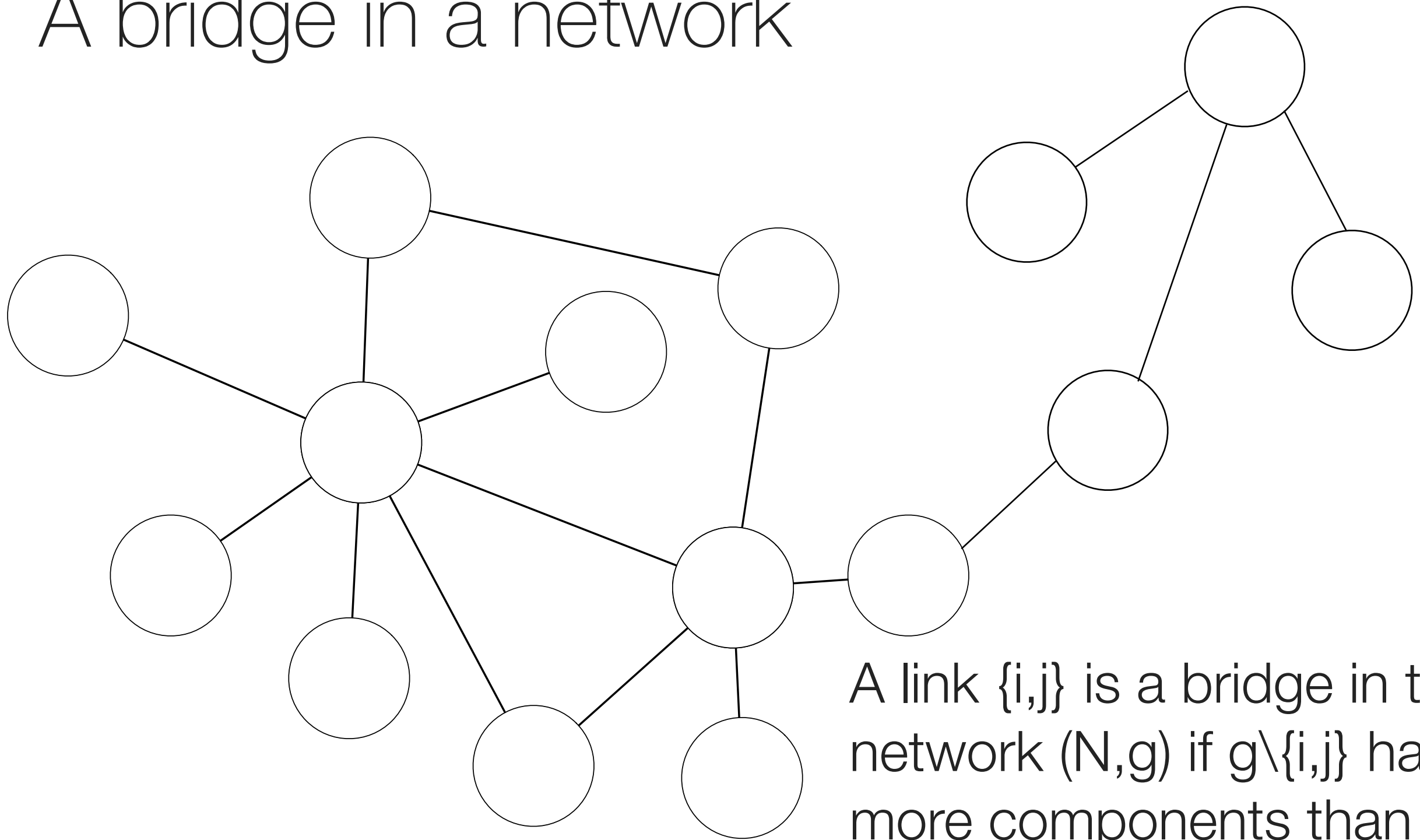
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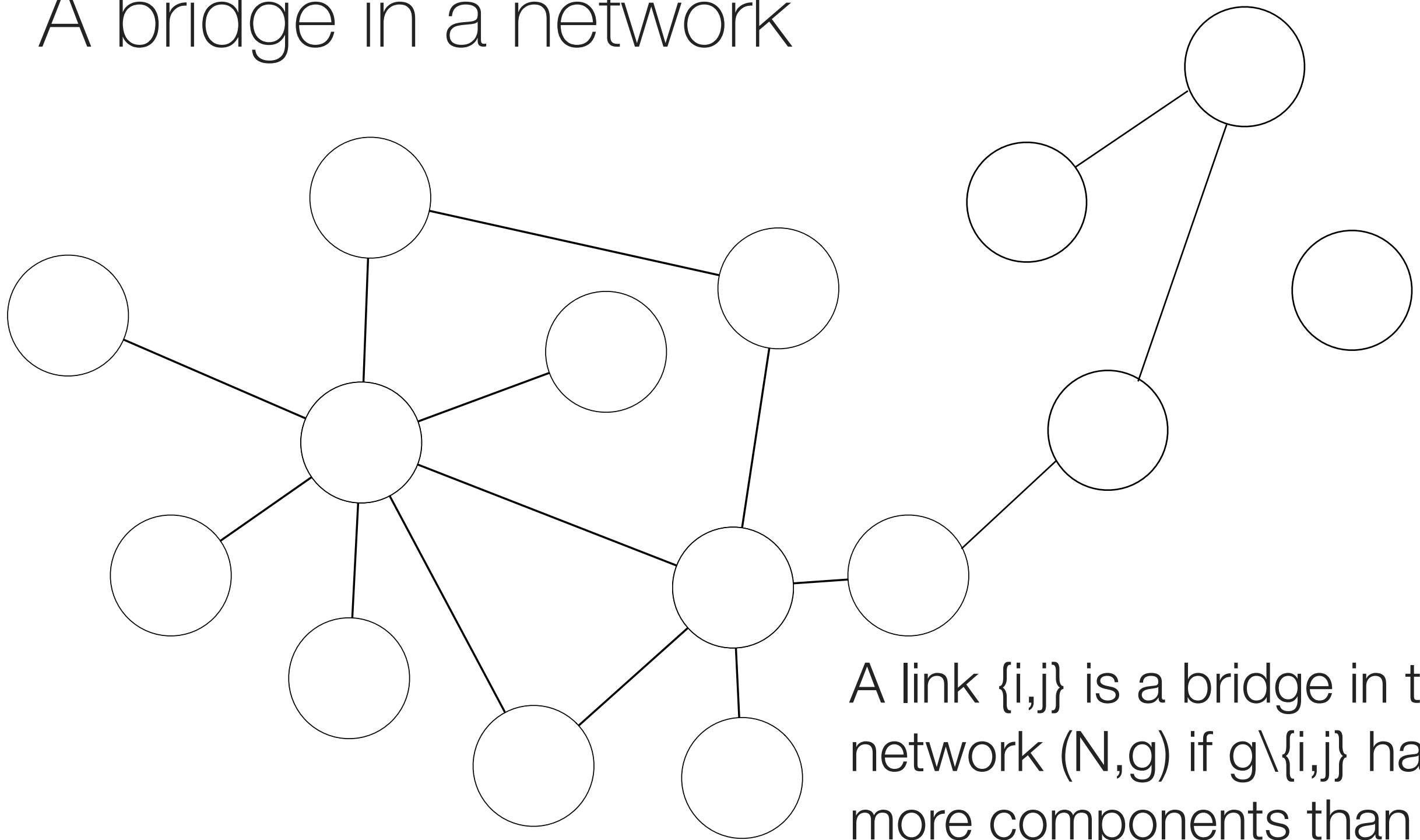


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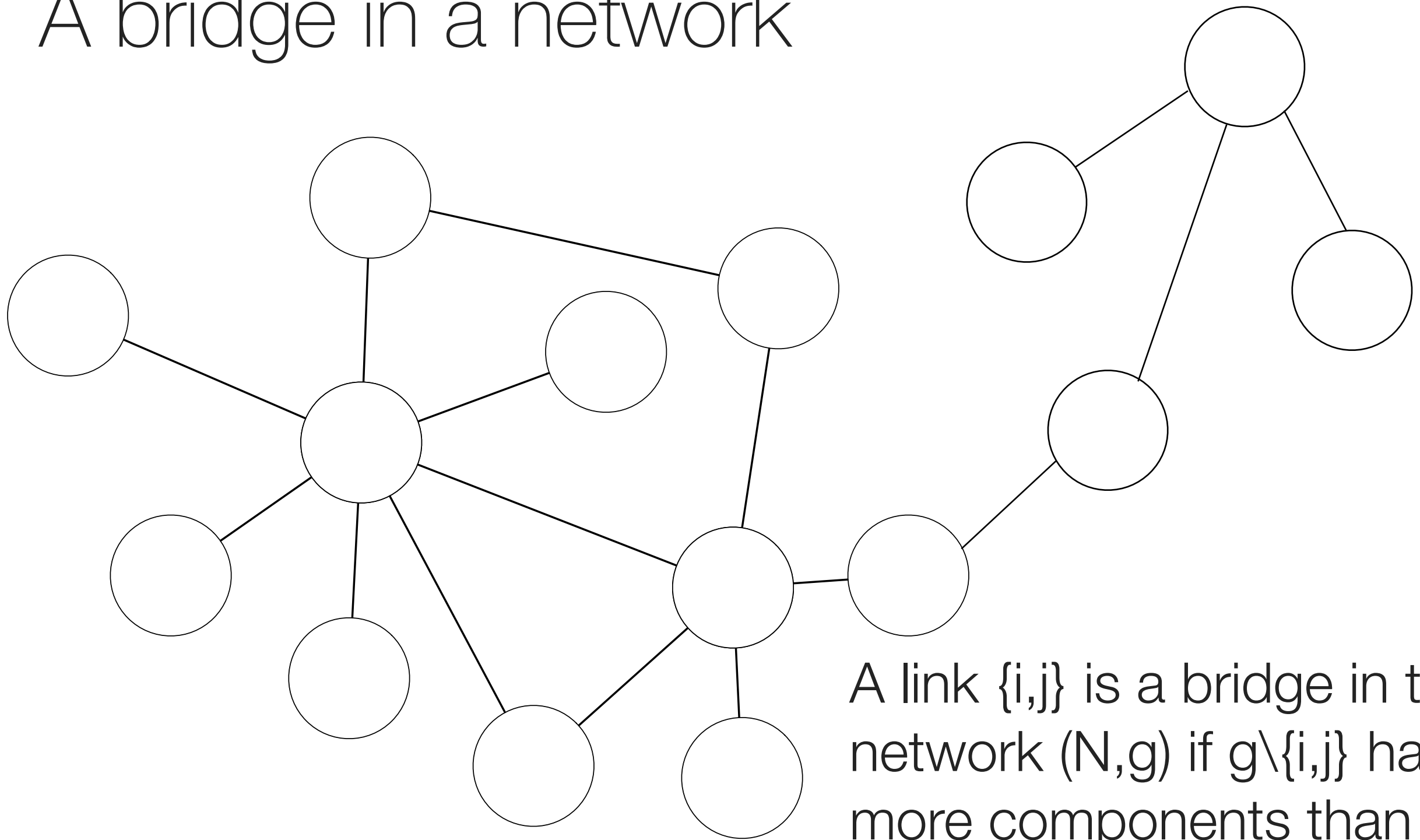
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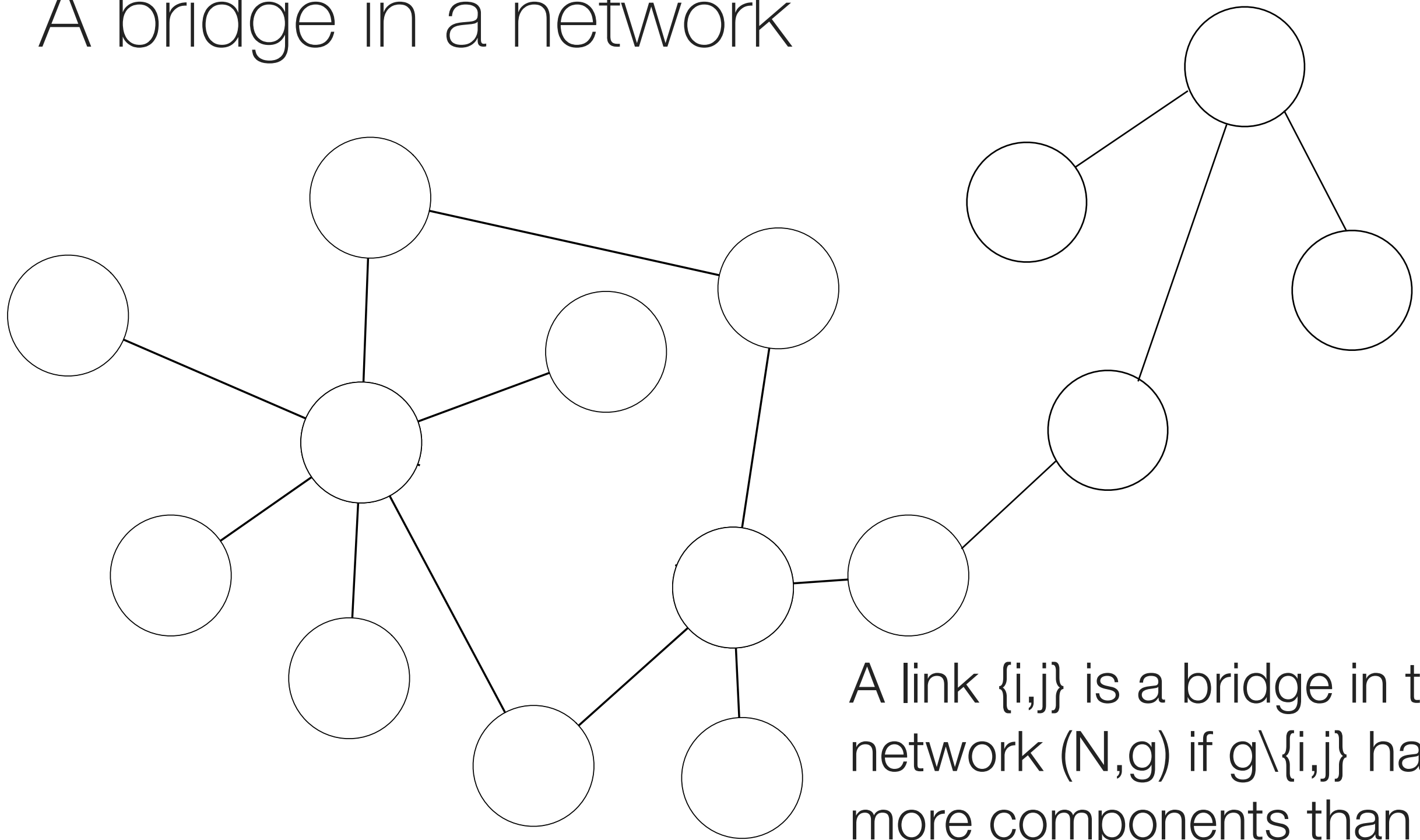
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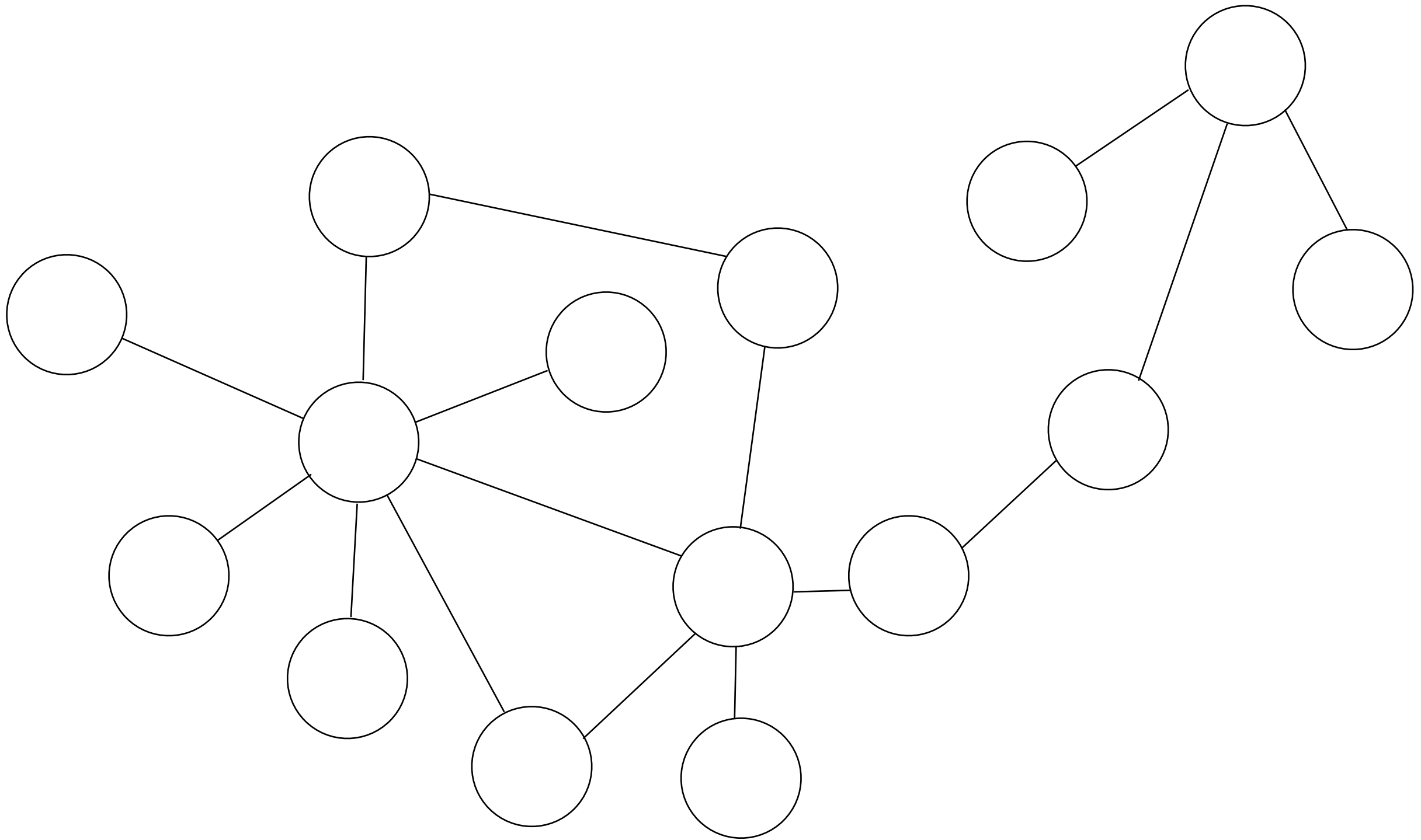
A link  $\{i,j\}$  is a bridge in the network  $(N,g)$  if  $g \setminus \{i,j\}$  has more components than  $(N,g)$

# A bridge in a network

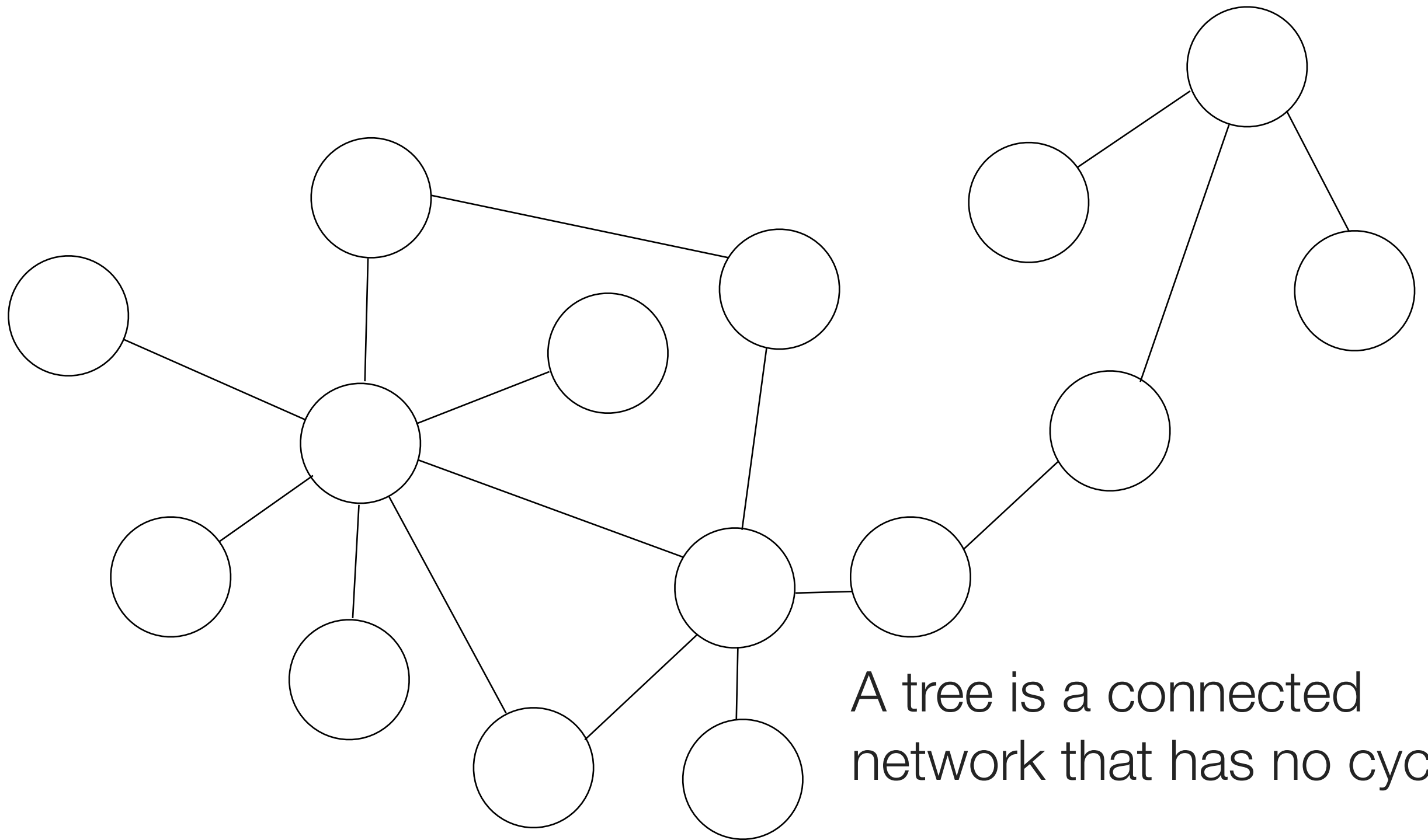


A link  $\{i,j\}$  is a bridge in the network  $(N,g)$  if  $g \setminus \{i,j\}$  has more components than  $(N,g)$

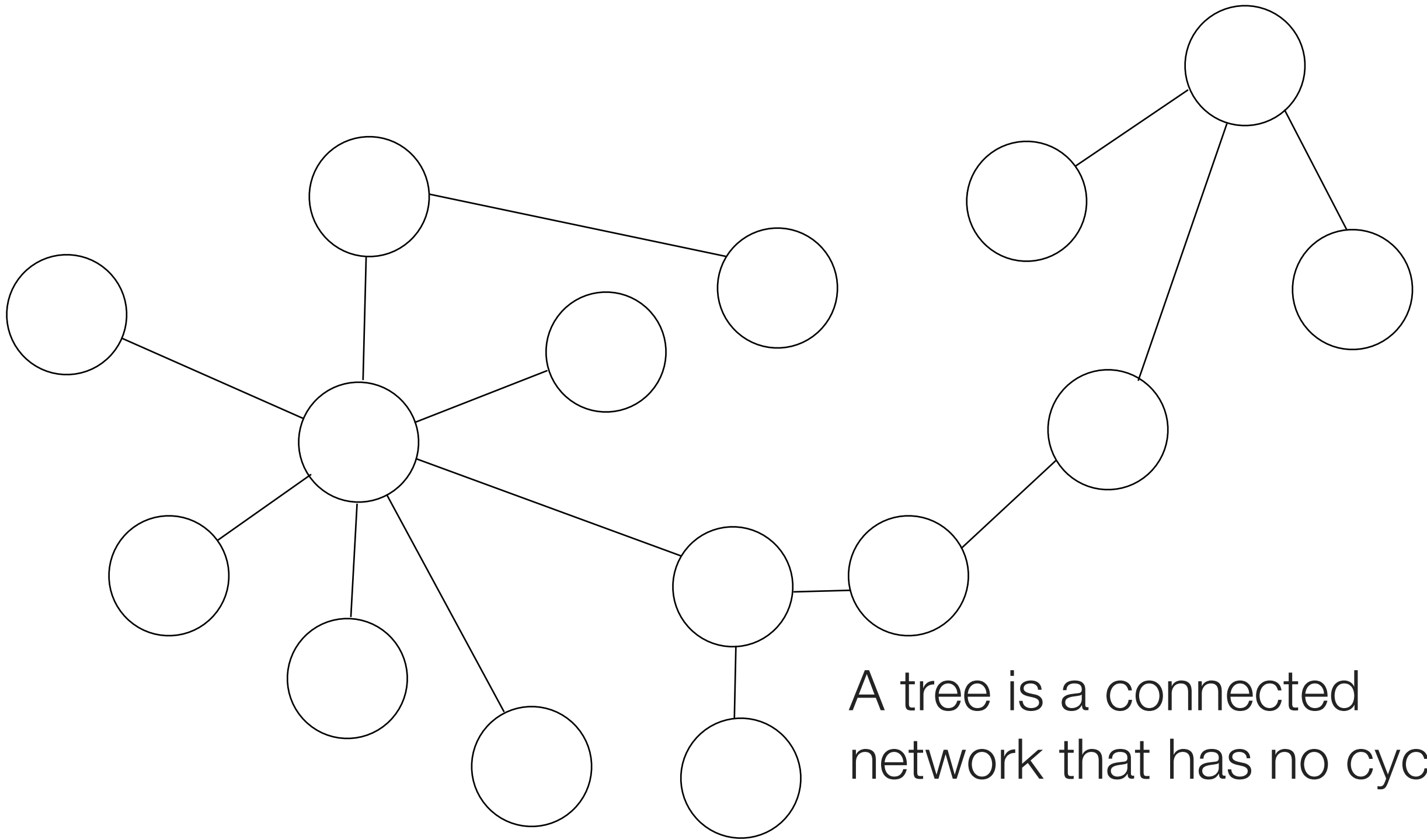
# Trees, stars, circles, complete networks



# Trees, stars, circles, complete networks

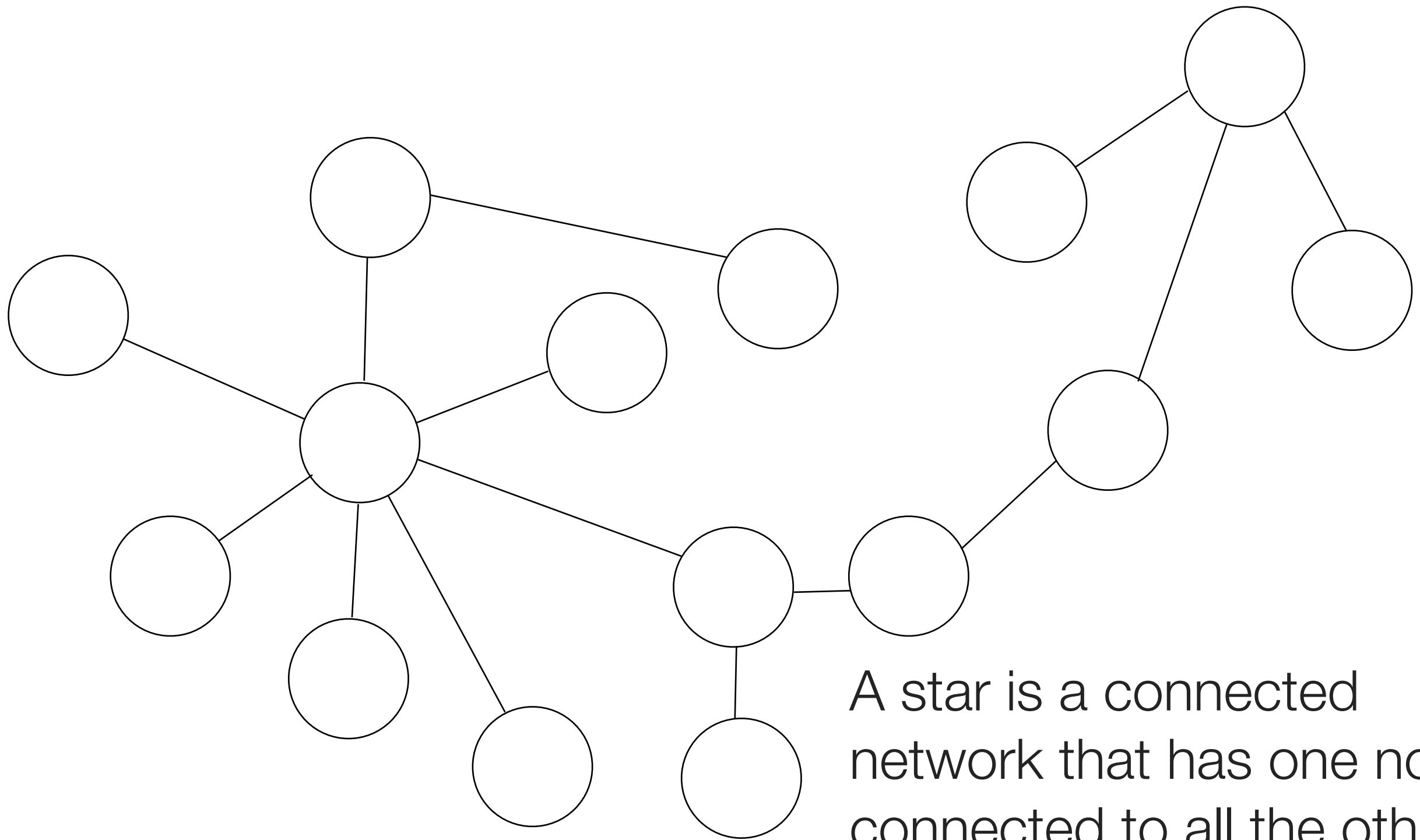


# Trees, stars, circles, complete networks



A tree is a connected network that has no cycles

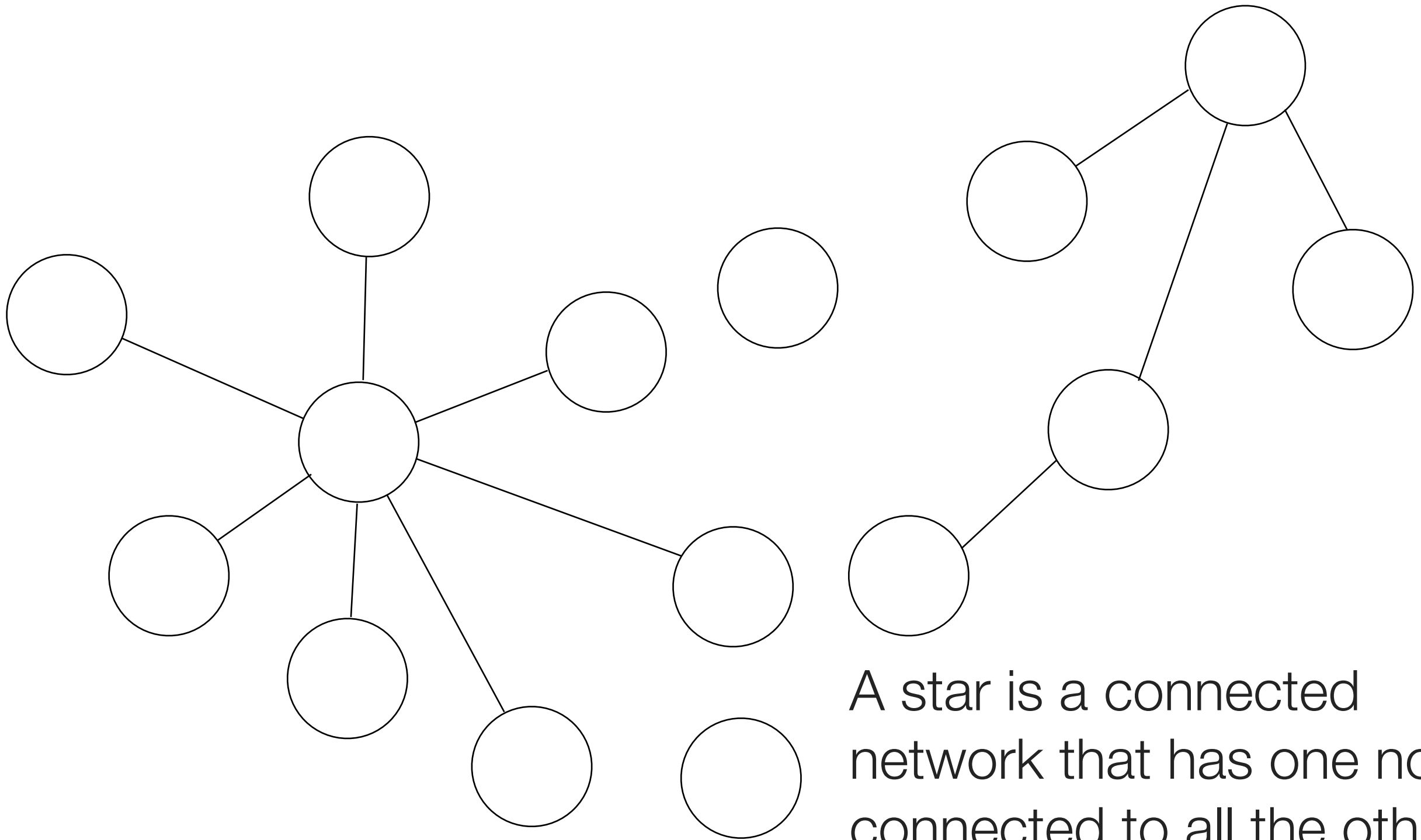
# Trees, stars, circles, complete networks



A star is a connected network that has one node connected to all the other nodes

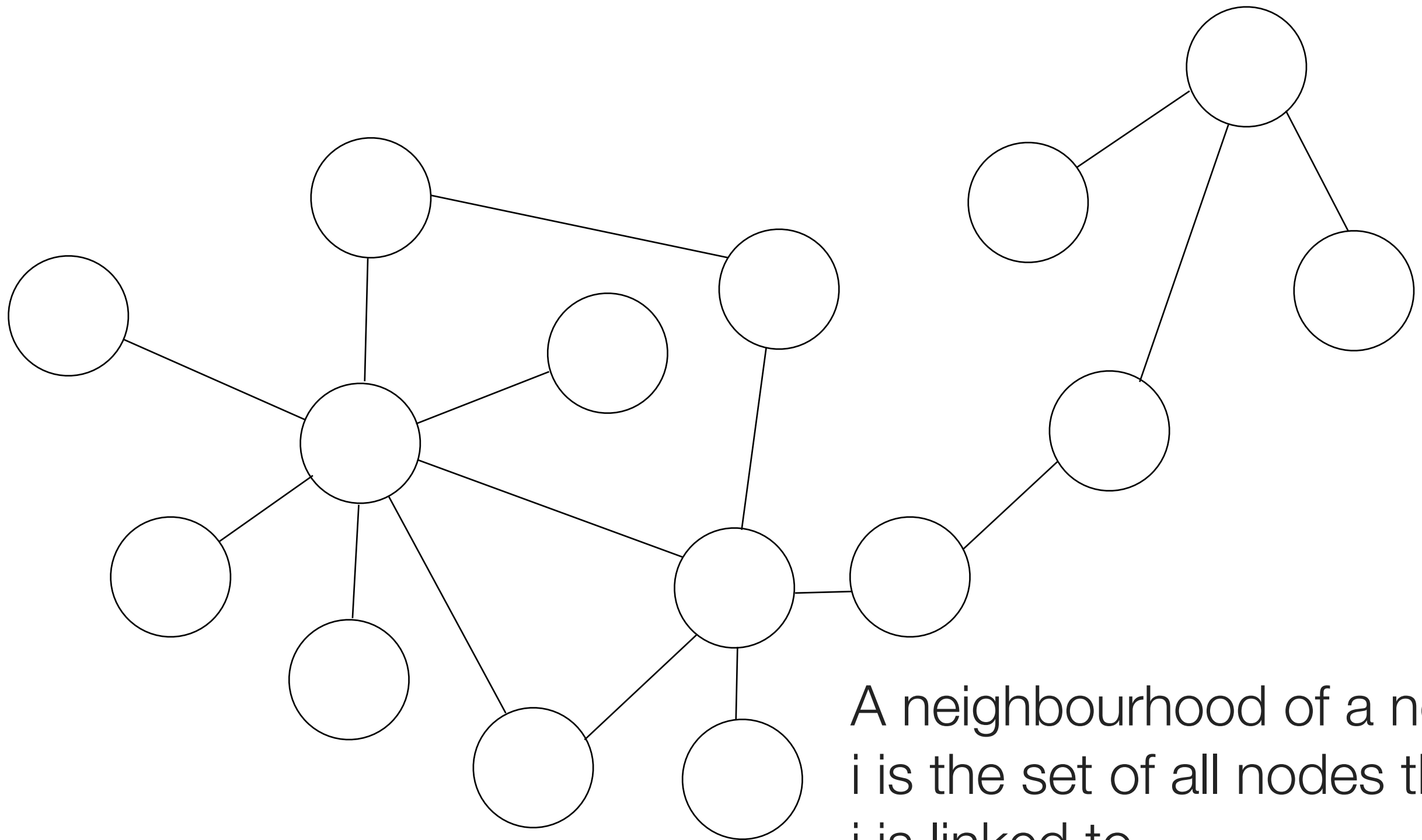


# Trees, stars, circles, complete networks



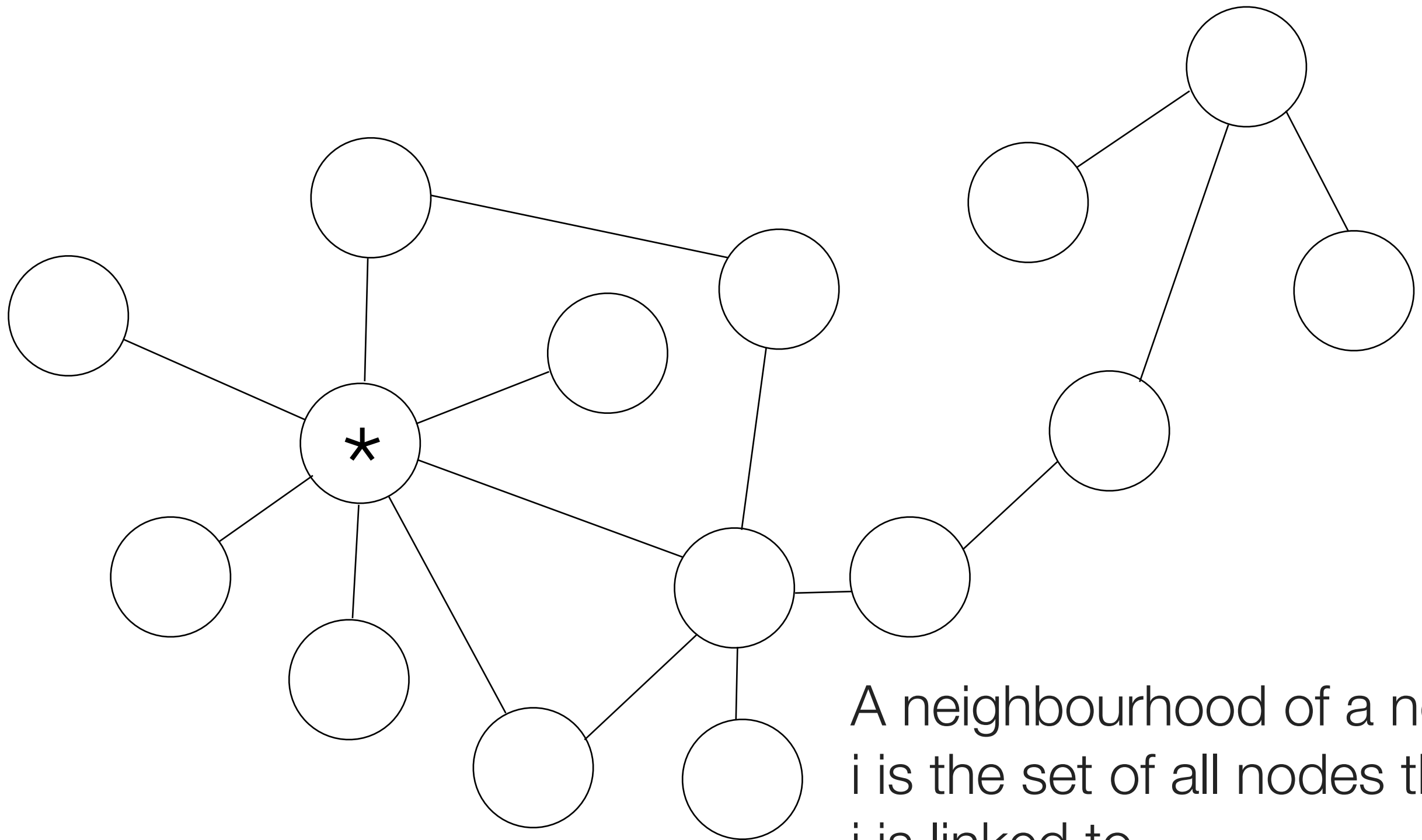
A star is a connected network that has one node connected to all the other nodes

# Neighbourhood



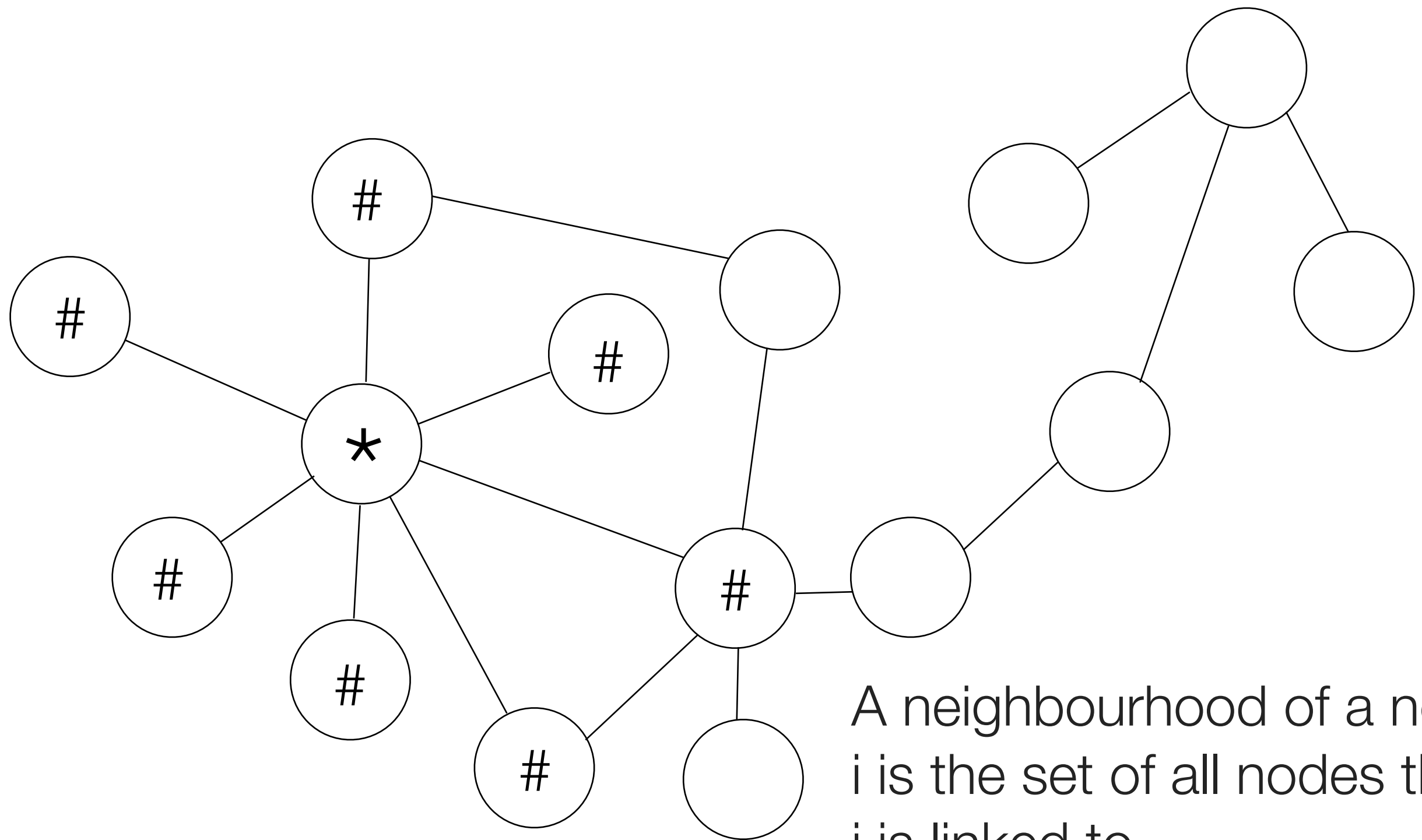
A neighbourhood of a node  $i$  is the set of all nodes that  $i$  is linked to

# Neighbourhood



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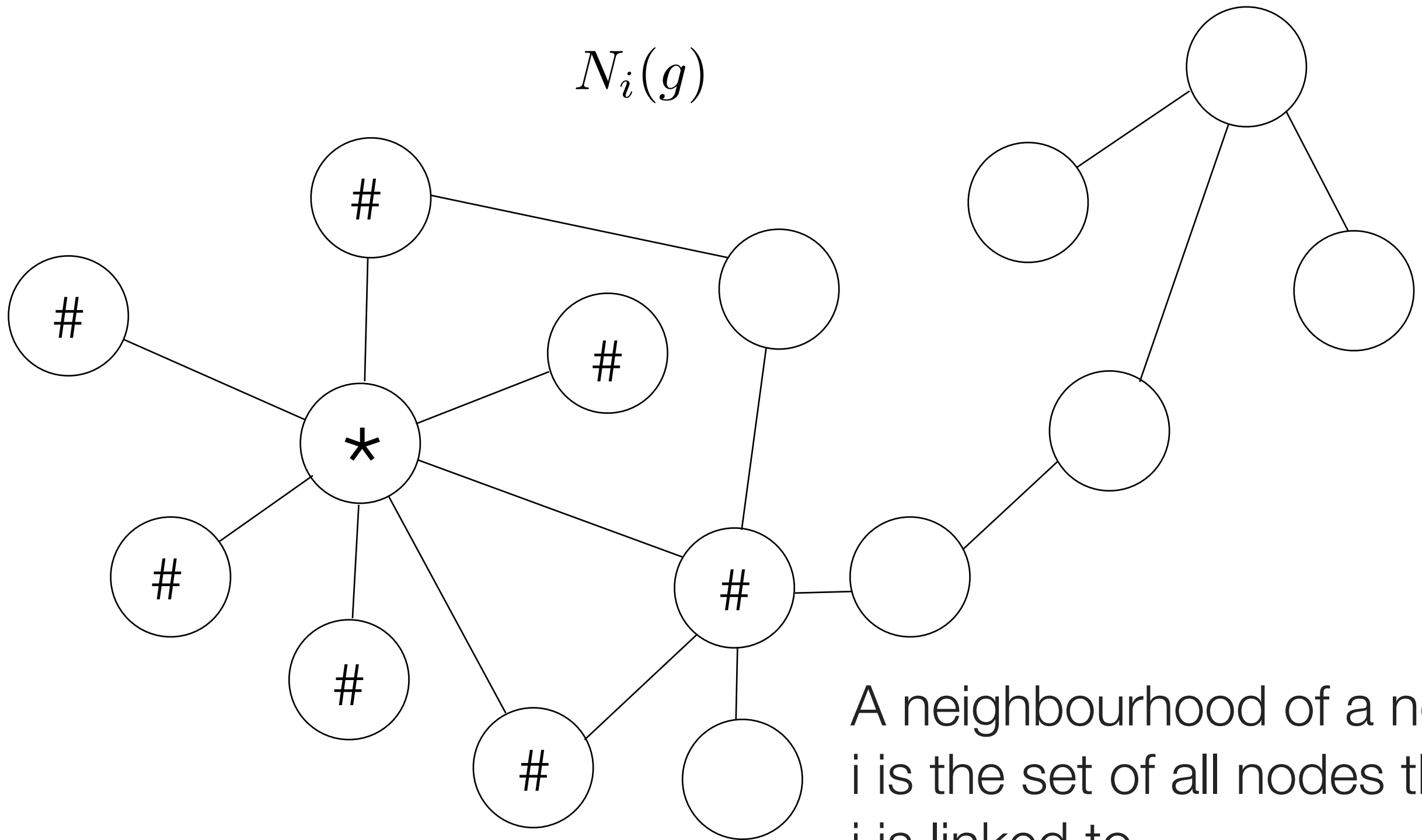
# Neighbourhood



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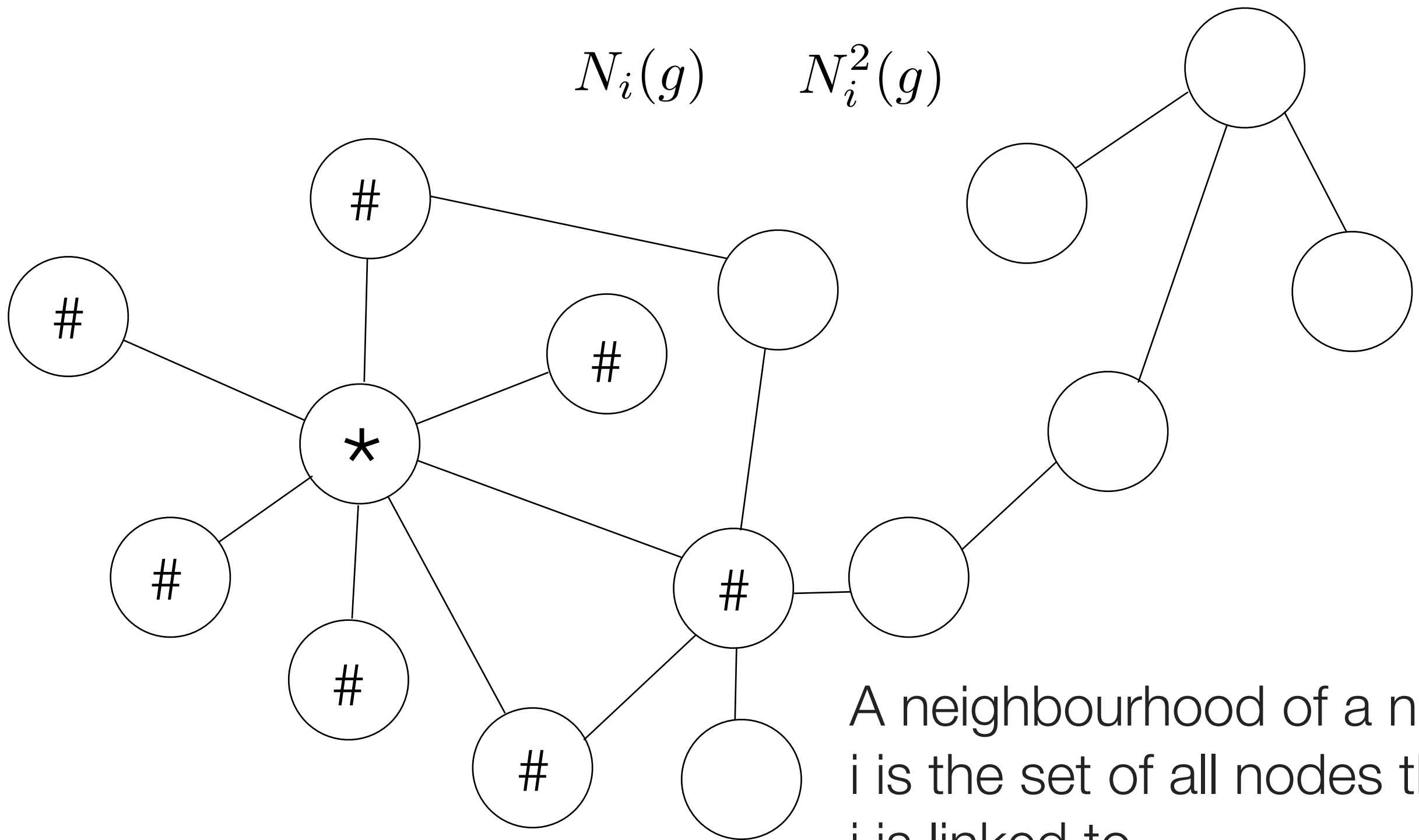
$$N_i(g)$$



A neighbourhood of a node  $i$  is the set of all nodes that  $i$  is linked to

# Neighbourhood

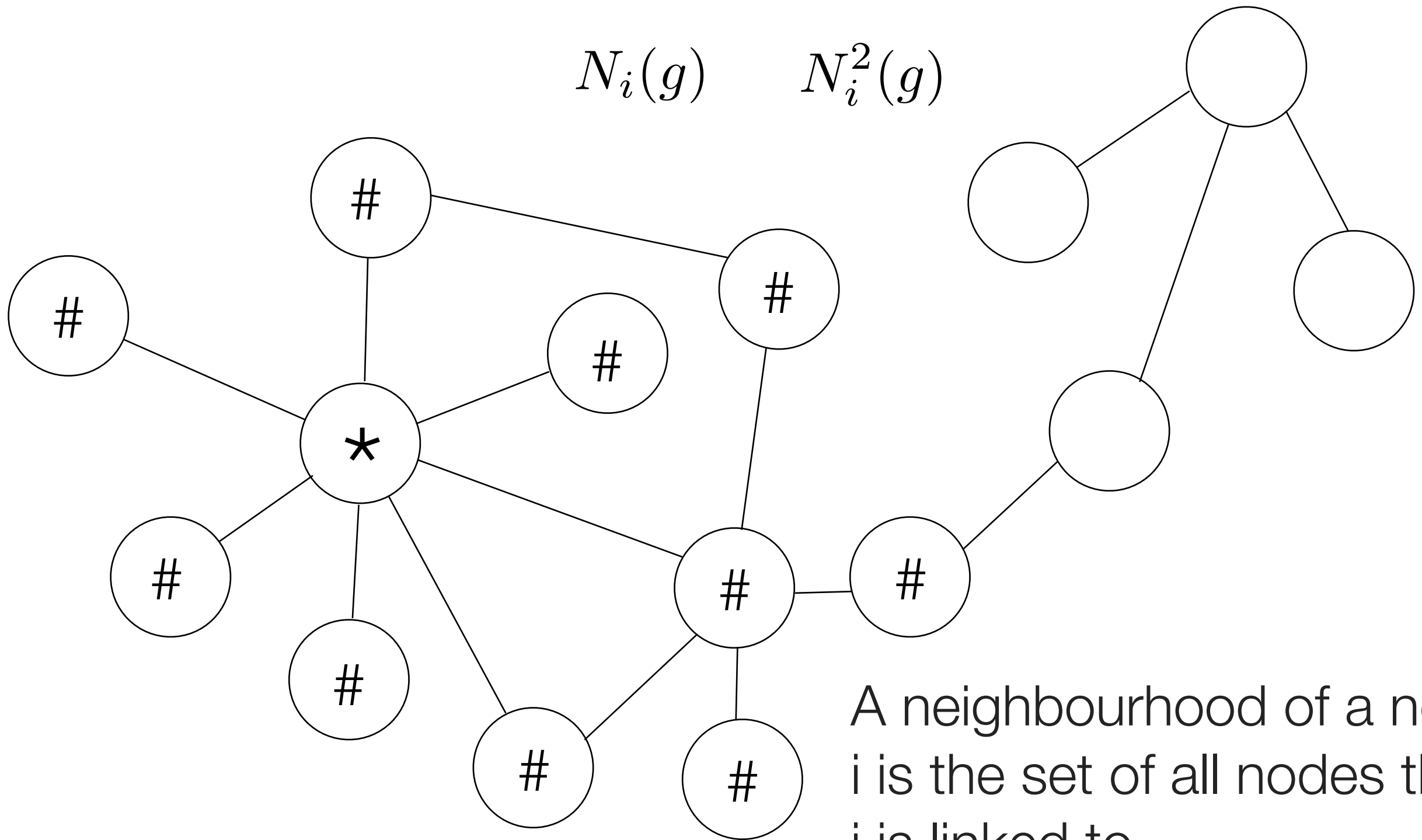
$$N_i(g) \quad N_i^2(g)$$



A neighbourhood of a node  $i$  is the set of all nodes that  $i$  is linked to

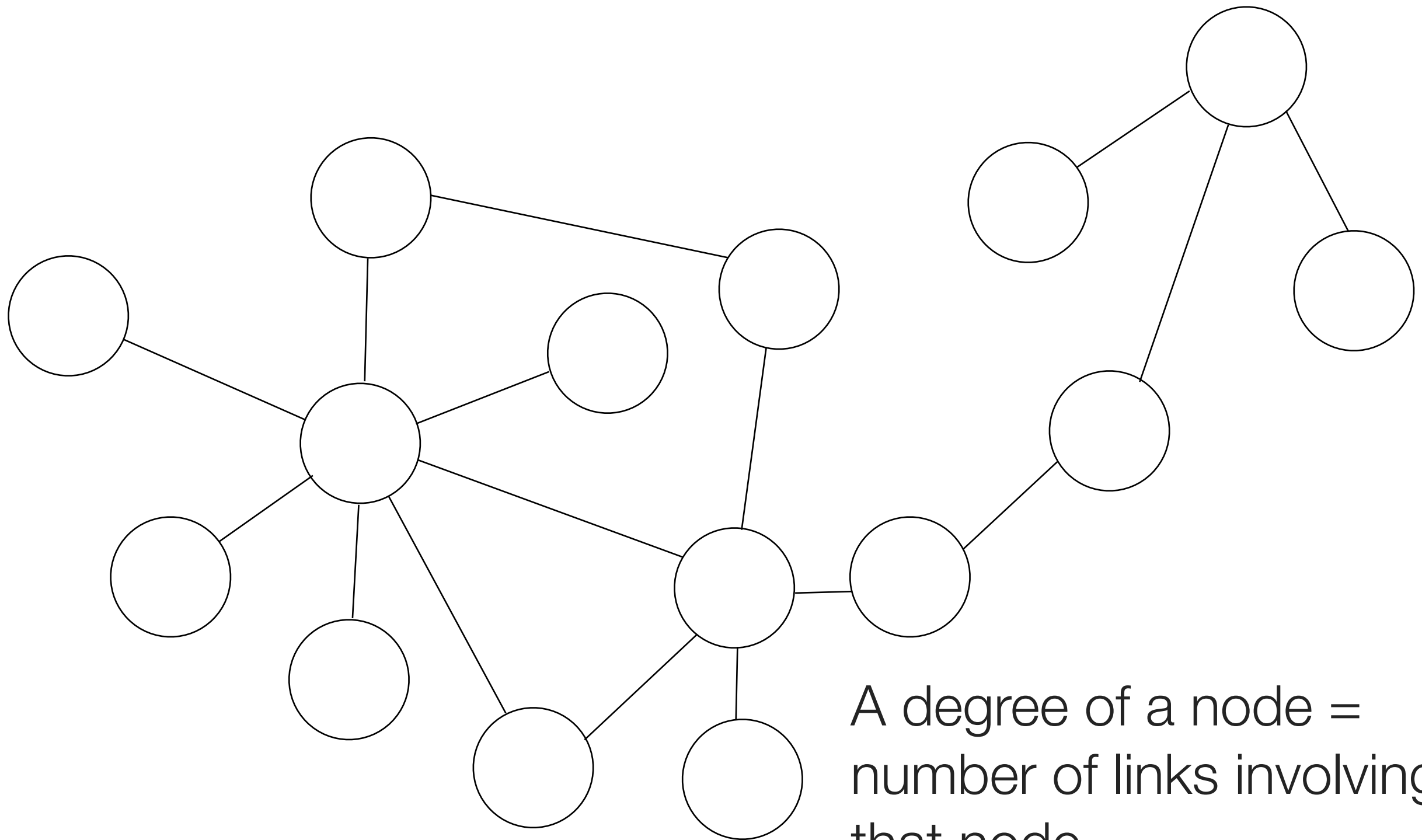
# Neighbourhood

$$N_i(g) \quad N_i^2(g)$$



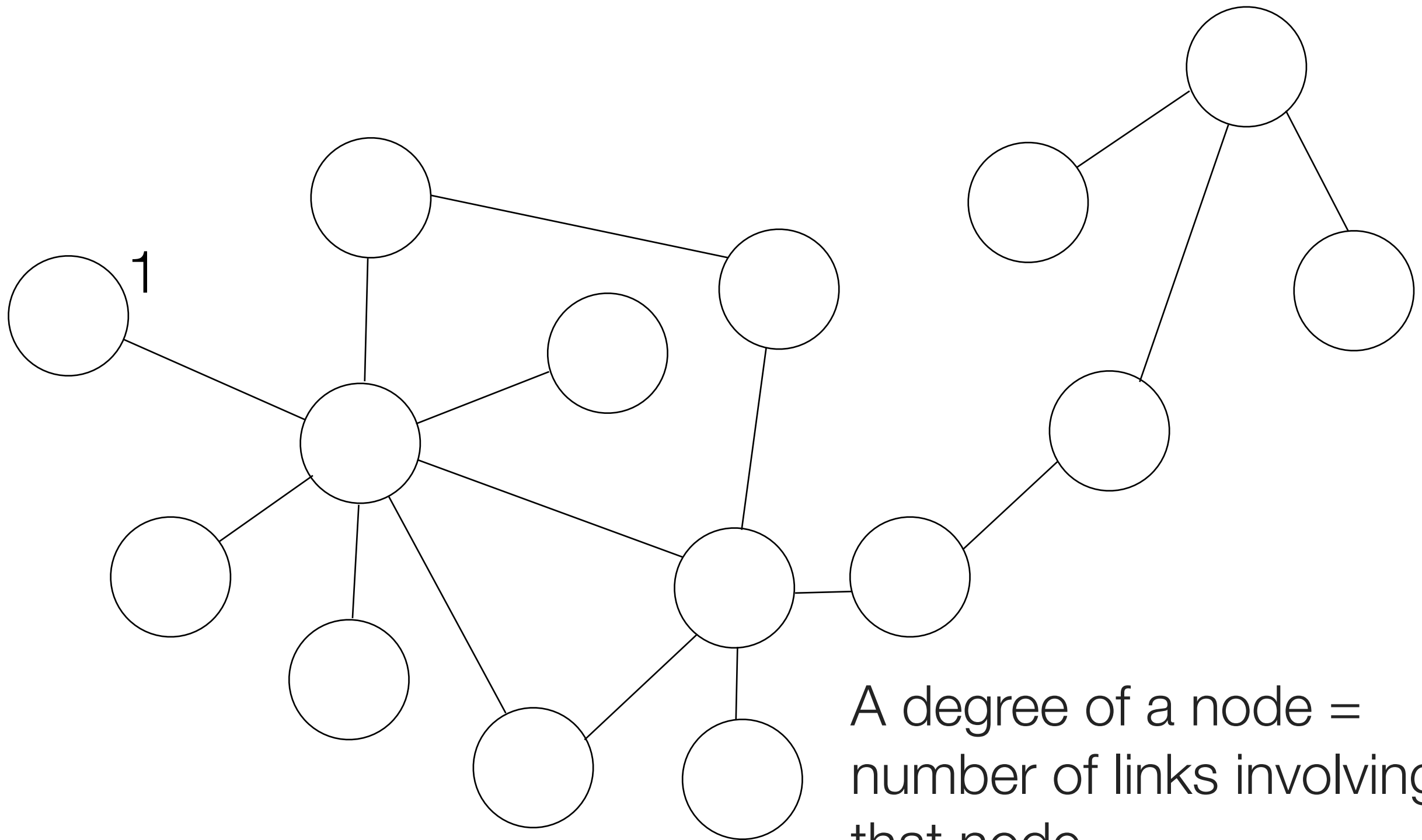
A neighbourhood of a node  $i$  is the set of all nodes that  $i$  is linked to

# Degree, network density

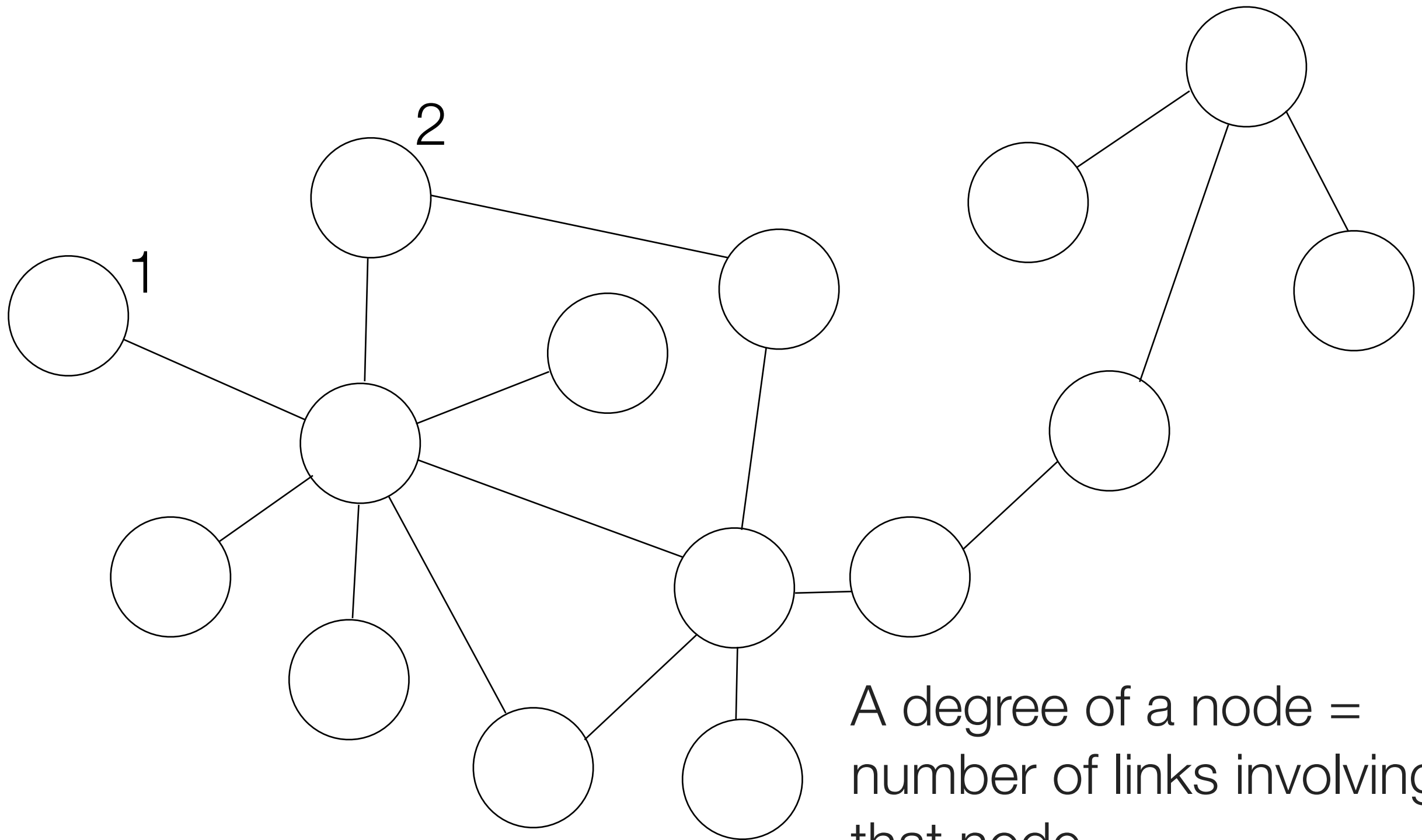




# Degree, network density

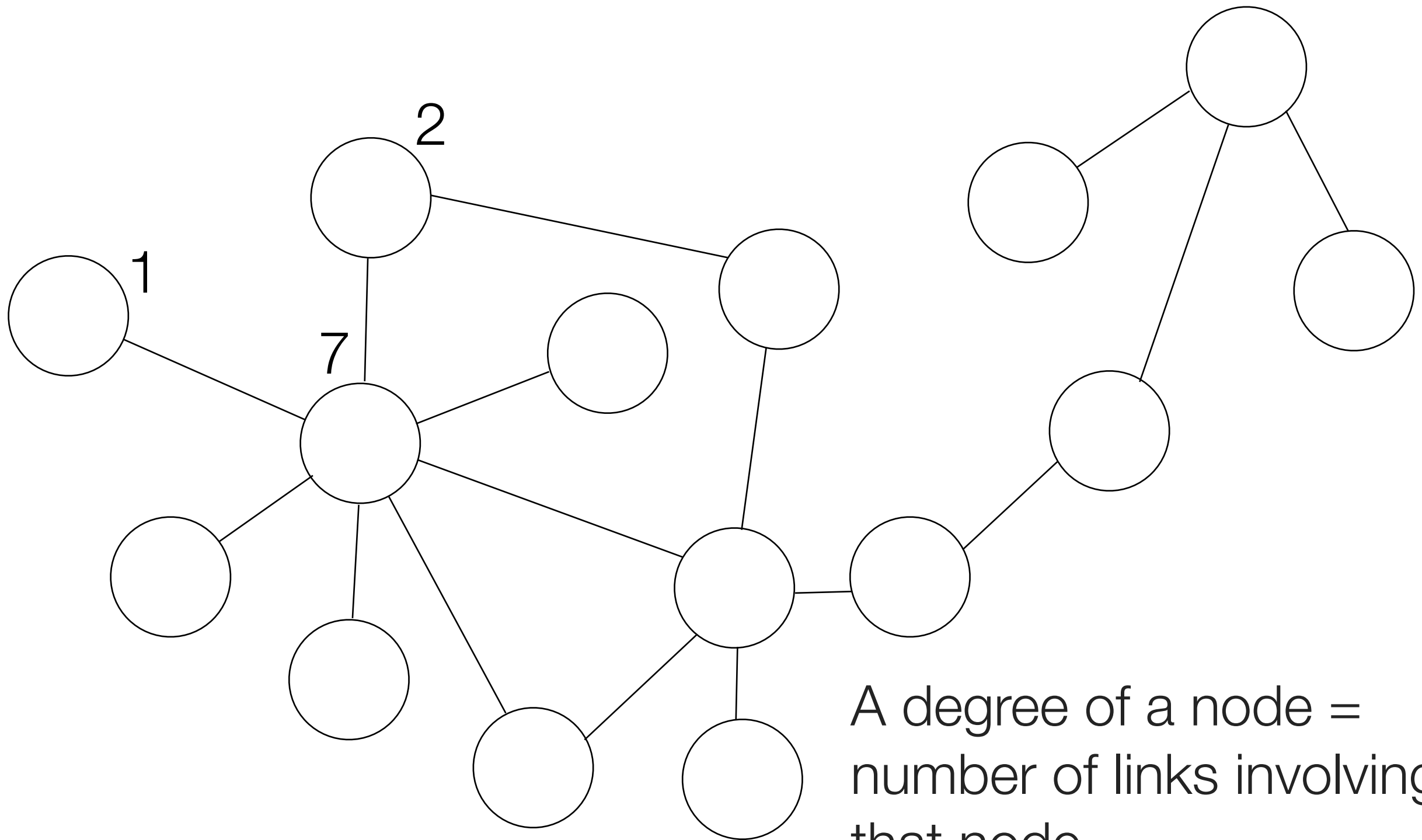


# Degree, network density



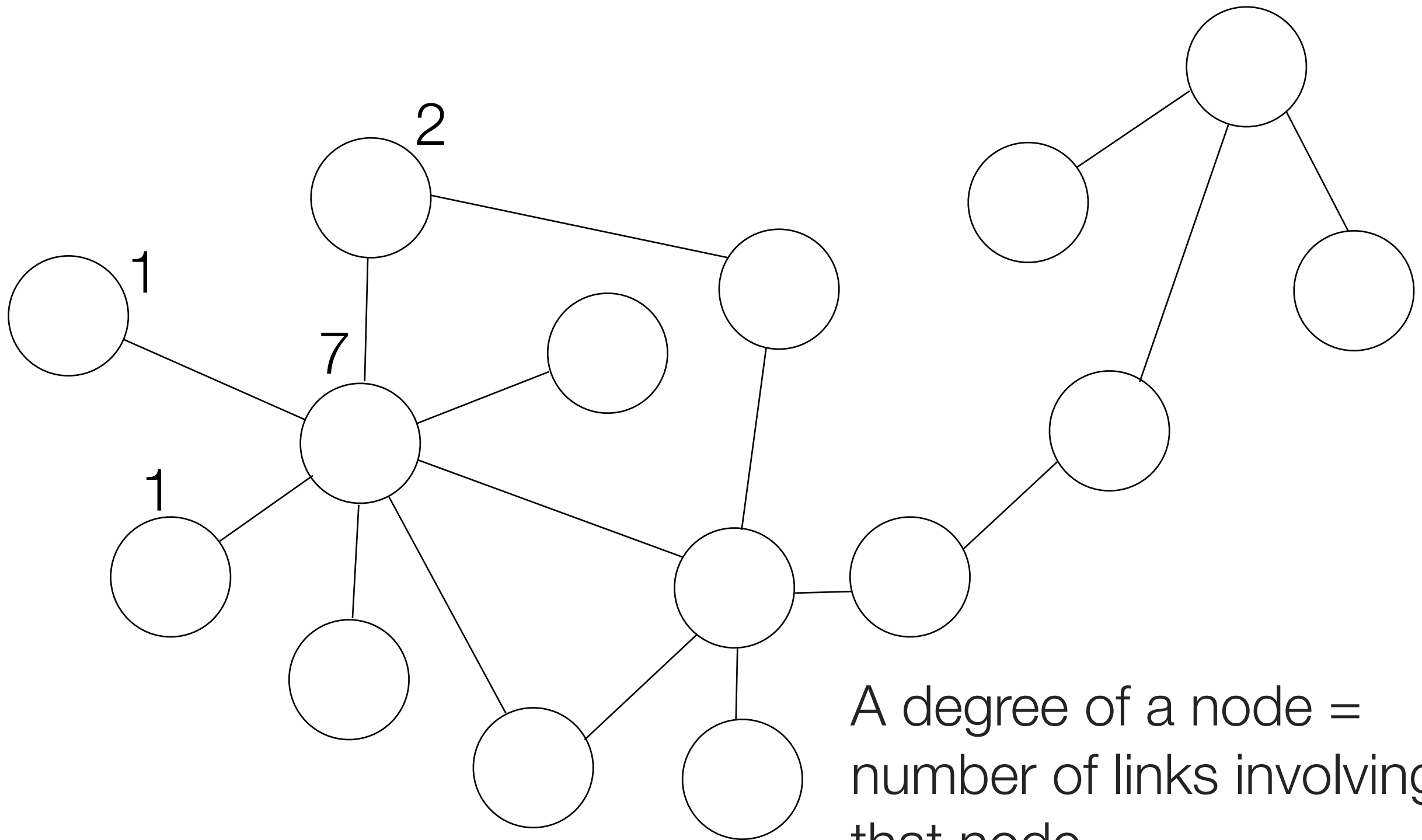
A degree of a node =  
number of links involving  
that node

# Degree, network density



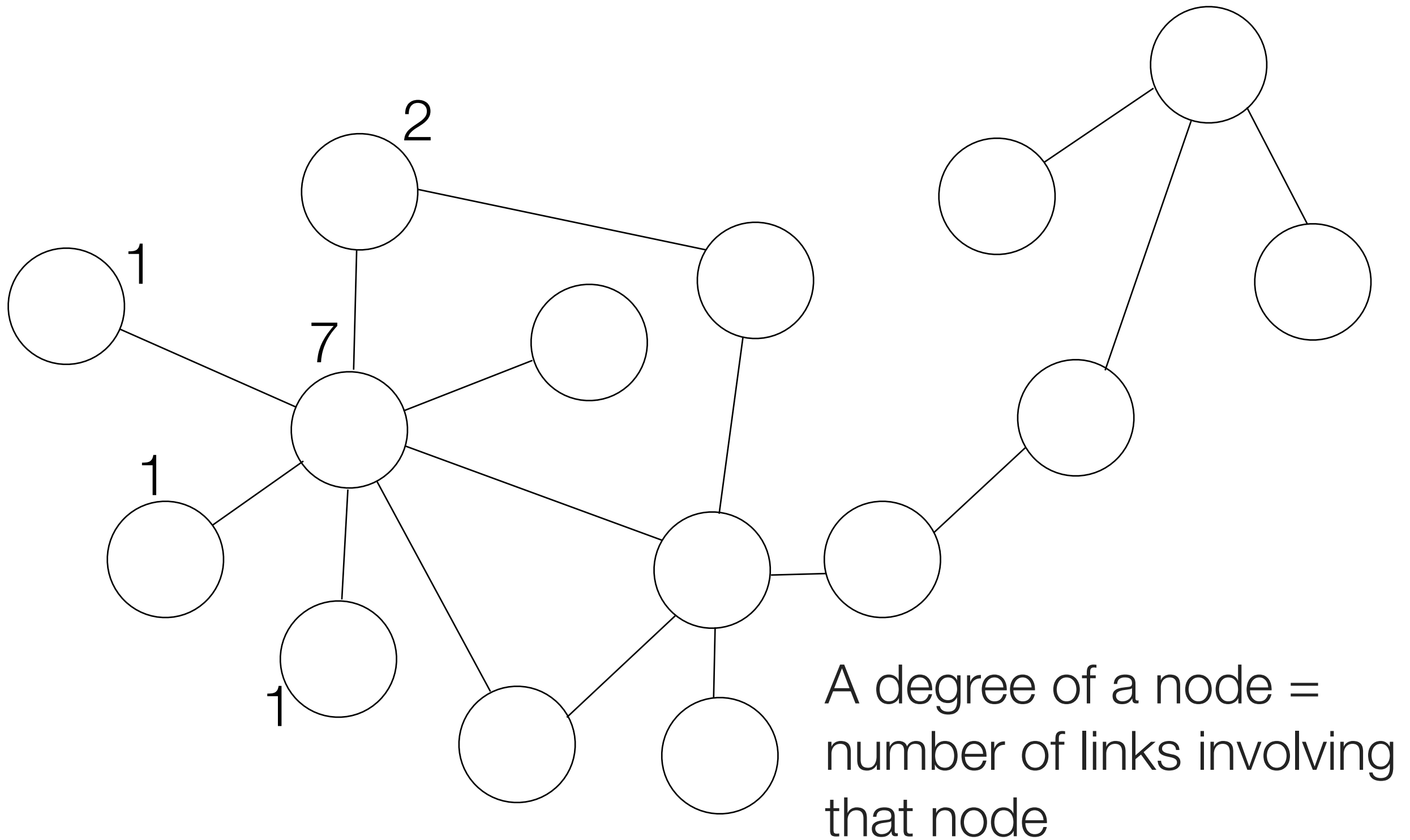
A degree of a node =  
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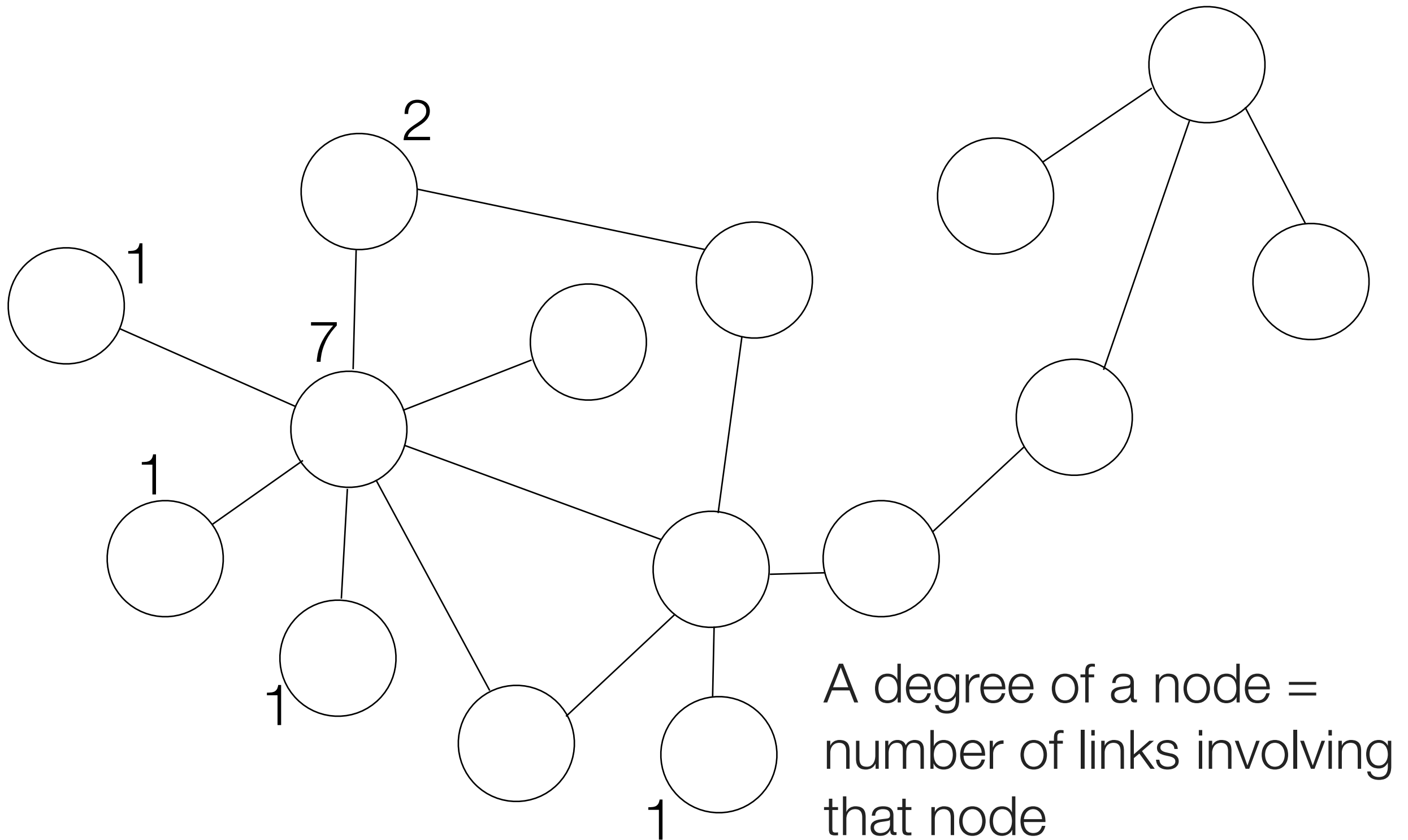


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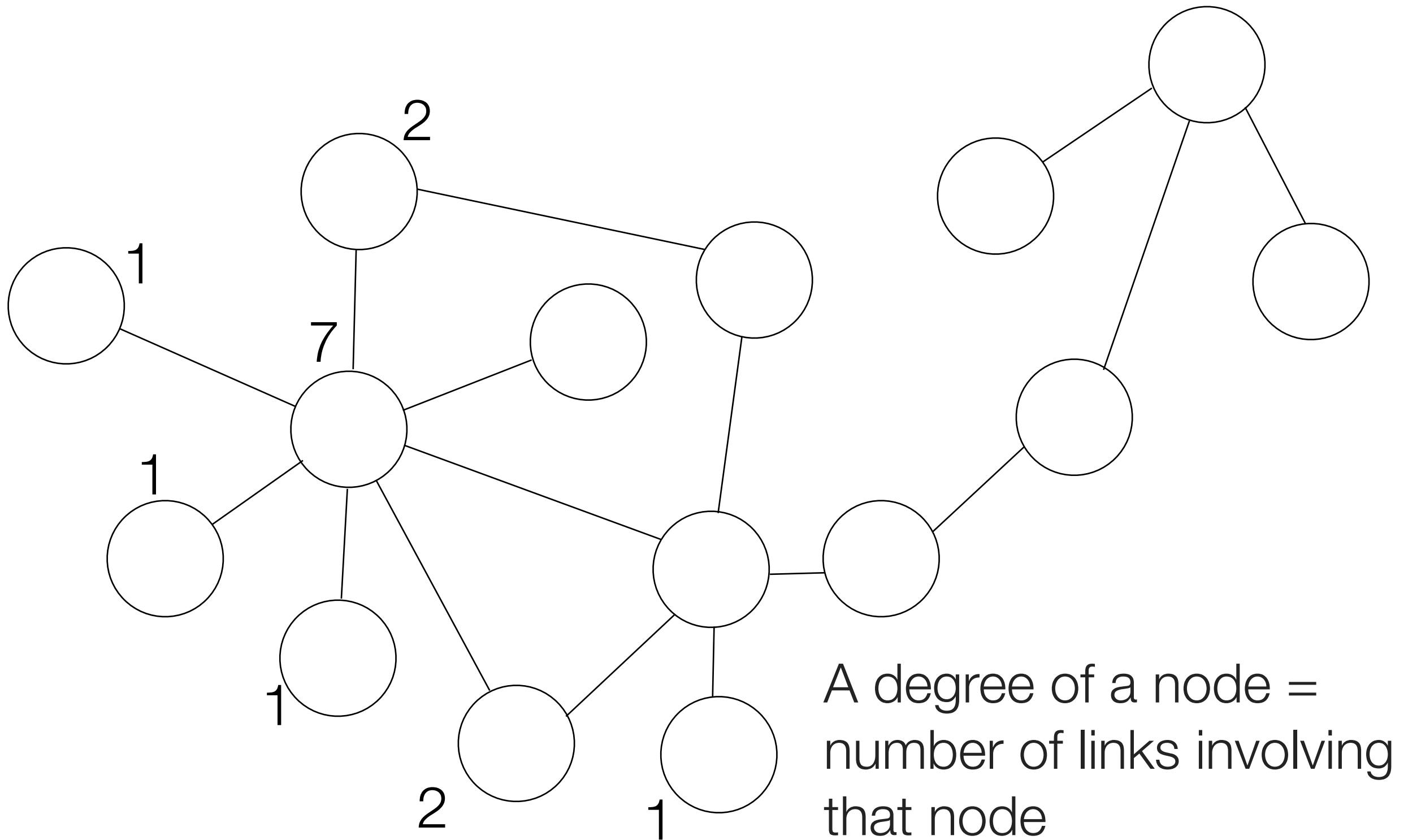
# Degree, network density



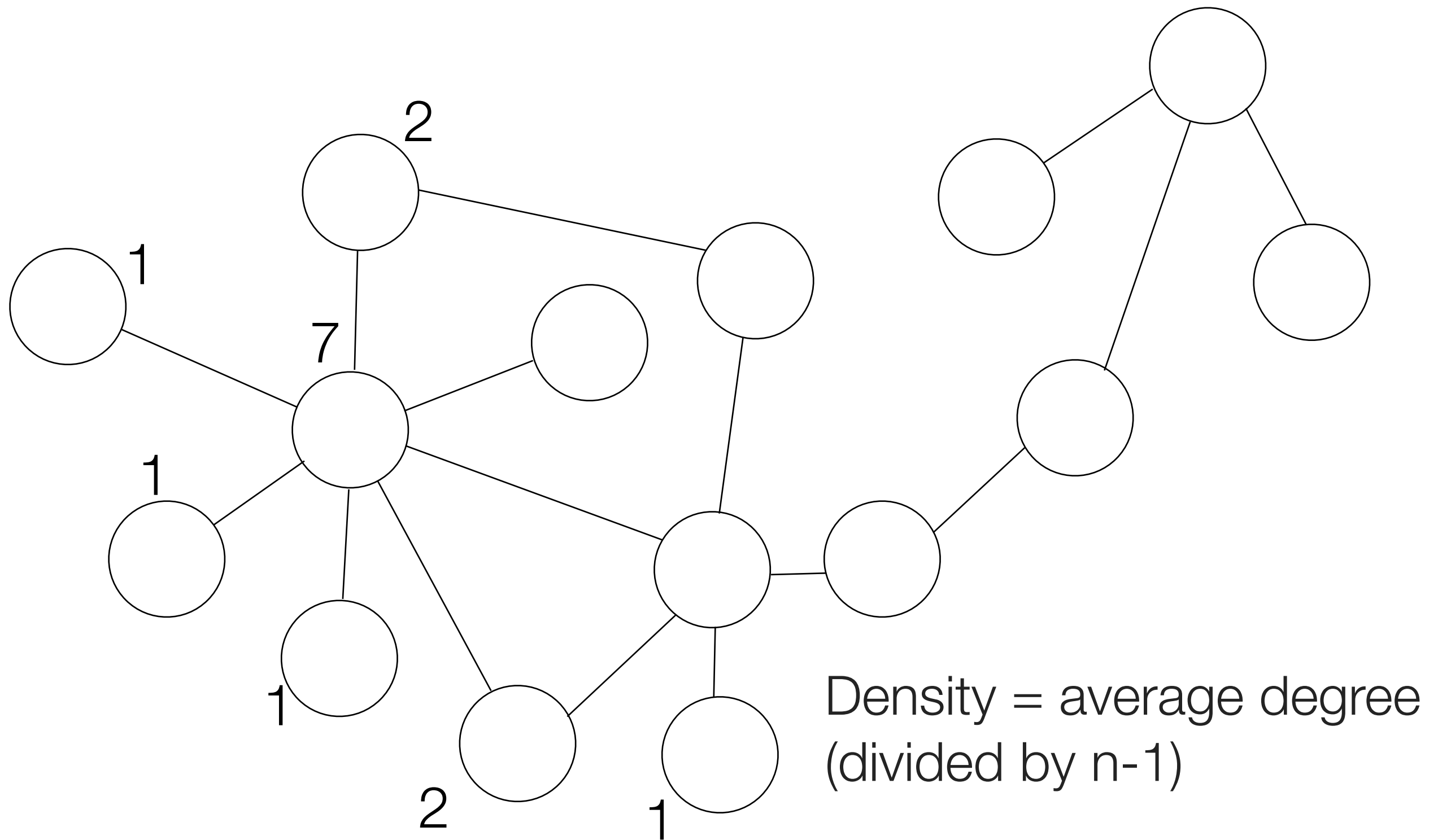
# Degree, network density



# Degree, network density



# Degree, network density





# Degree distribution

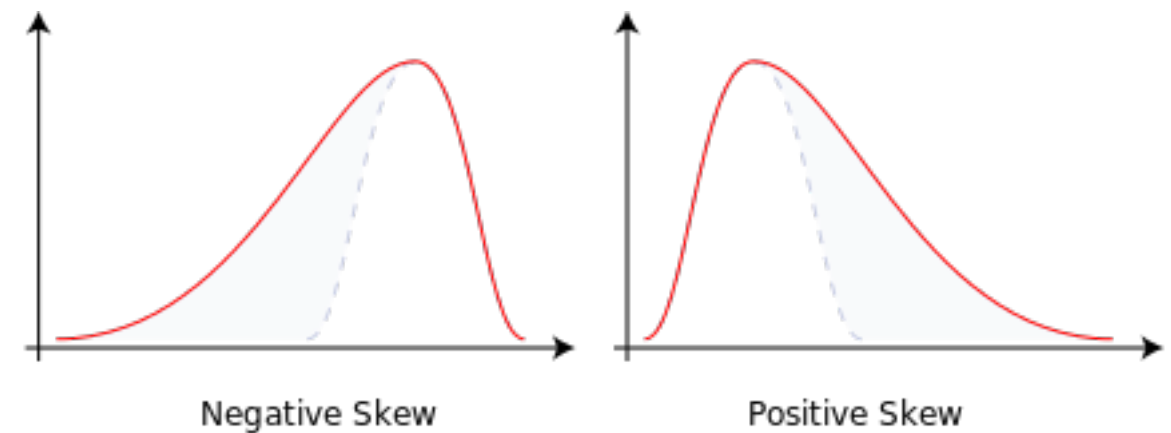
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# Degree distribution

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Regular network = all nodes have the same degree

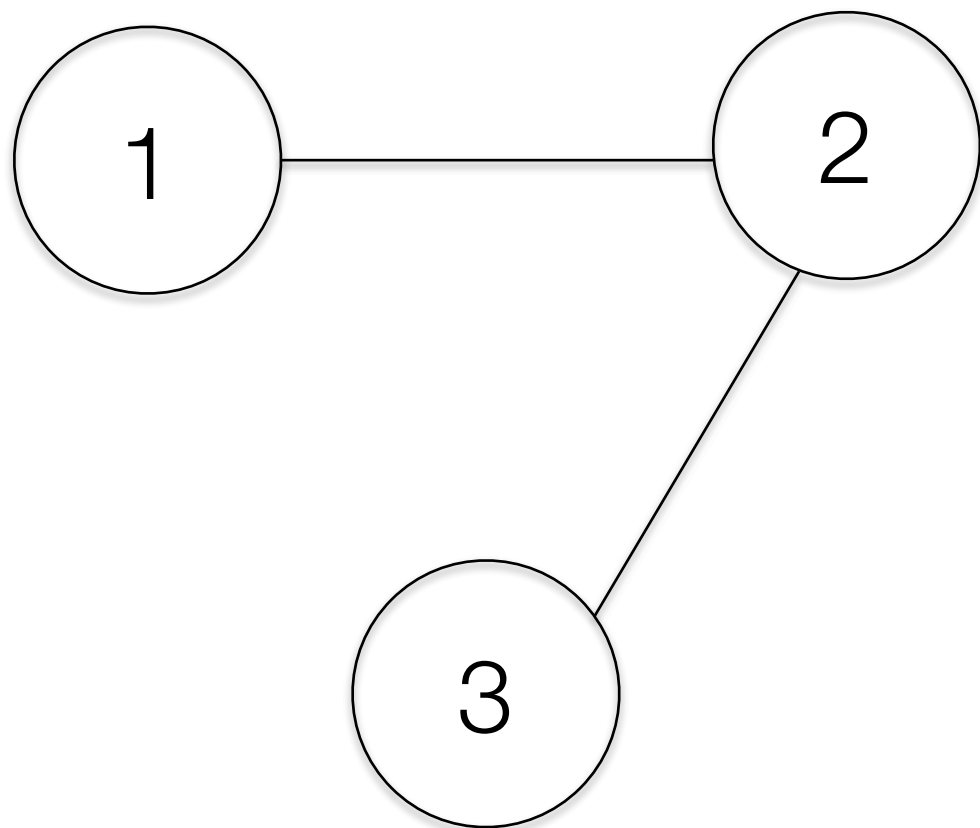
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The degree distribution of a network is a description of the relative frequency of nodes that have different degrees.

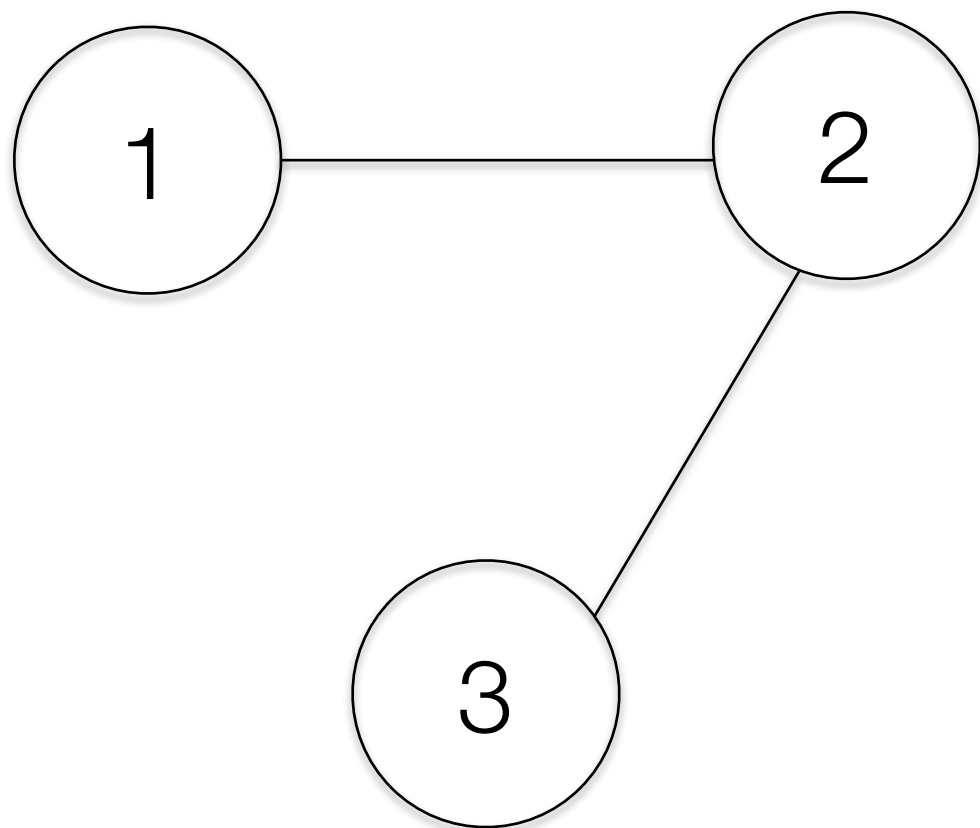
Regular network = all nodes have the same degree

# Random graphs



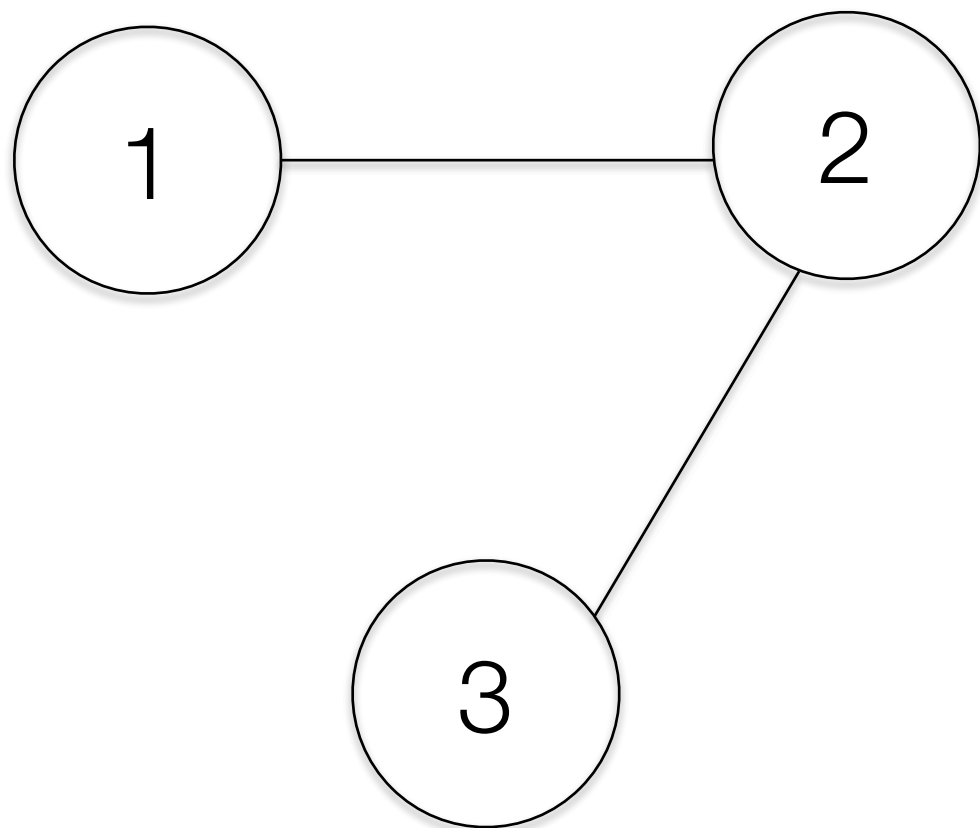
$$g = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

# Random graphs



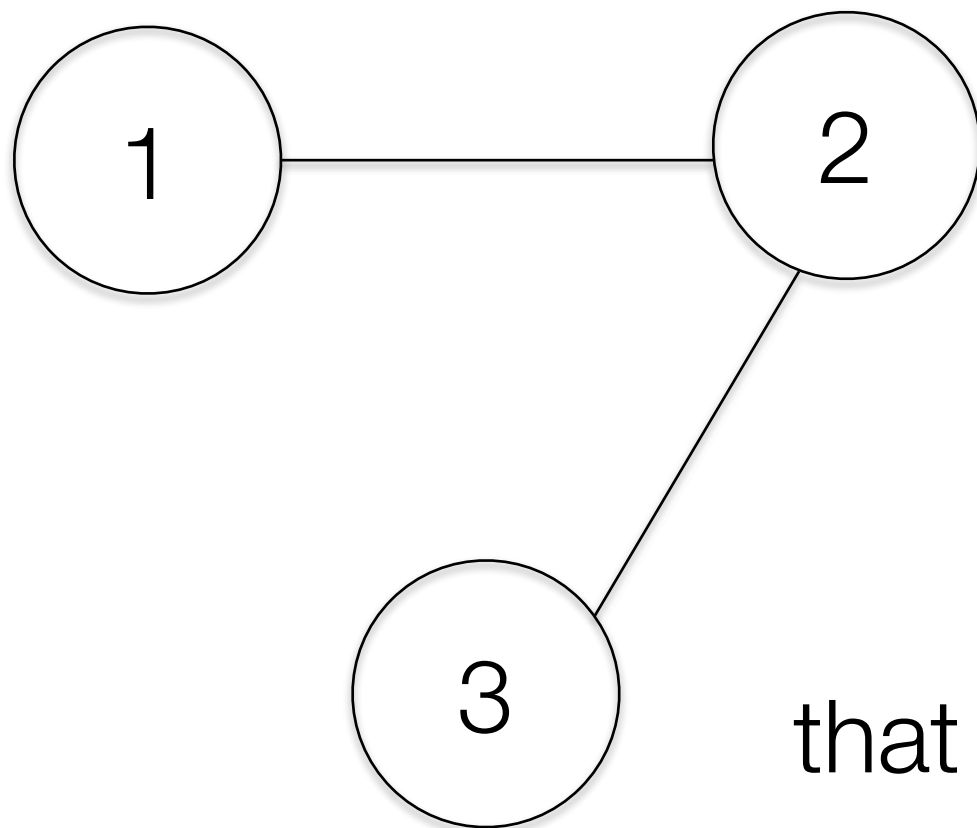
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# Random graphs



$$g = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$g_{ij}$  is now the probability  
that an edge exists between  $i$  and  $j$

# Properties of social networks

## Colloquium

### Random graph models of social networks

M. E. J. Newman<sup>\*†</sup>, D. J. Watts<sup>‡</sup>, and S. H. Strogatz<sup>§</sup>

<sup>\*</sup>Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501; <sup>†</sup>Department of Sociology, Columbia University, 1180 Amsterdam Avenue, New York, NY 10027; and <sup>§</sup>Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, NY 14853-1503

**We describe some new exactly solvable models of the structure of social networks, based on random graphs with arbitrary degree distributions. We give models both for simple unipartite networks, such as acquaintance networks, and bipartite networks, such as affiliation networks. We compare the predictions of our models to data for a number of real-world social networks and find that in some cases, the models are in remarkable agreement with the data, whereas in others the agreement is poorer, perhaps indicating the presence of additional social structure in the network that is not captured by the random graph.**

**A** social network is a set of people or groups of people, “actors” in the jargon of the field, with some pattern of interactions or “ties” between them (1, 2). Friendships among a group of individuals, business relationships between companies, and intermarriages between families are all examples of networks that have been studied in the past. Network analysis has

actually connected by a very short chain of intermediate acquaintances. He found this chain to be of typical length of only about six, a result which has passed into folklore by means of John Guare’s 1990 play *Six Degrees of Separation* (10). It has since been shown that many networks have a similar small-world property (11–14).

It is worth noting that the phrase “small world” has been used to mean a number of different things. Early on, sociologists used the phrase both in the conversational sense of two strangers who discover that they have a mutual friend—i.e., that they are separated by a path of length two—and to refer to any short path between individuals (8, 9). Milgram talked about the “small-world problem,” meaning the question of how two people can have a short connecting path of acquaintances in a network that has other social structure such as insular communities or geographical and cultural barriers. In more recent work, D.J.W. and S.H.S. (11) have used the phrase “small-world network” to mean



# Properties of social networks

# Properties of social networks

- “small-world” property - maximal degree of separation is low (6)

# Proper



---

An Experimental Study of the Small World Problem  
Author(s): Jeffrey Travers and Stanley Milgram  
Source: *Sociometry*, Vol. 32, No. 4 (Dec., 1969), pp. 425-443  
Published by: [American Sociological Association](#)  
Stable URL: <http://www.jstor.org/stable/2786545>  
Accessed: 23/09/2010 13:05

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Letter | Published: 04 June 1998

## Collective dynamics of ‘small-world’ networks

Duncan J. Watts✉ & Steven H. Strogatz

*Nature* **393**, 440–442 (04 June 1998) | [Download Citation](#) ↓

# Properties of social networks

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---

## Statistical mechanics of complex networks

Réka Albert and Albert-László Barabási  
Rev. Mod. Phys. **74**, 47 – Published 30 January 2002

# Properties of social networks

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*Review*

# **A Survey on Information Diffusion in Online Social Networks: Models and Methods**

**Mei Li \*, Xiang Wang, Kai Gao and Shanshan Zhang**

School of Information Science and Engineering, Hebei University of Science and Technology, Shijiazhuang 050018, China; wangxiang@hebust.edu.cn (X.W.); gaokai@hebust.edu.cn (K.G.); zshanshanmiss@gmail.com (S.Z.)

\* Correspondence: limei@hebust.edu.cn; Tel.: +86-139-3317-5921

Received: 16 August 2017; Accepted: 22 September 2017; Published: 29 September 2017

# Diffusion as infection

Phys Rev Lett. 2001 Apr 2;86(14):3200-3.

## **Epidemic spreading in scale-free networks.**

Pastor-Satorras R<sup>1</sup>, Vespignani A.

### **Author information**

### **Abstract**

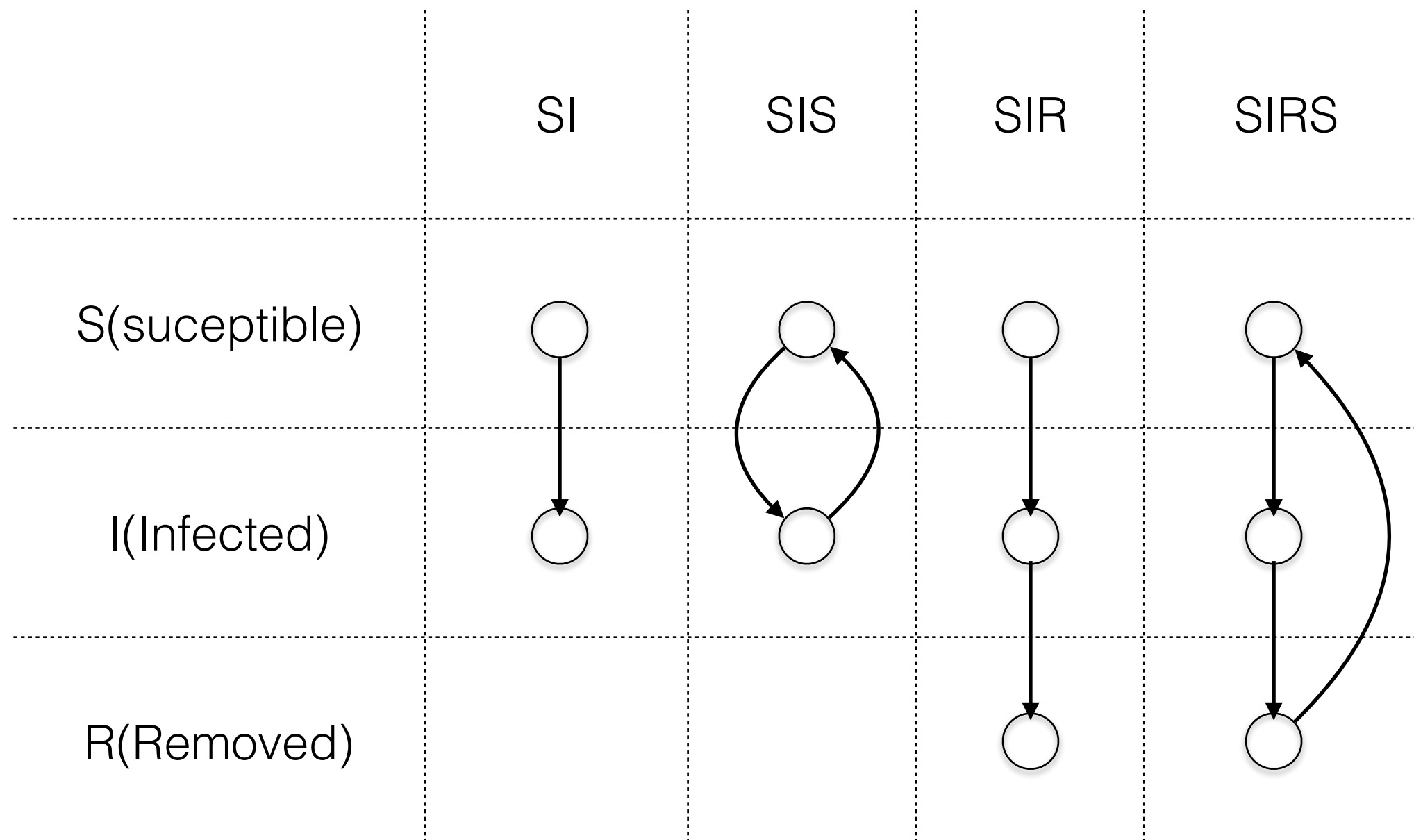
The Internet has a very complex connectivity recently modeled by the class of scale-free networks. This feature, which appears to be very efficient for a communications network, favors at the same time the spreading of computer viruses. We analyze real data from computer virus infections and find the average lifetime and persistence of viral strains on the Internet. We define a dynamical model for the spreading of infections on scale-free networks, finding the absence of an epidemic threshold and its associated critical behavior. This new epidemiological framework rationalizes data of computer viruses and could help in the understanding of other spreading phenomena on communication and social networks.

PMID: 11290142 DOI: [10.1103/PhysRevLett.86.3200](https://doi.org/10.1103/PhysRevLett.86.3200)

[Indexed for MEDLINE]



# Diffusion as infection - basic models



# Centrality measures - micro view

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- degree - how connected a node is

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- closeness - how easily a node can reach other nodes



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# Centrality measures - micro view

- degree - how connected a node is
- closeness - how easily a node can reach other nodes
- betweenness - how important a node is in terms of connecting other nodes
- neighbours' characteristics - how important, central, or influential a node's neighbours are



**American Journal of Sociology**

Volume 78, Number 6 | May, 1973

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## **The Strength of Weak Ties**

**Mark S. Granovetter**

**Abstract**

**Cited by**

**PDF**

# The strength of weak ties

# The strength of weak ties

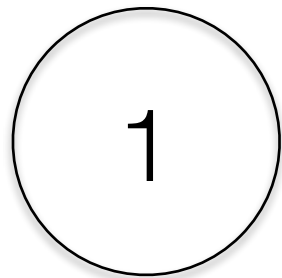
- The stronger the tie (edge) between A and B, the larger the proportion of nodes that are linked to both A and to B

# The strength of weak ties

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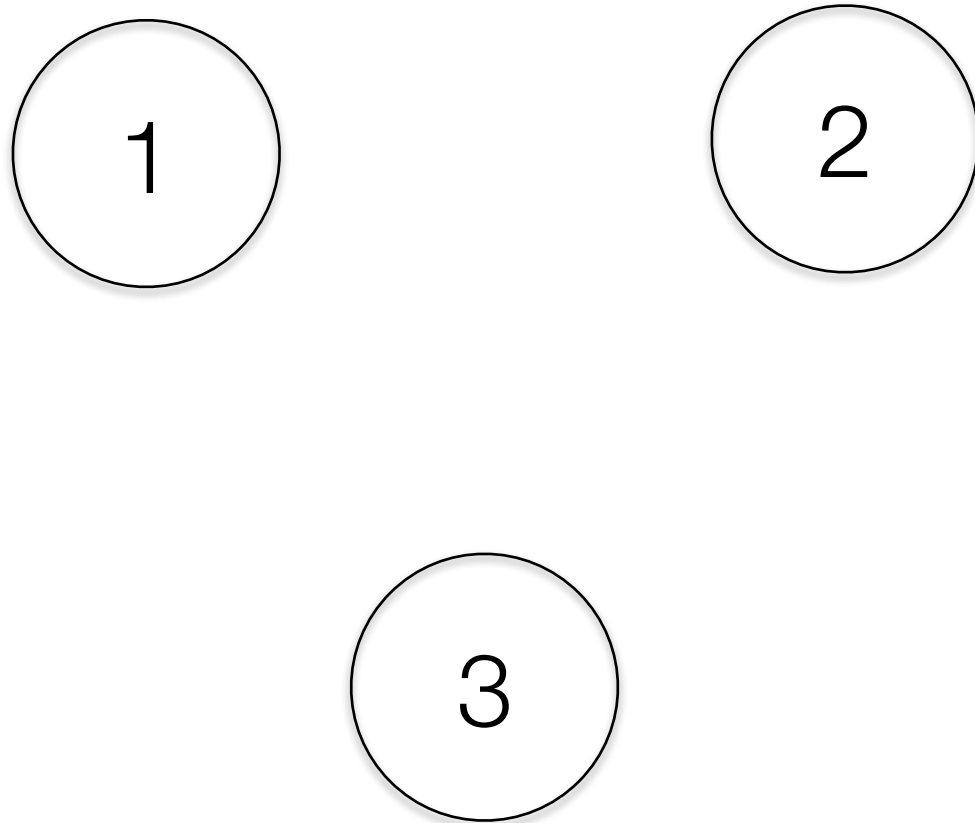
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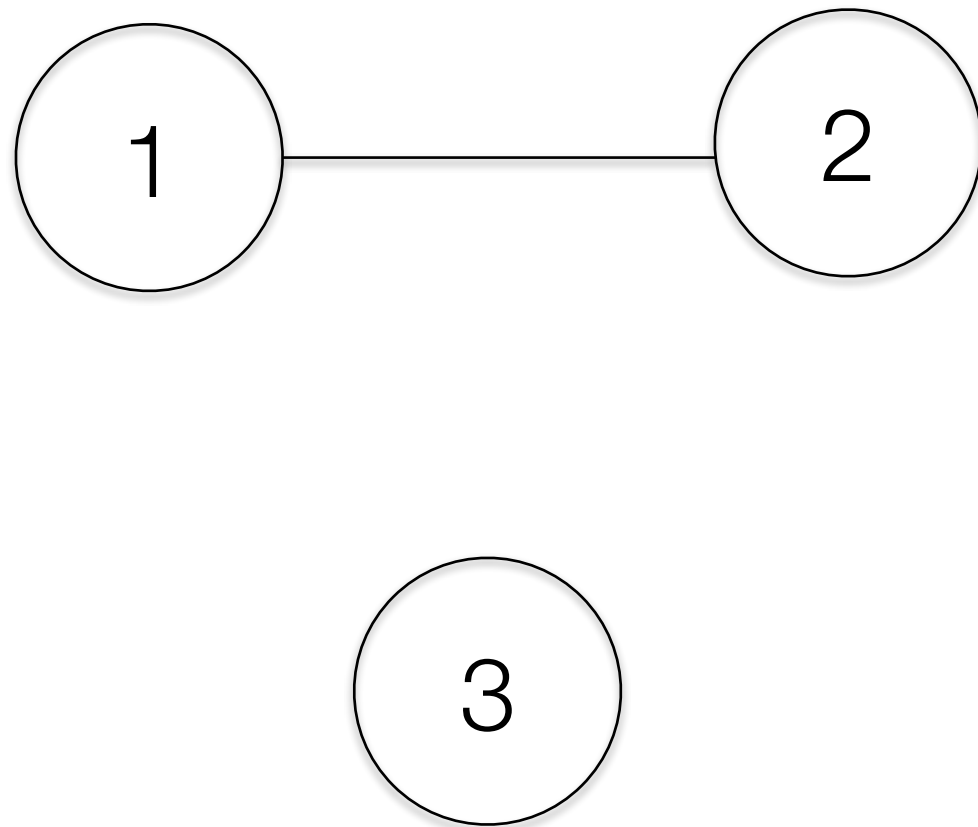
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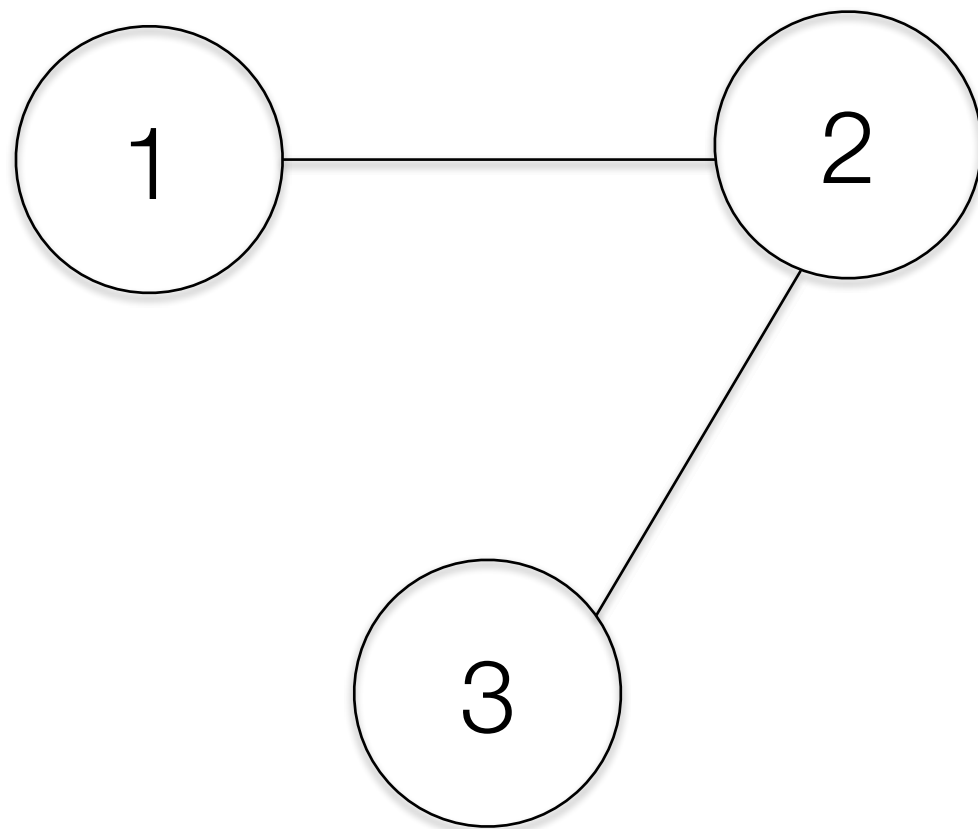
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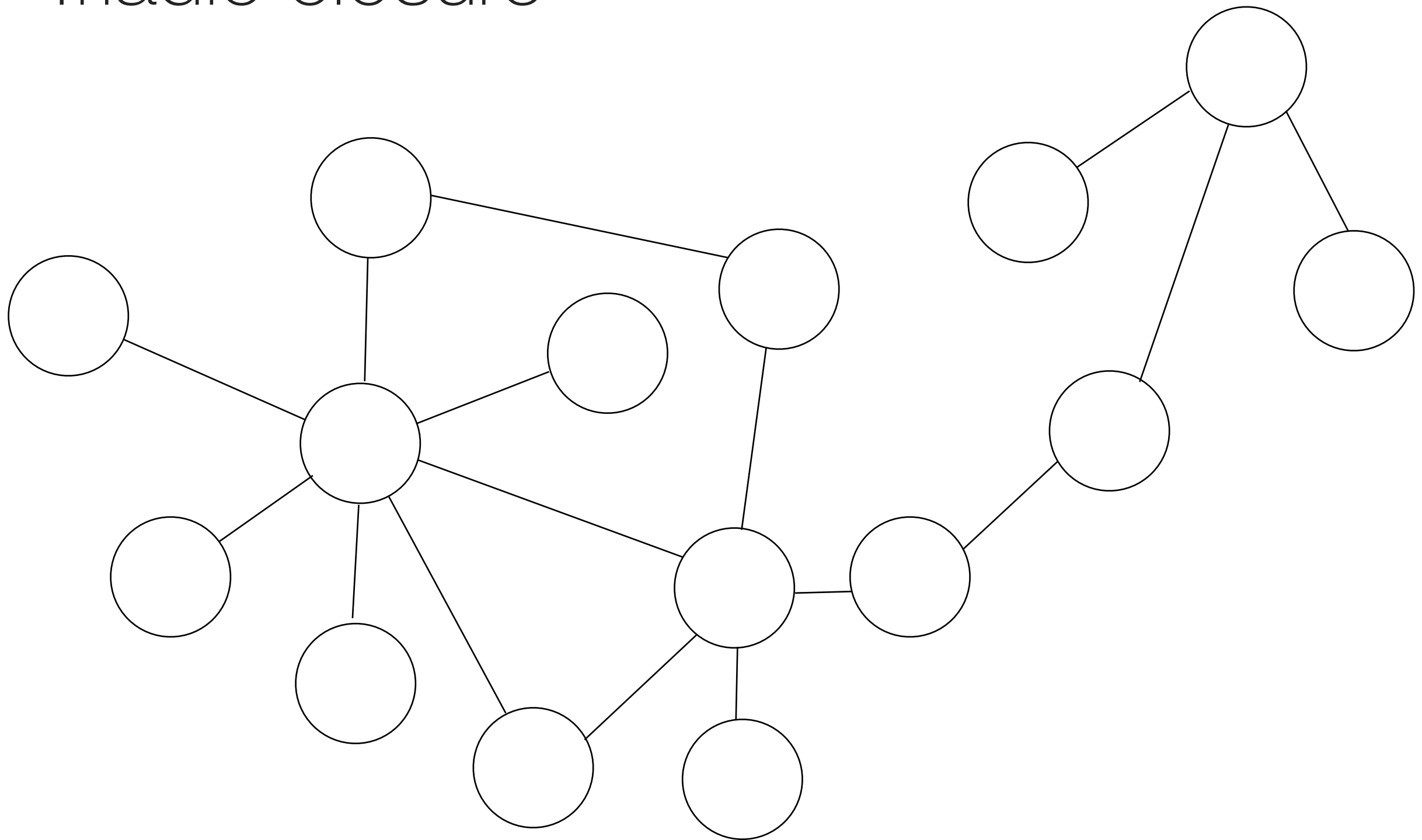


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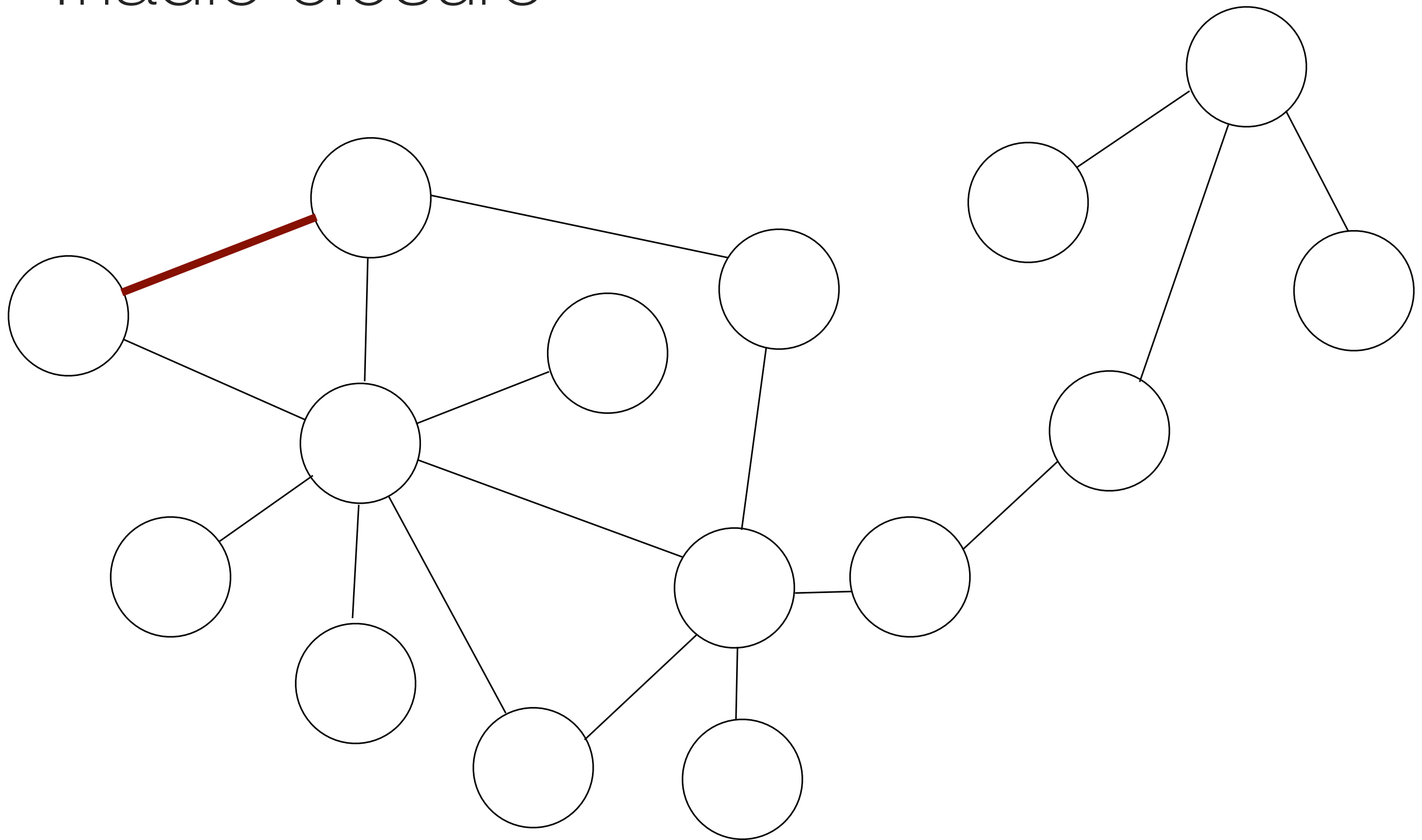
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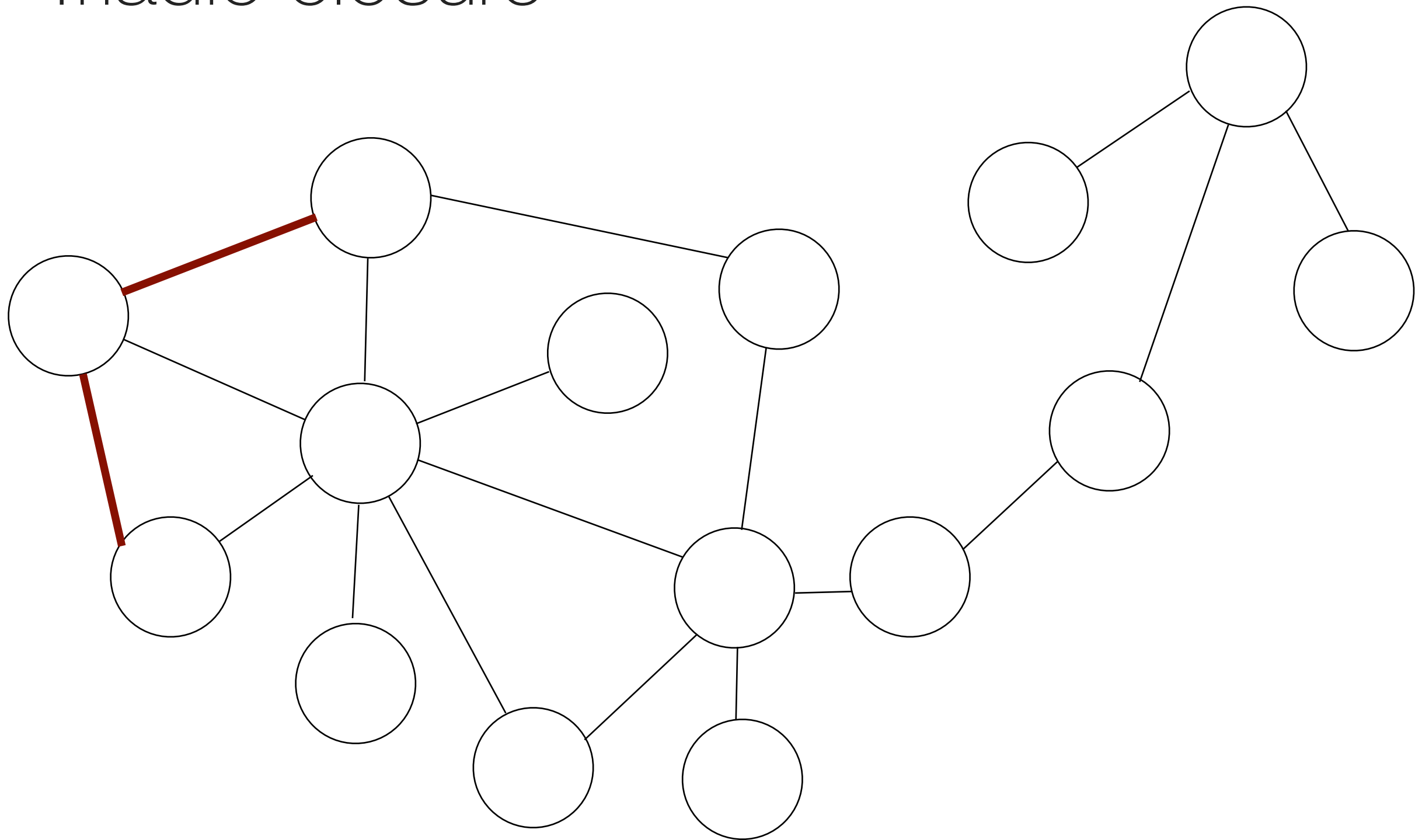
# Triadic closure



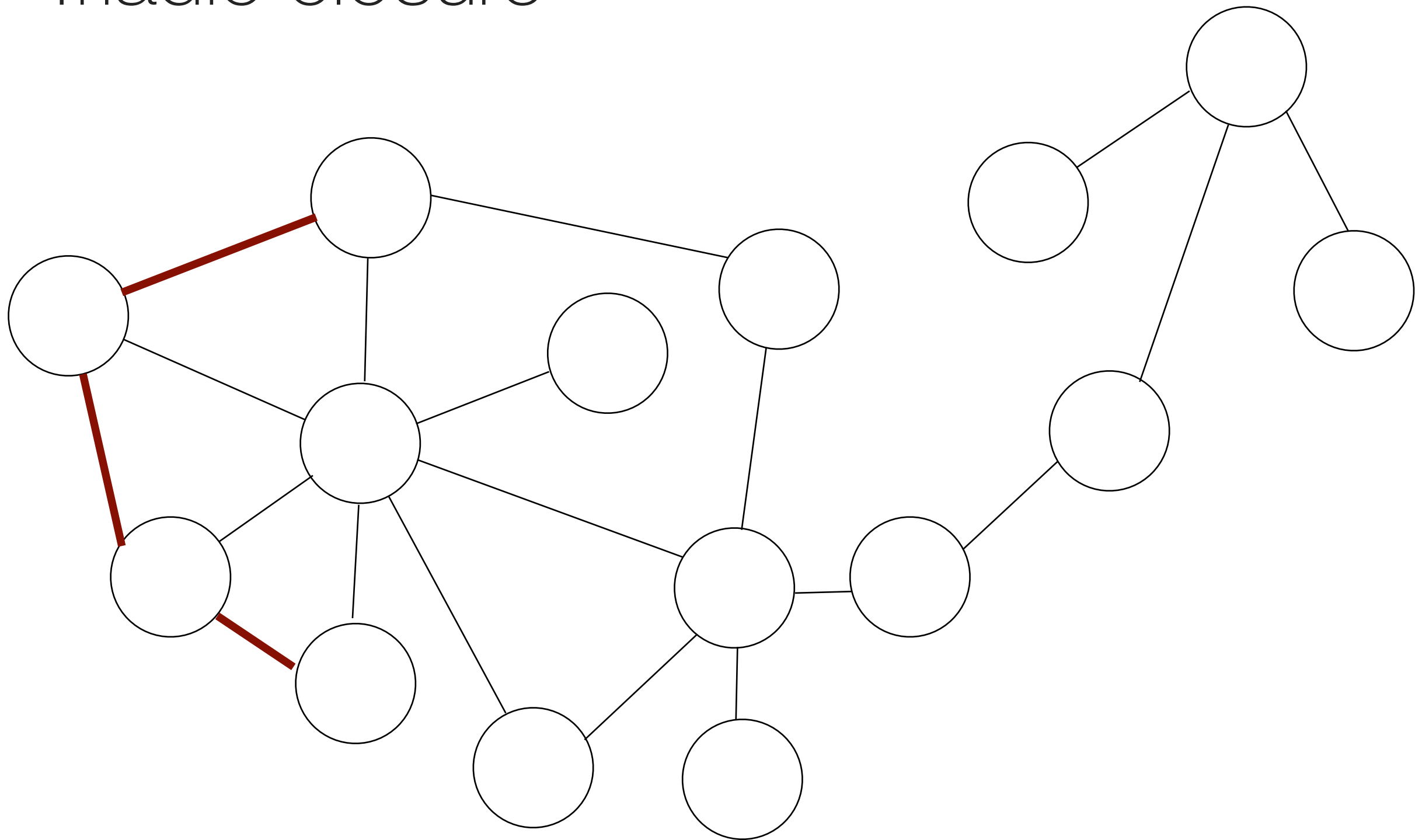
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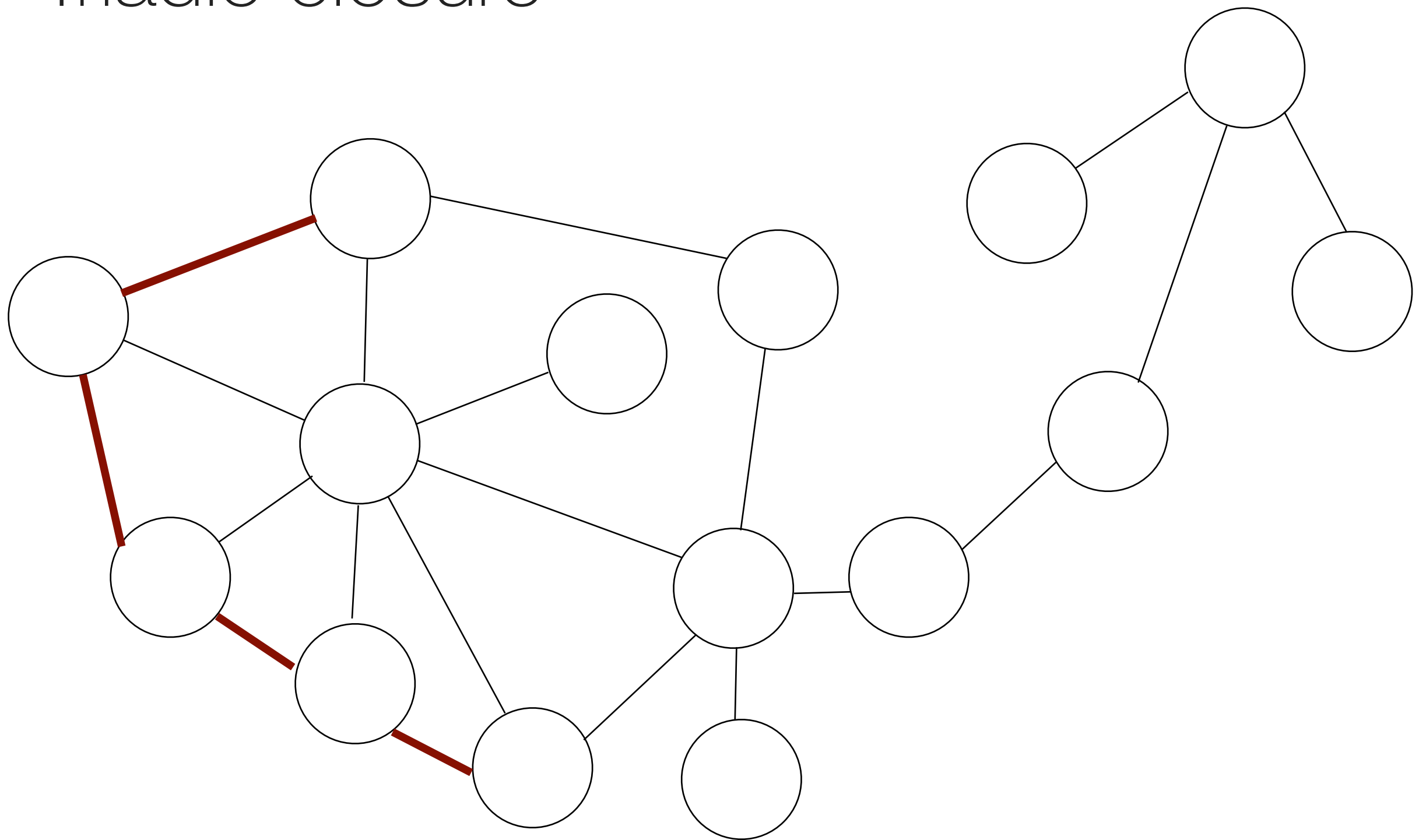
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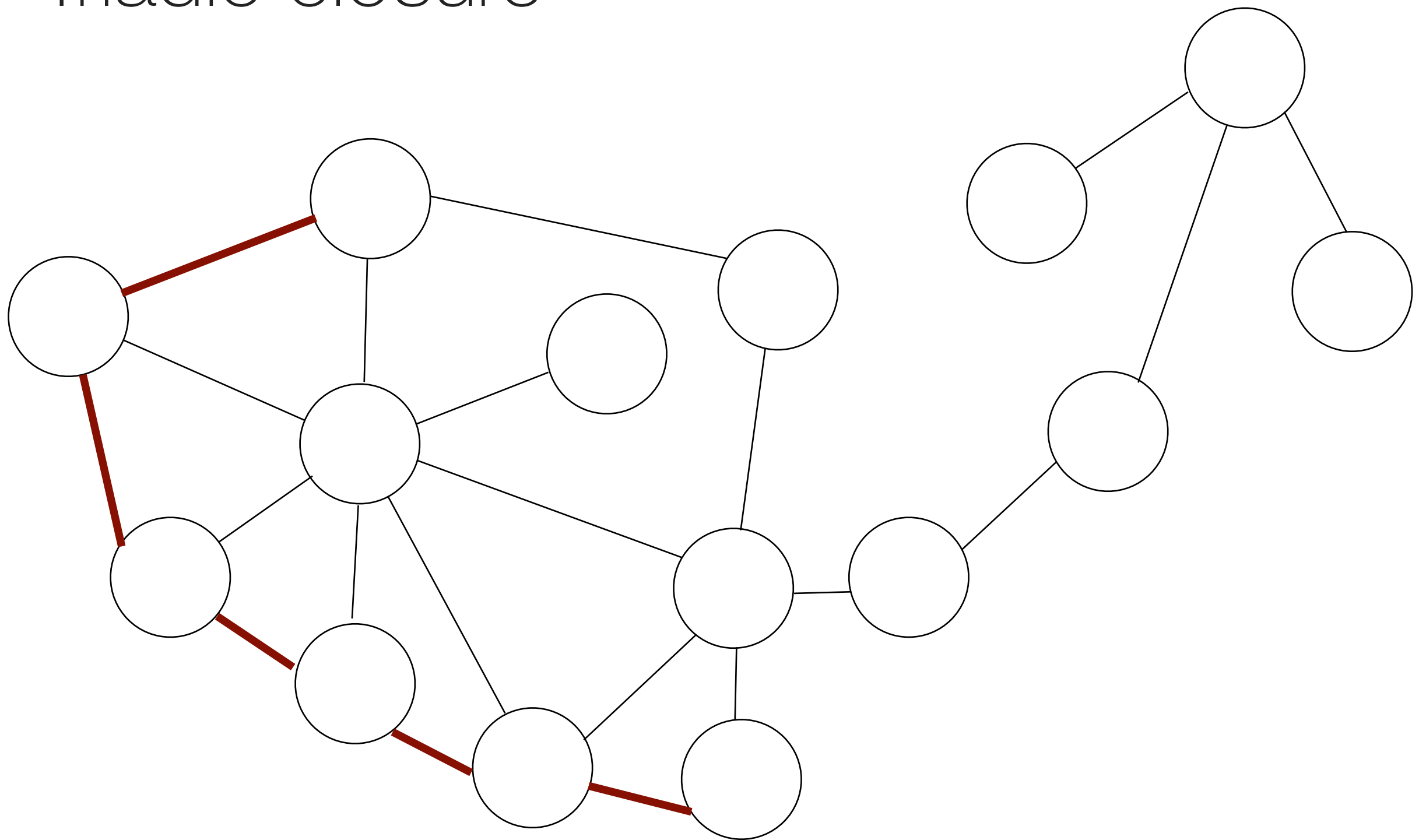


# Triadic closure

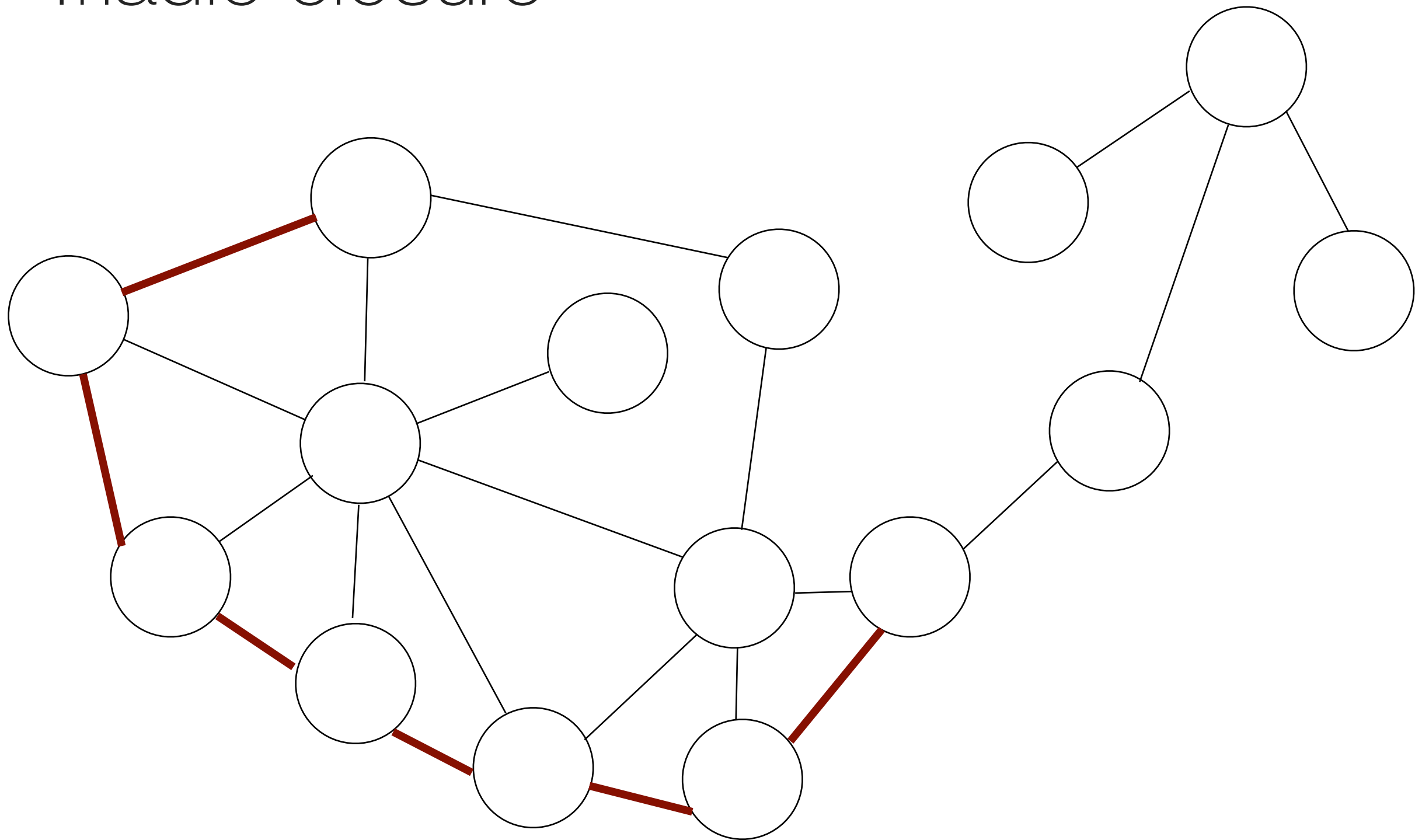




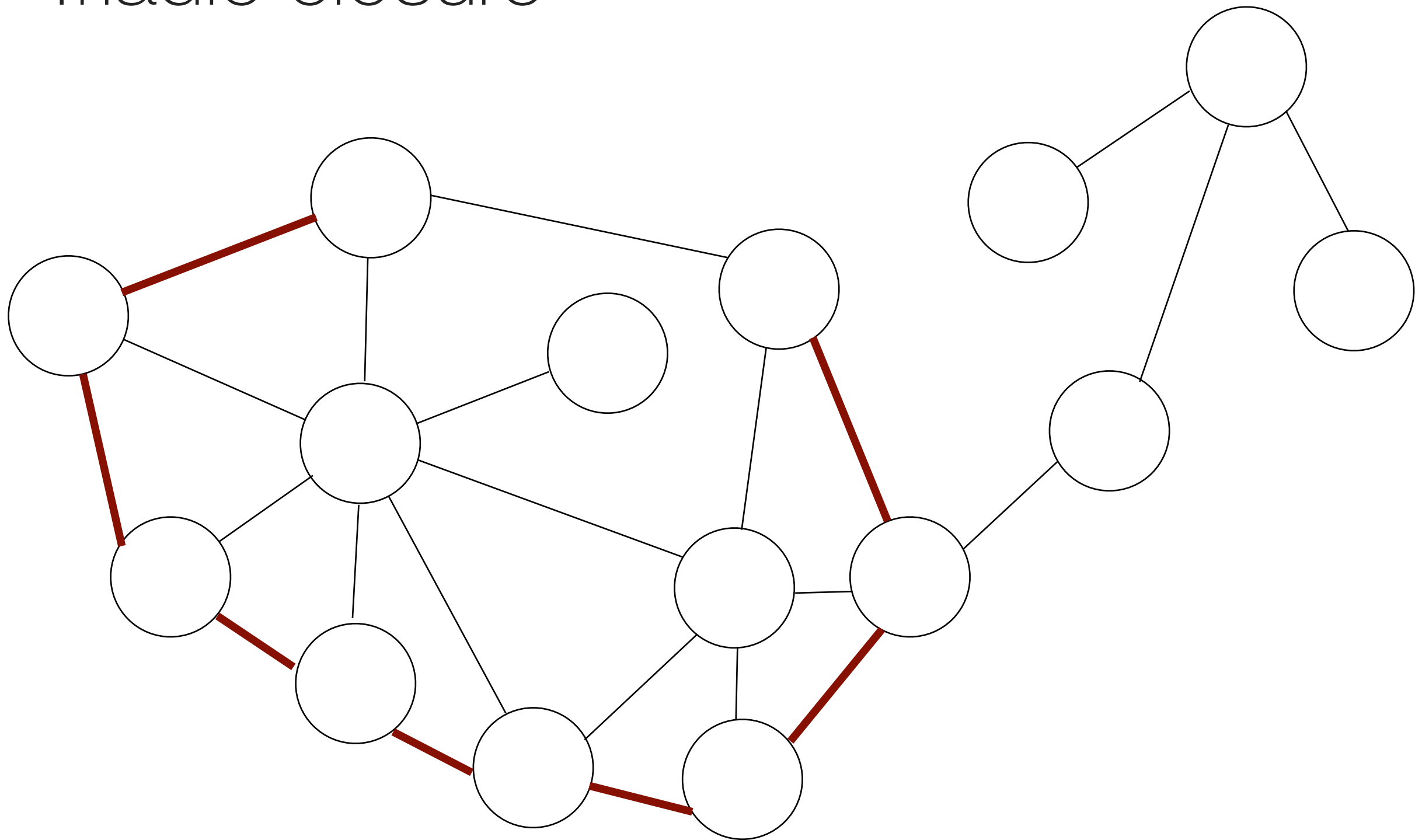
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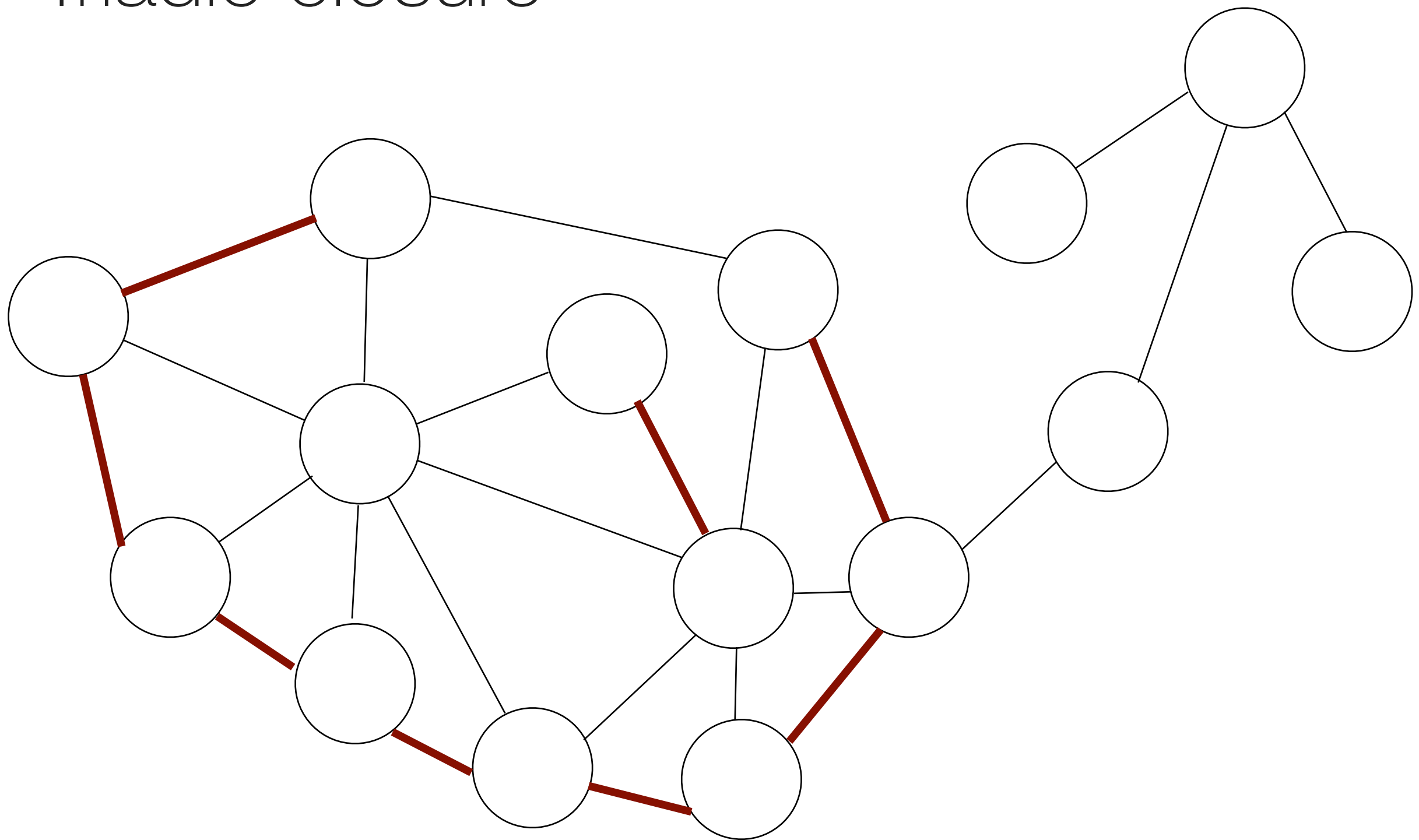
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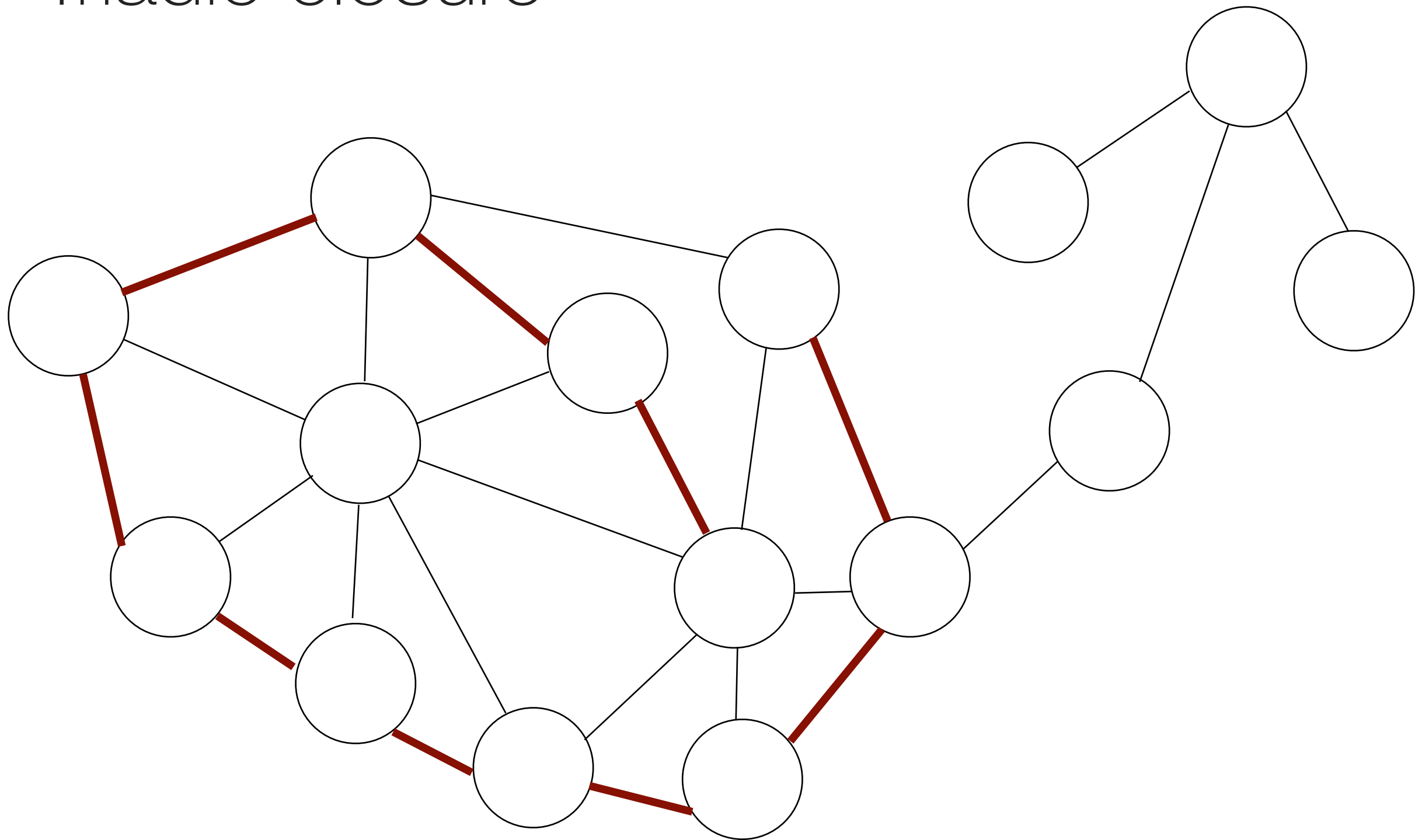
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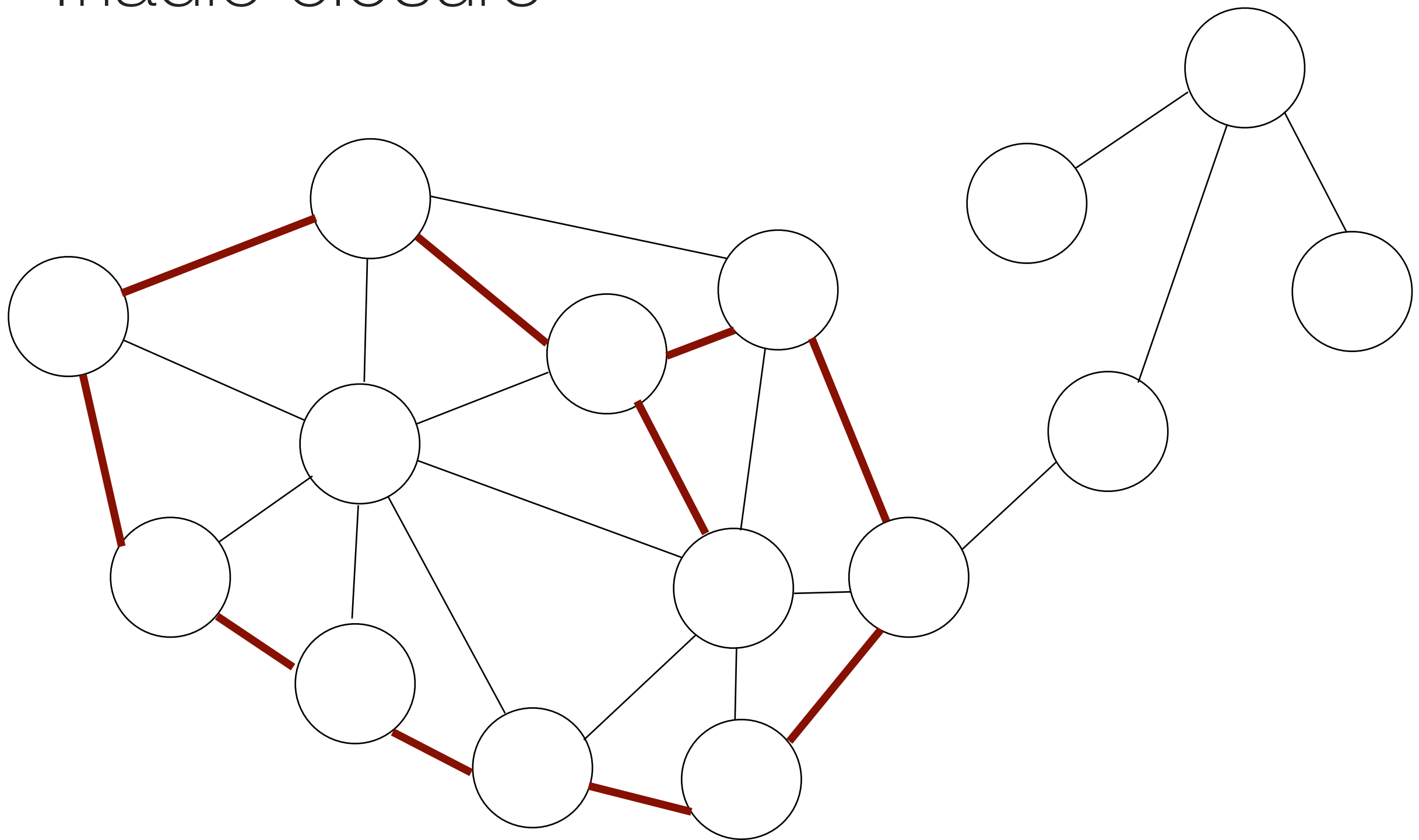
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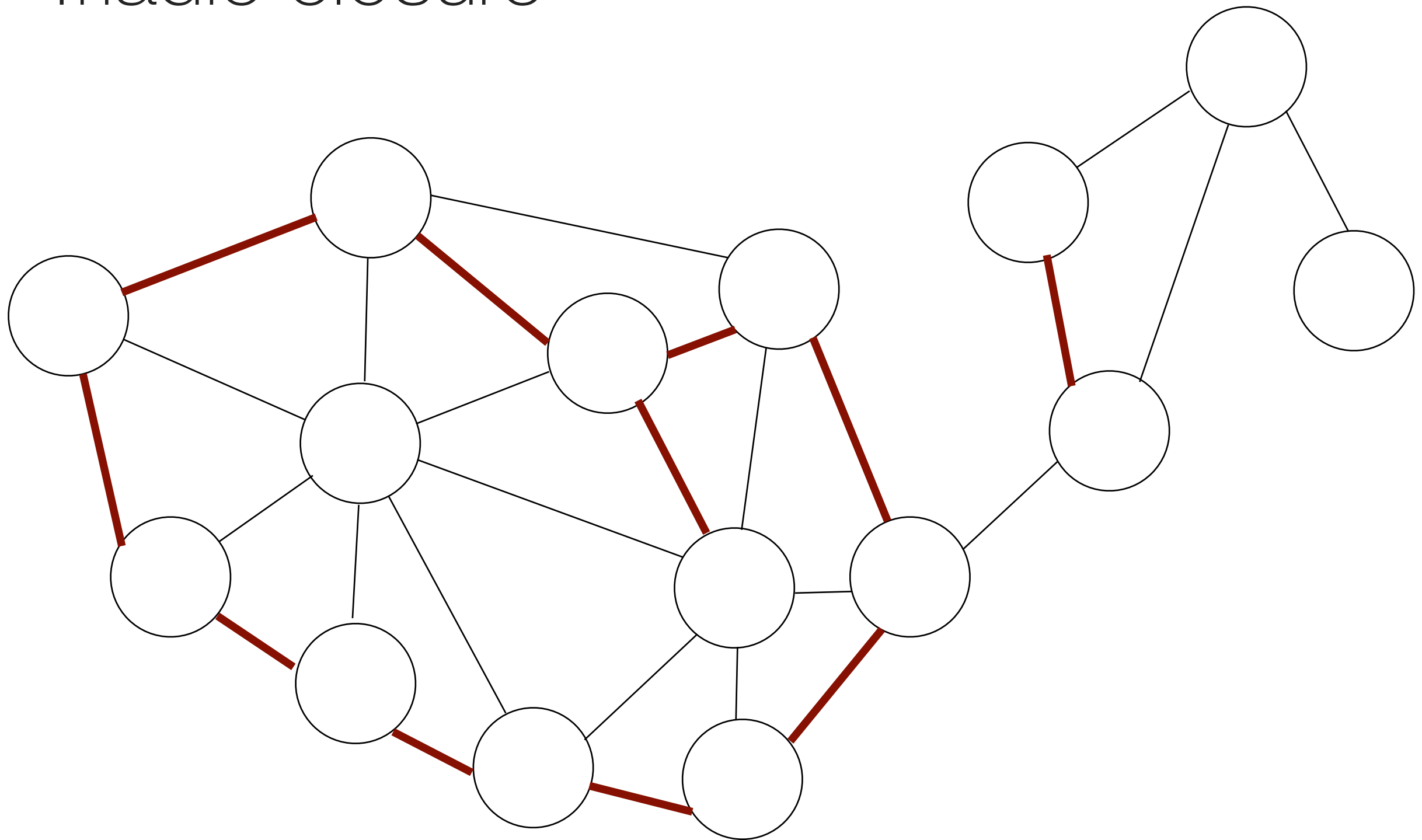
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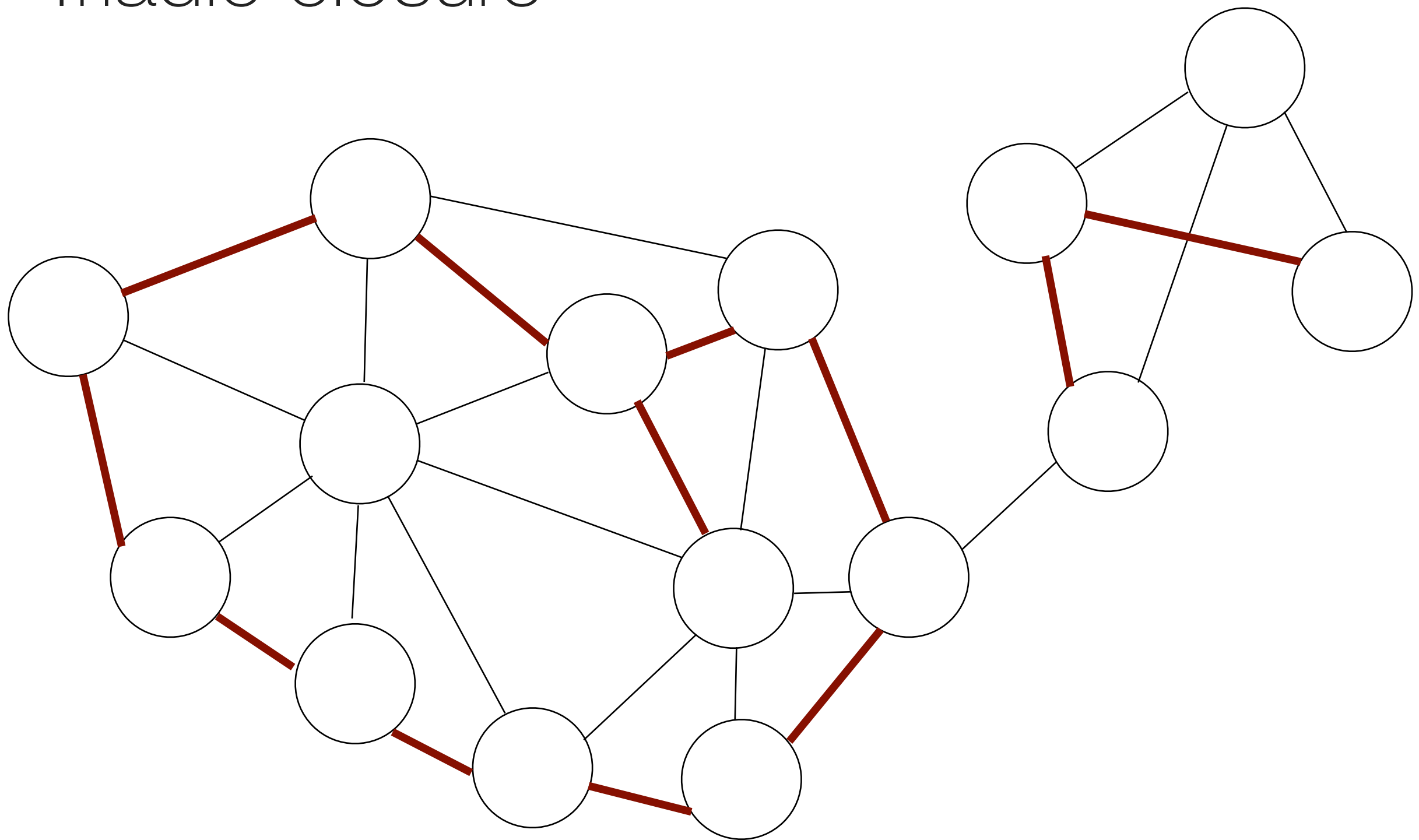
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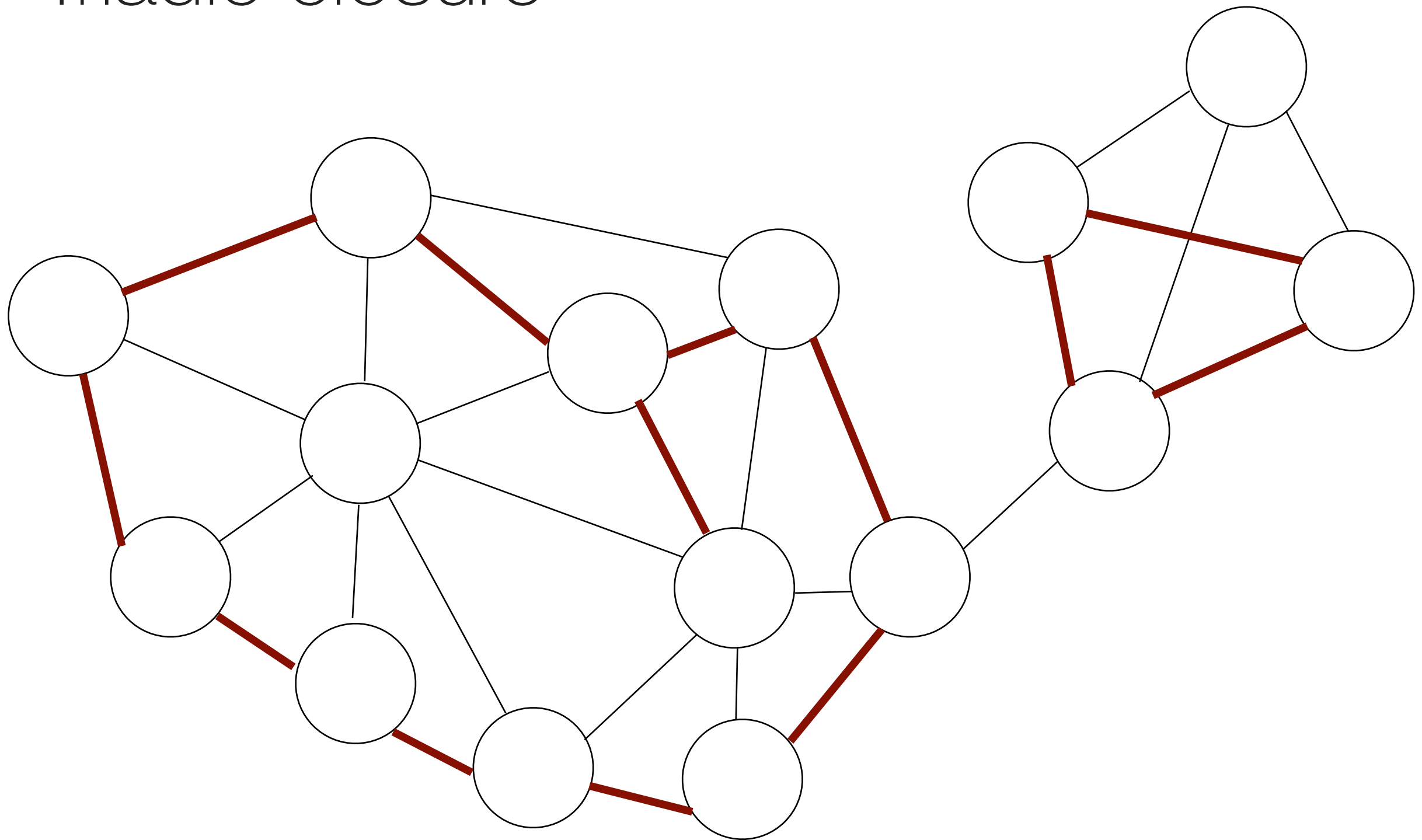


# Triadic closure

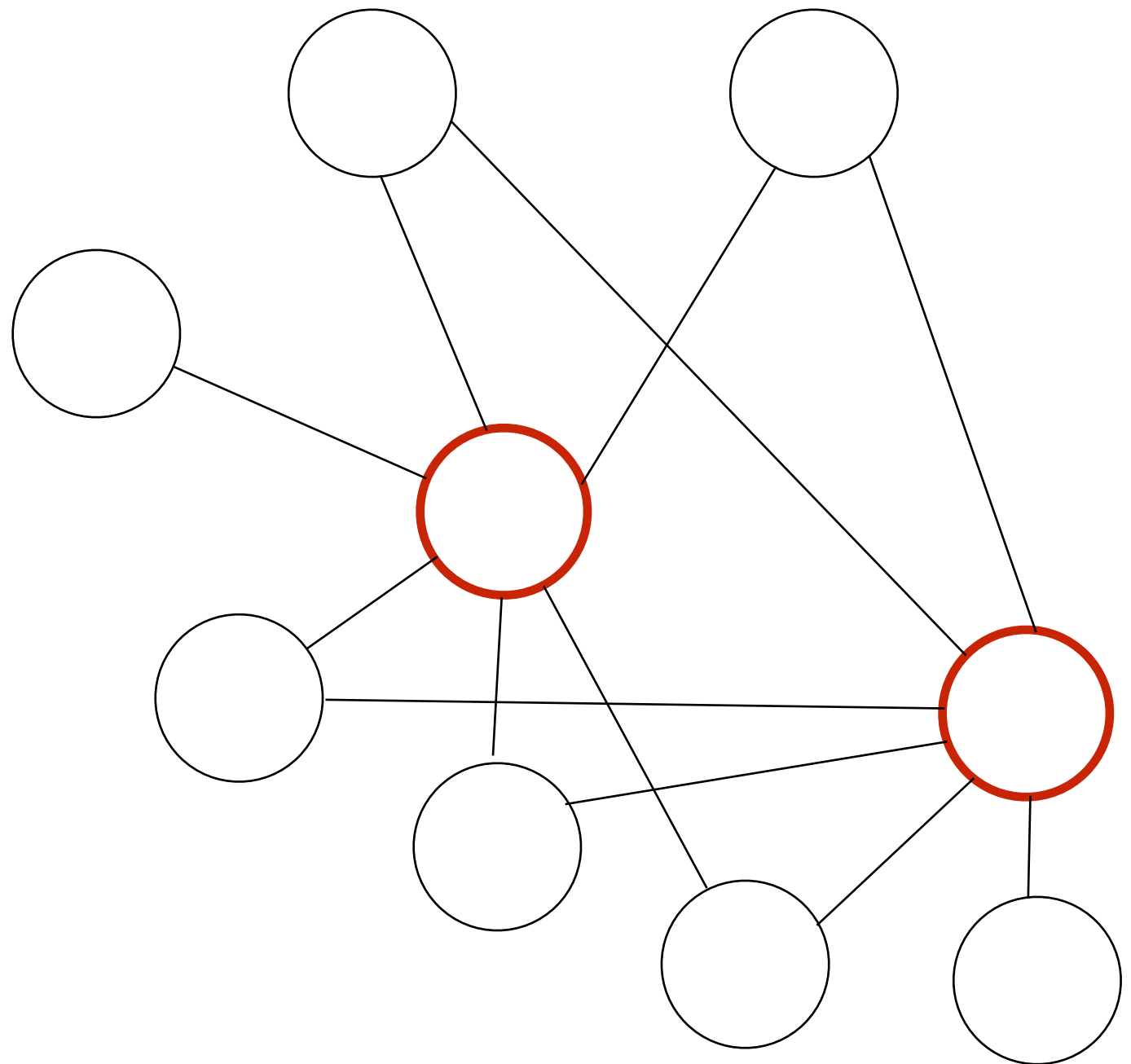




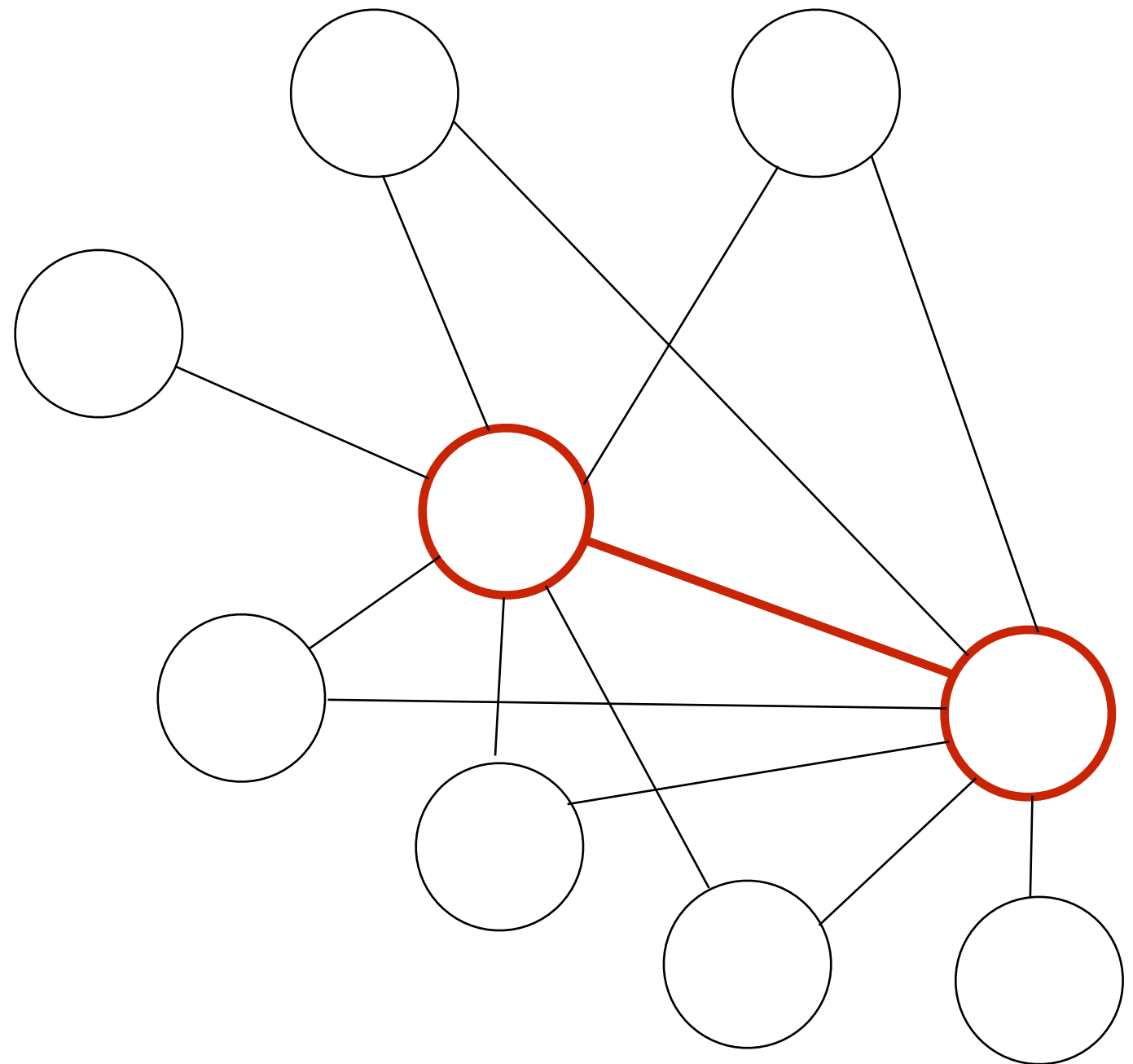
# Triadic closure



# Triadic closure + homophily

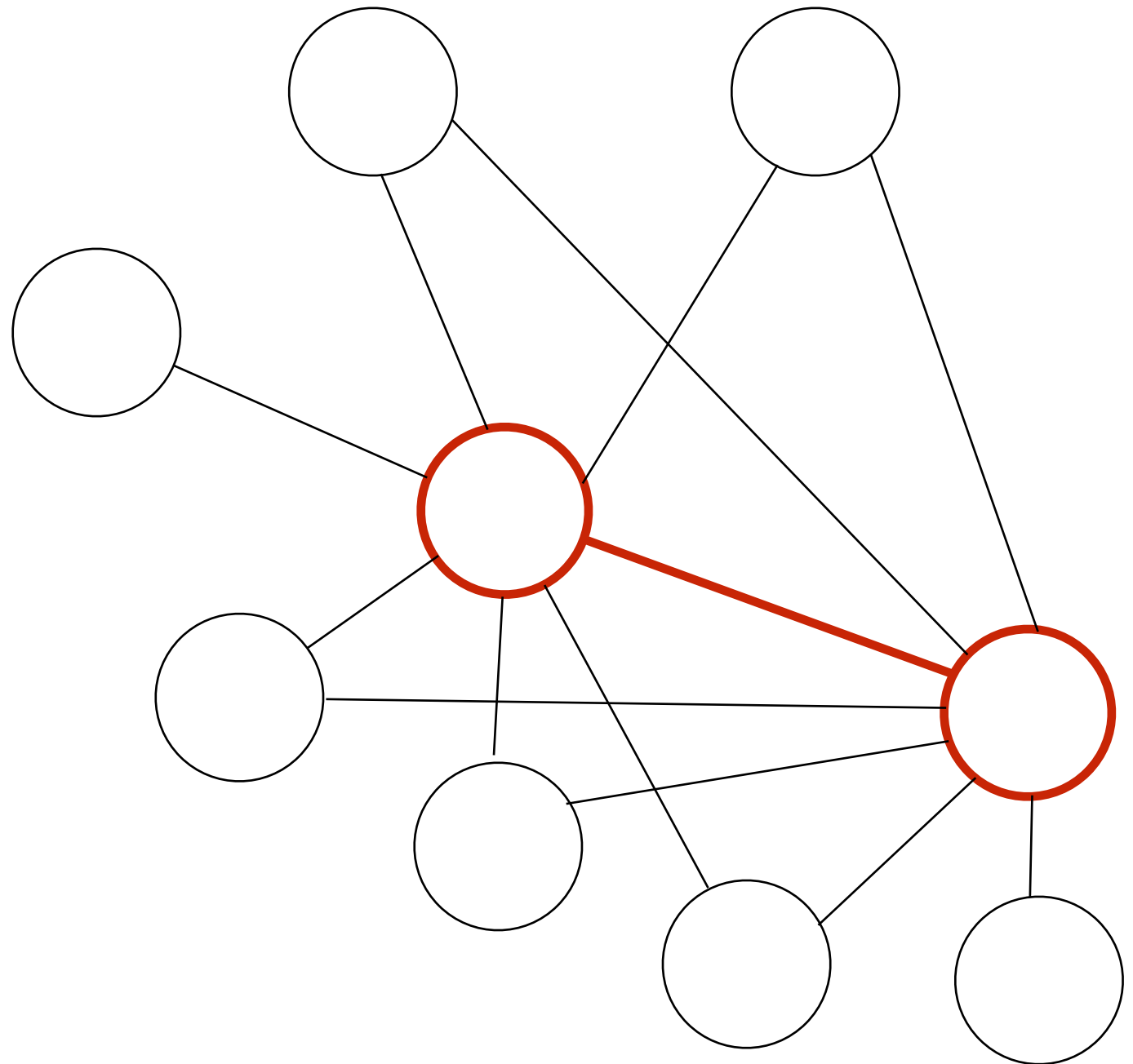


# Triadic closure + homophily

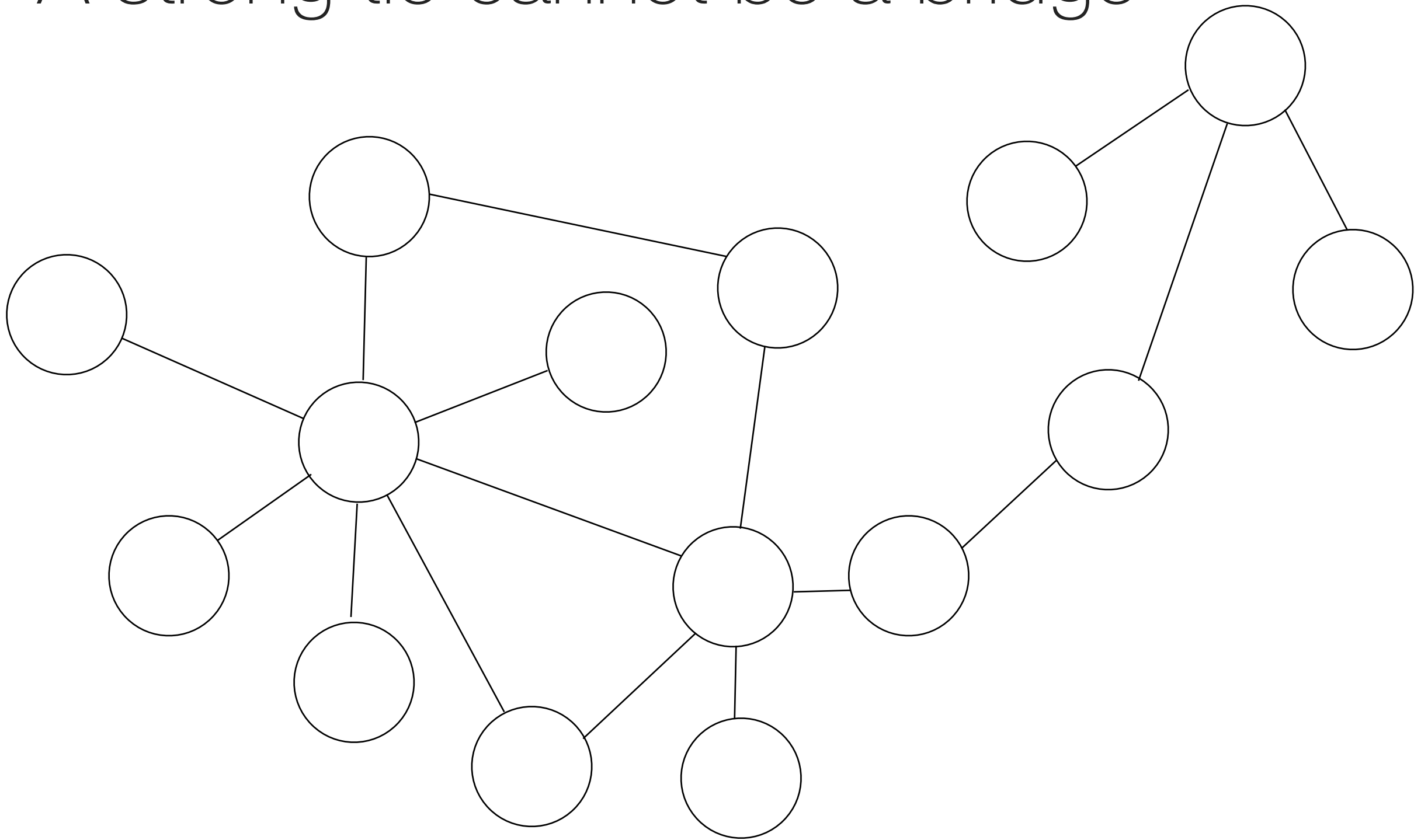


# Triadic closure + homophily

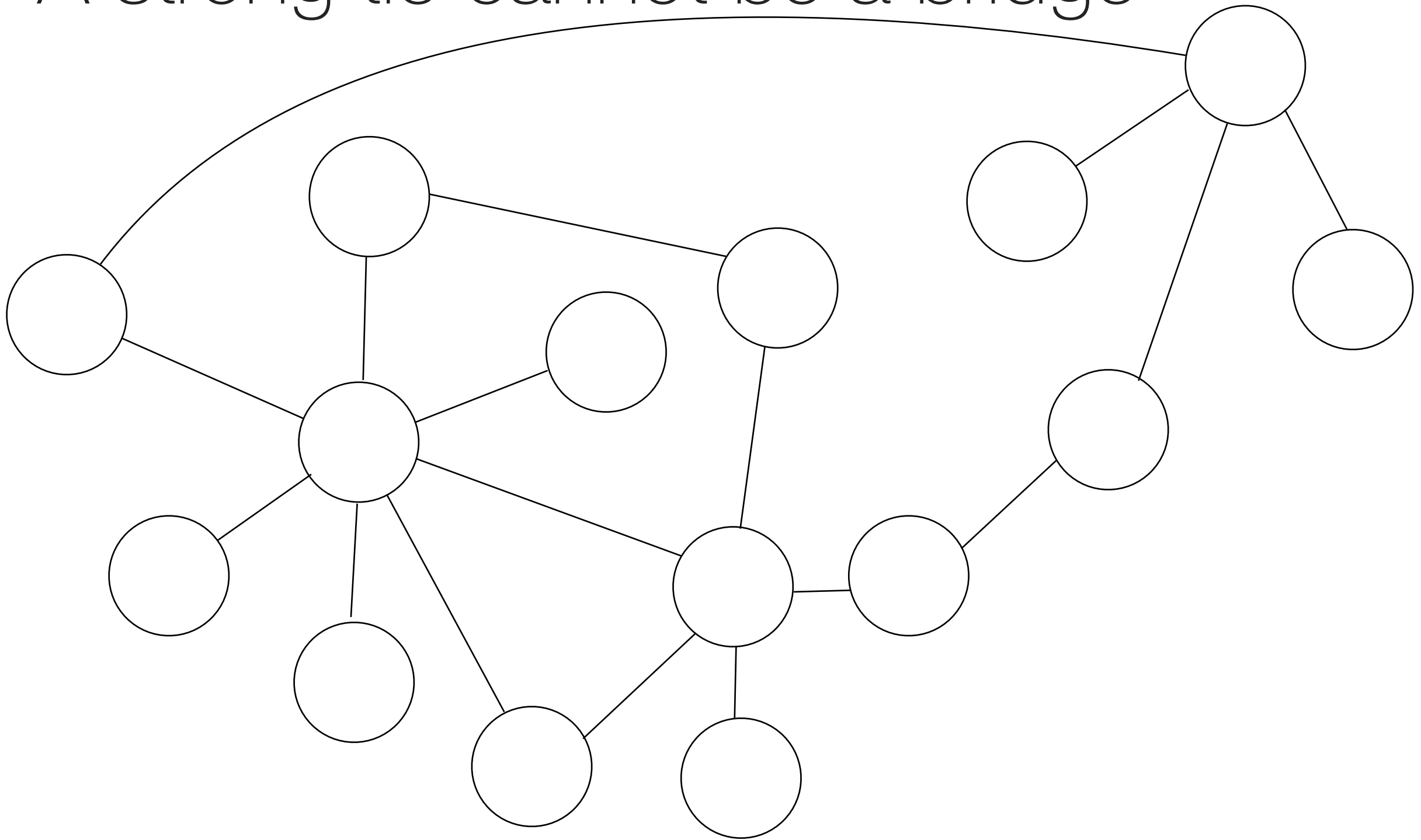
- Influence spreads wider if it spread via weak ties



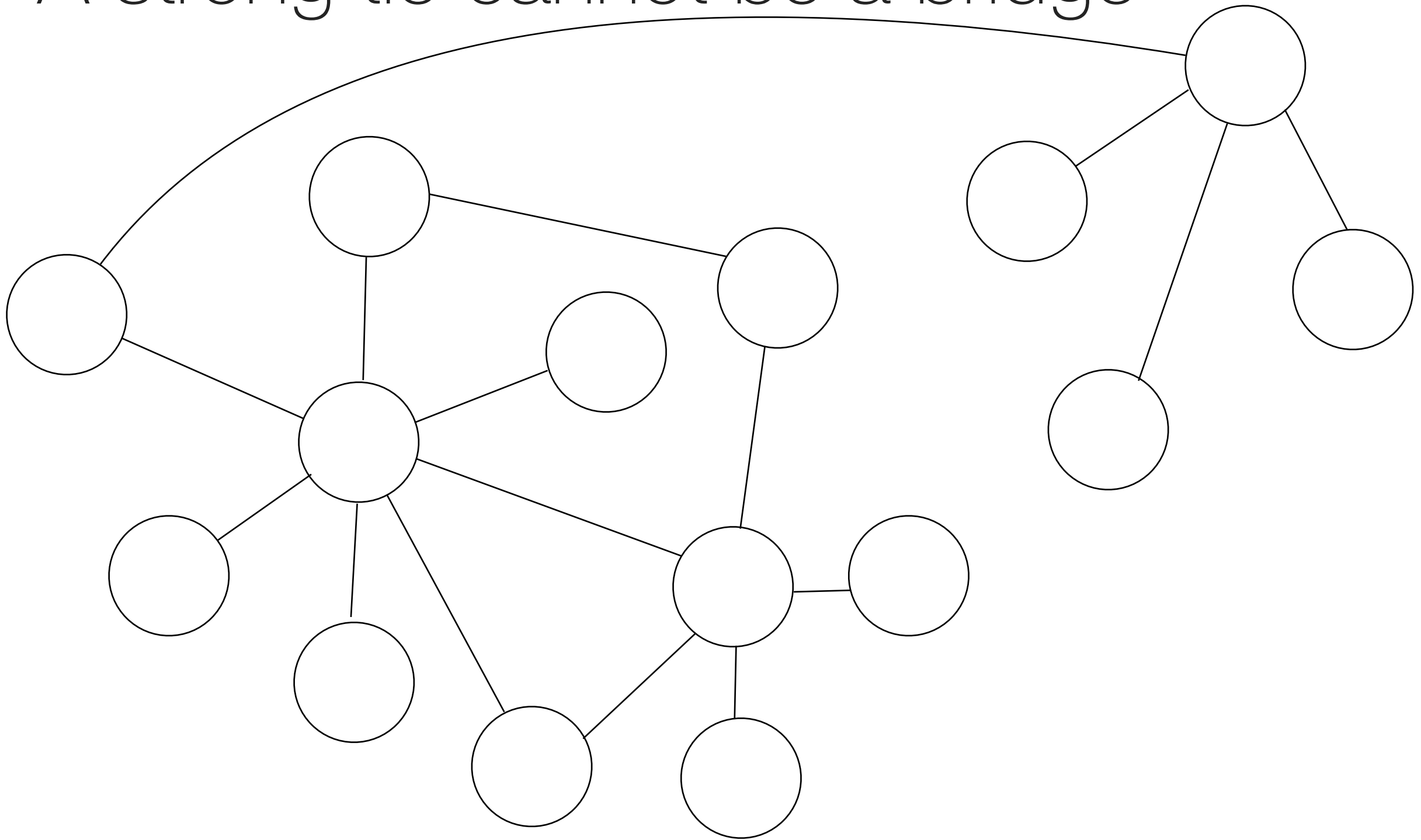
# A strong tie cannot be a bridge



# A strong tie cannot be a bridge



# A strong tie cannot be a bridge



# Simmelian ties



# Simmelian ties

THE TIES THAT TORTURE:  
SIMMELIAN TIE ANALYSIS IN ORGANIZATIONS

David Krackhardt

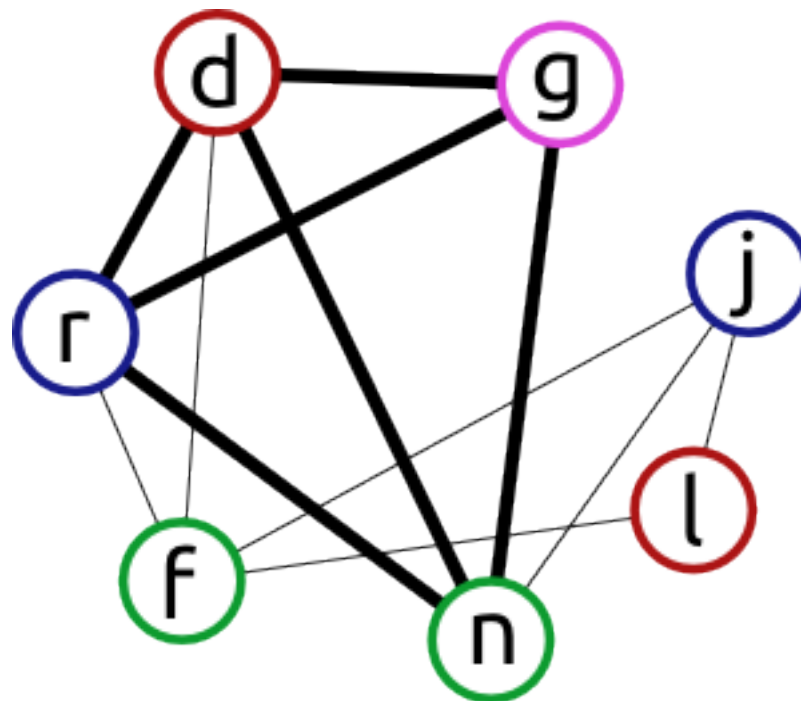
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# Simmelian ties



# Simmelian ties

# Simmelian ties



# Theory of social power

## A formal theory of social power.

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Journal Article

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[French Jr., John R. P.](#)

### Citation

French, J. R. P., Jr. (1956). A formal theory of social power. *Psychological Review*, 63(3), 181-194.

<http://dx.doi.org/10.1037/h0046123>

### Abstract

"This theory illustrates a way by which many complex phenomena about groups can be deduced from a few simple postulates about interpersonal relations. By the application of digraph theory we are able to treat in detail the *patterns of relations* whose importance has long been noted by the field theorists." Three major postulates are presented as well as a variety of theorems dealing with the effects of the power structure of the group, the effects of communication patterns, the effects of patterns of opinion, and leadership. 32 references. (PsycINFO Database Record (c) 2016 APA, all rights reserved)

# DeGroot's Consensus

## Morris Herman DeGroot

<b>Born</b>	June 8, 1931 <a href="#">Scranton, Pennsylvania</a>
<b>Died</b>	November 2, 1989 (aged 58) <a href="#">Pittsburgh, Pennsylvania</a>
<b>Alma mater</b>	<a href="#">Roosevelt University</a> <a href="#">University of Chicago</a>
<b>Awards</b>	<a href="#">ASA Fellow (1966)</a> <sup>[1]</sup> <a href="#">IMS Fellow</a> <sup>[2]</sup> <a href="#">AAAS Fellow</a> <sup>[3]</sup>
<b>Scientific career</b>	
<b>Fields</b>	<a href="#">Statistics</a>
<b>Institutions</b>	<a href="#">Carnegie Mellon University</a>
<b>Doctoral advisor</b>	<a href="#">Leonard Jimmie Savage</a>
<b>Doctoral students</b>	<a href="#">Kathryn Chaloner</a>

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<b>Awards</b>	ASA Fellows IMS Fellows AAAS Fellow
<b>Fields</b>	Statistical Decision Theory
<b>Institutions</b>	Carnegie Mellon University
<b>Doctoral advisor</b>	Leonard J. Savage
<b>Doctoral students</b>	Kathryn Chaloner

## Reaching a Consensus

Morris H. DeGroot

*Journal of the American Statistical Association*, Volume 69, Issue 345 (Mar., 1974), 118-121.



# DeGroot's Consensus



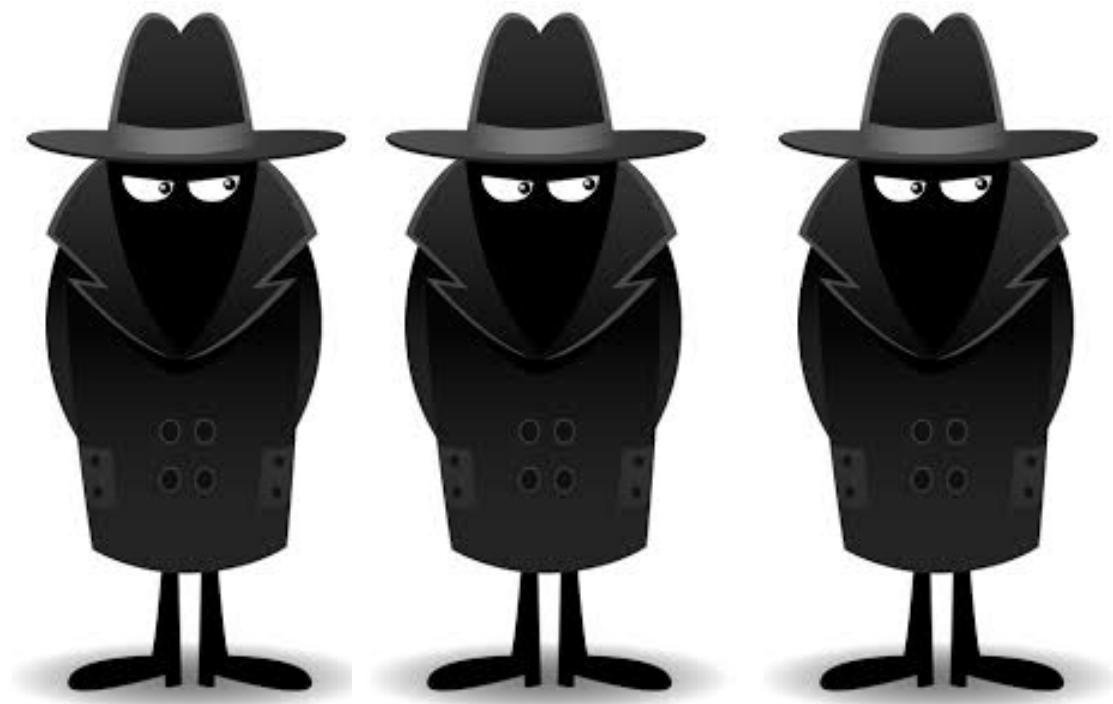
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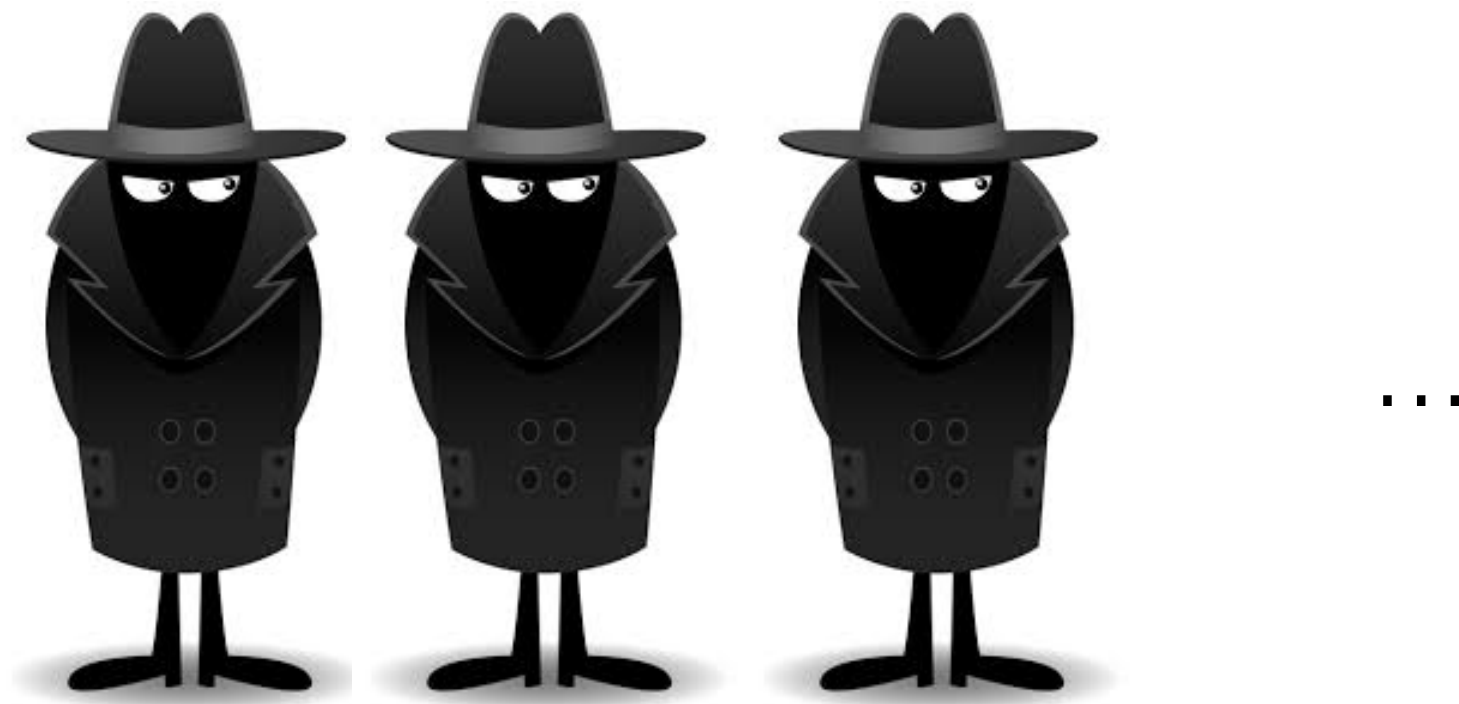
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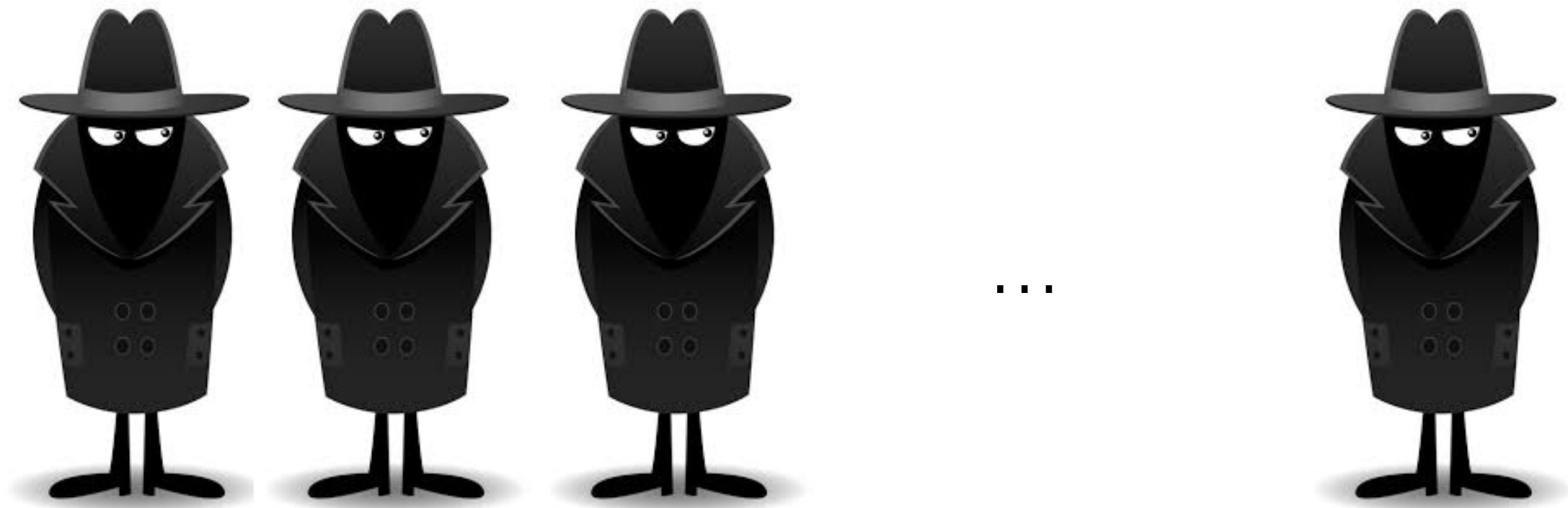
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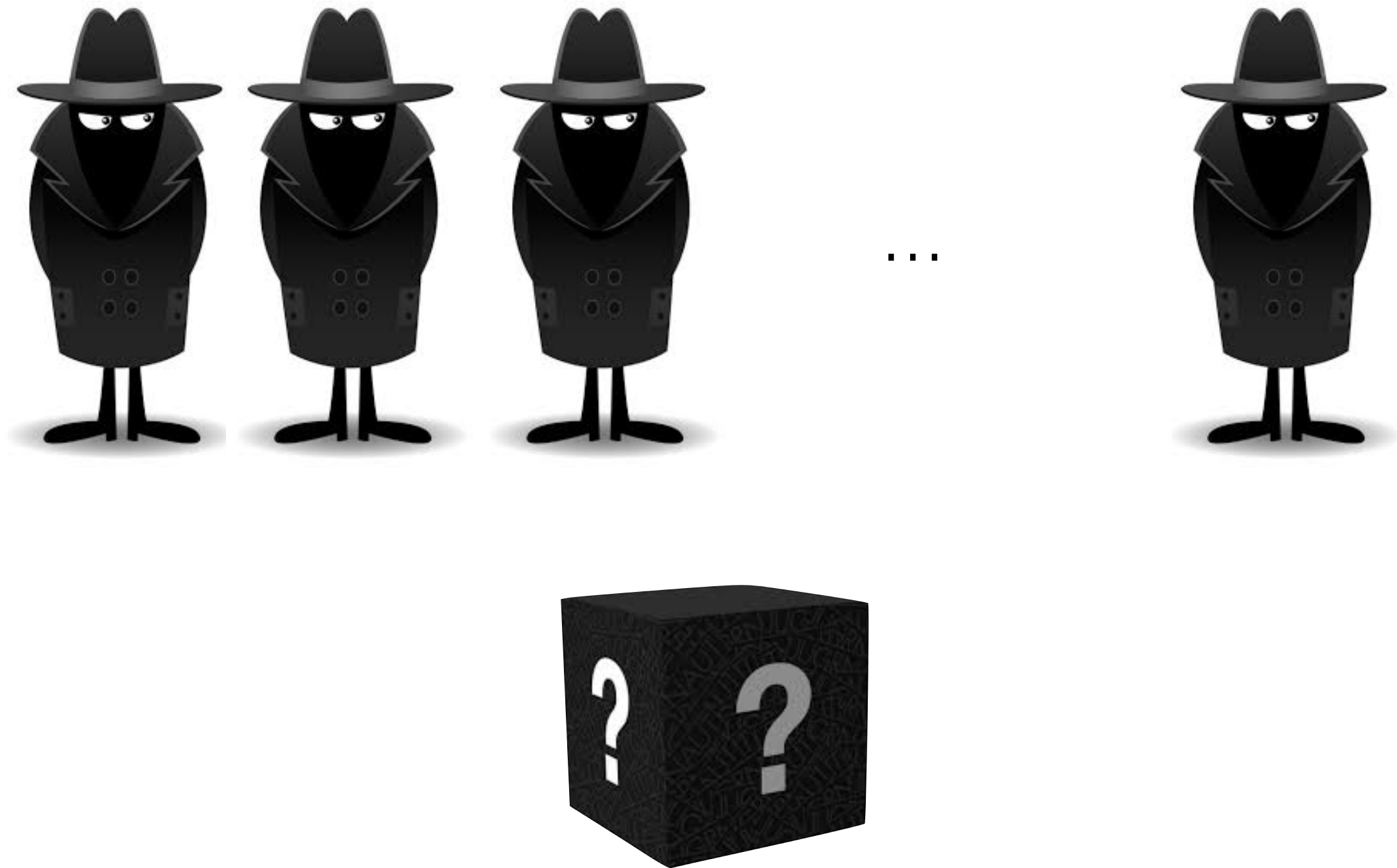
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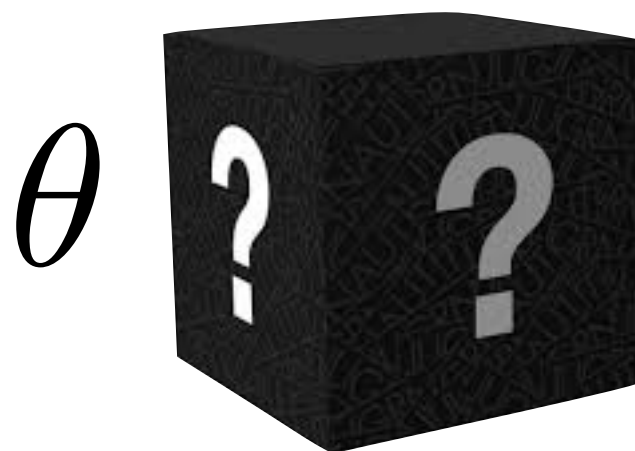
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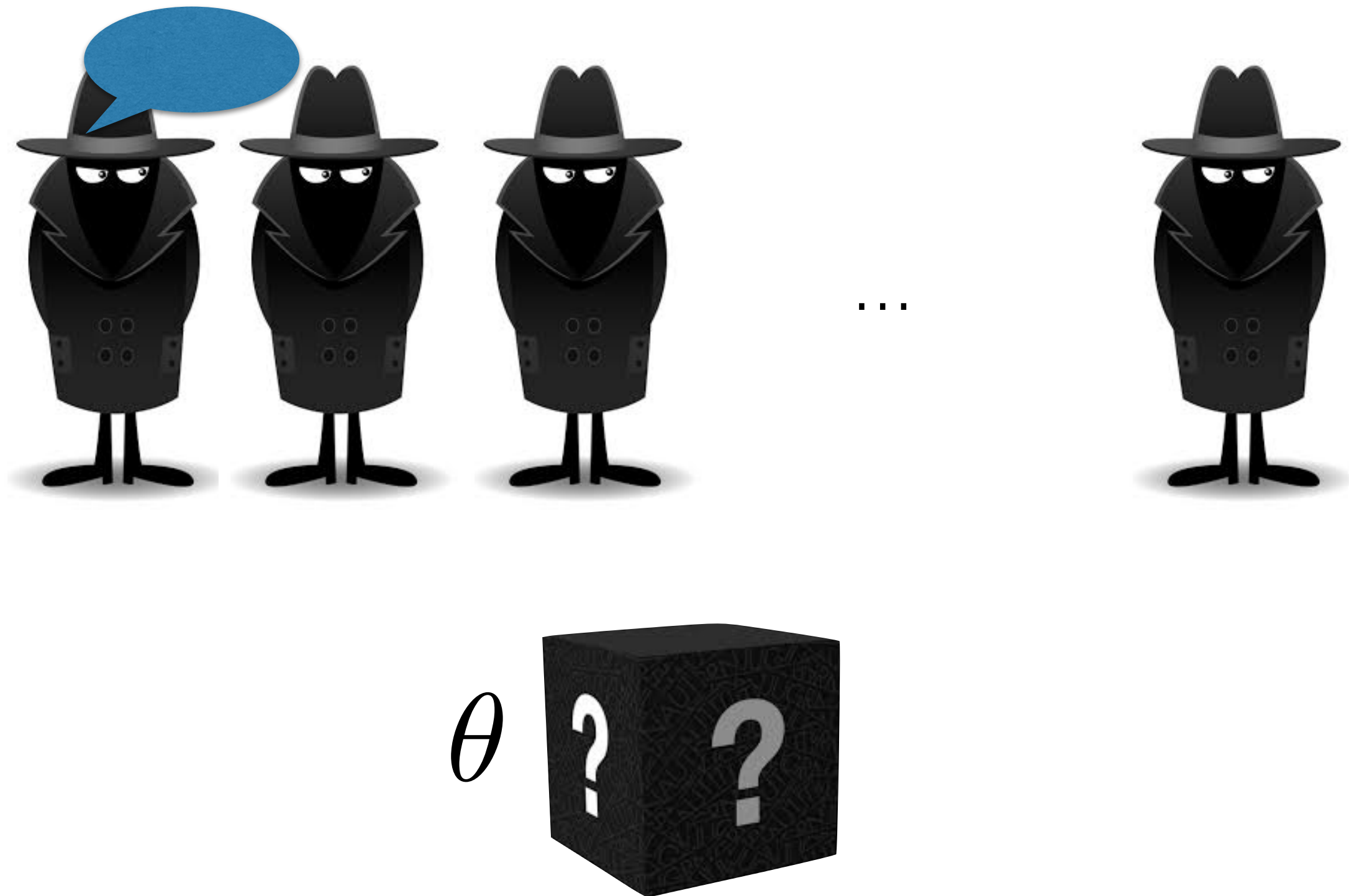
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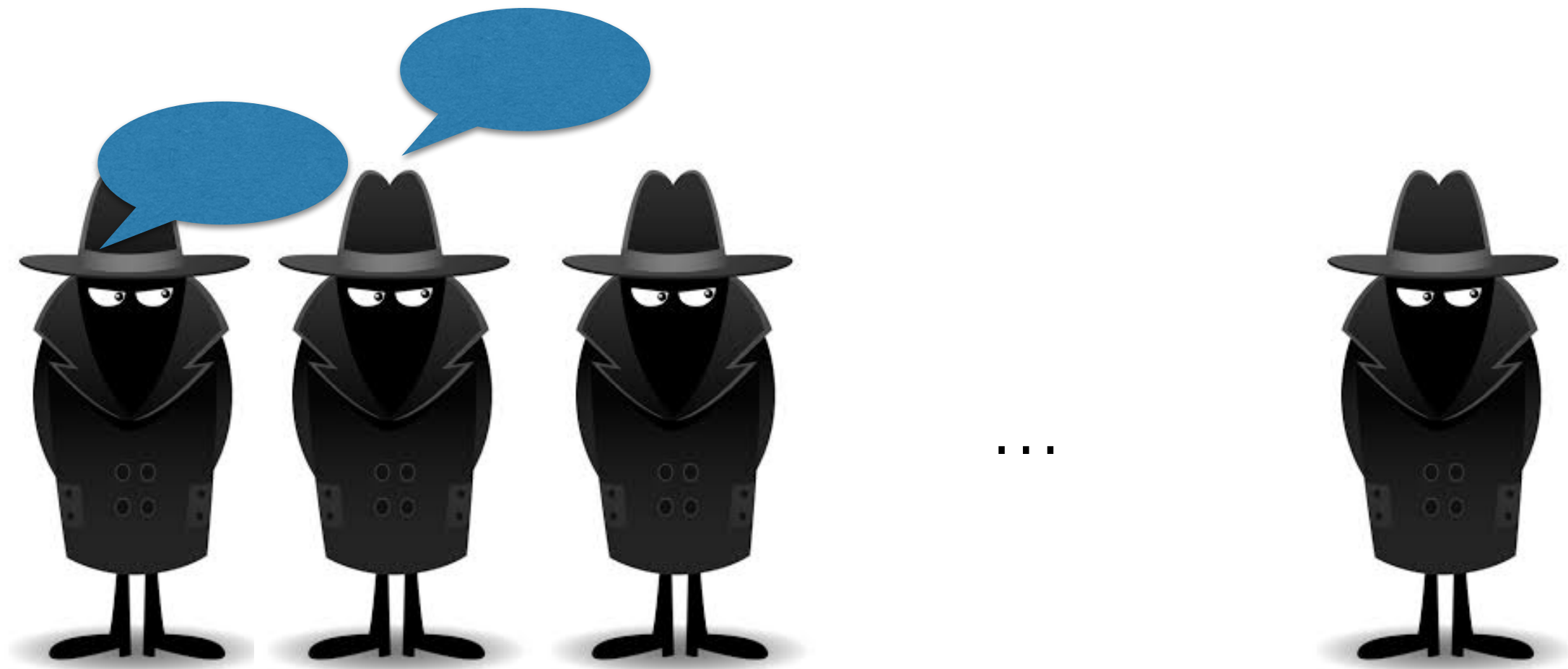


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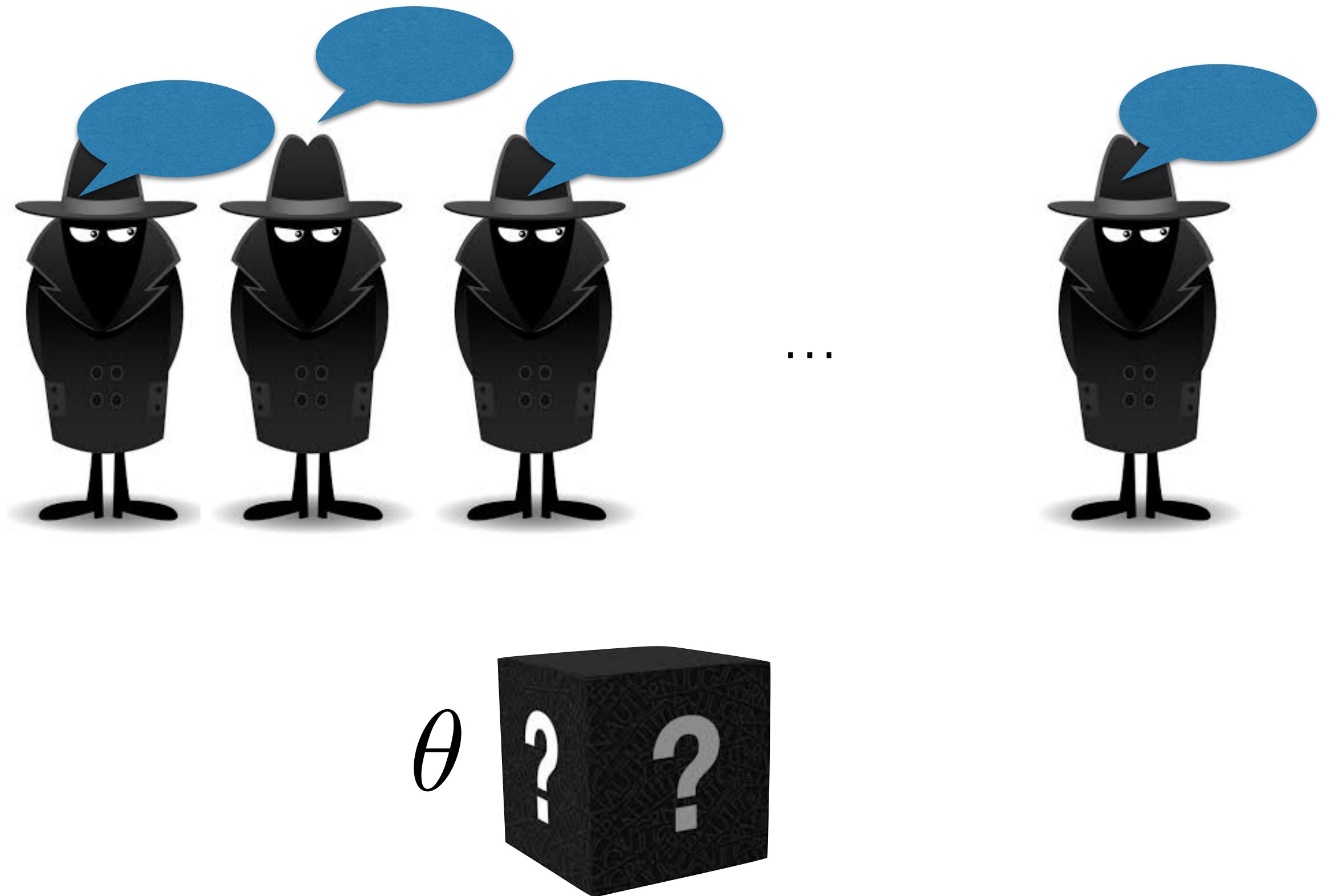
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$$\lim_{n \rightarrow \infty} F_{in} = F^* \text{ for } i = 1, \dots, k$$

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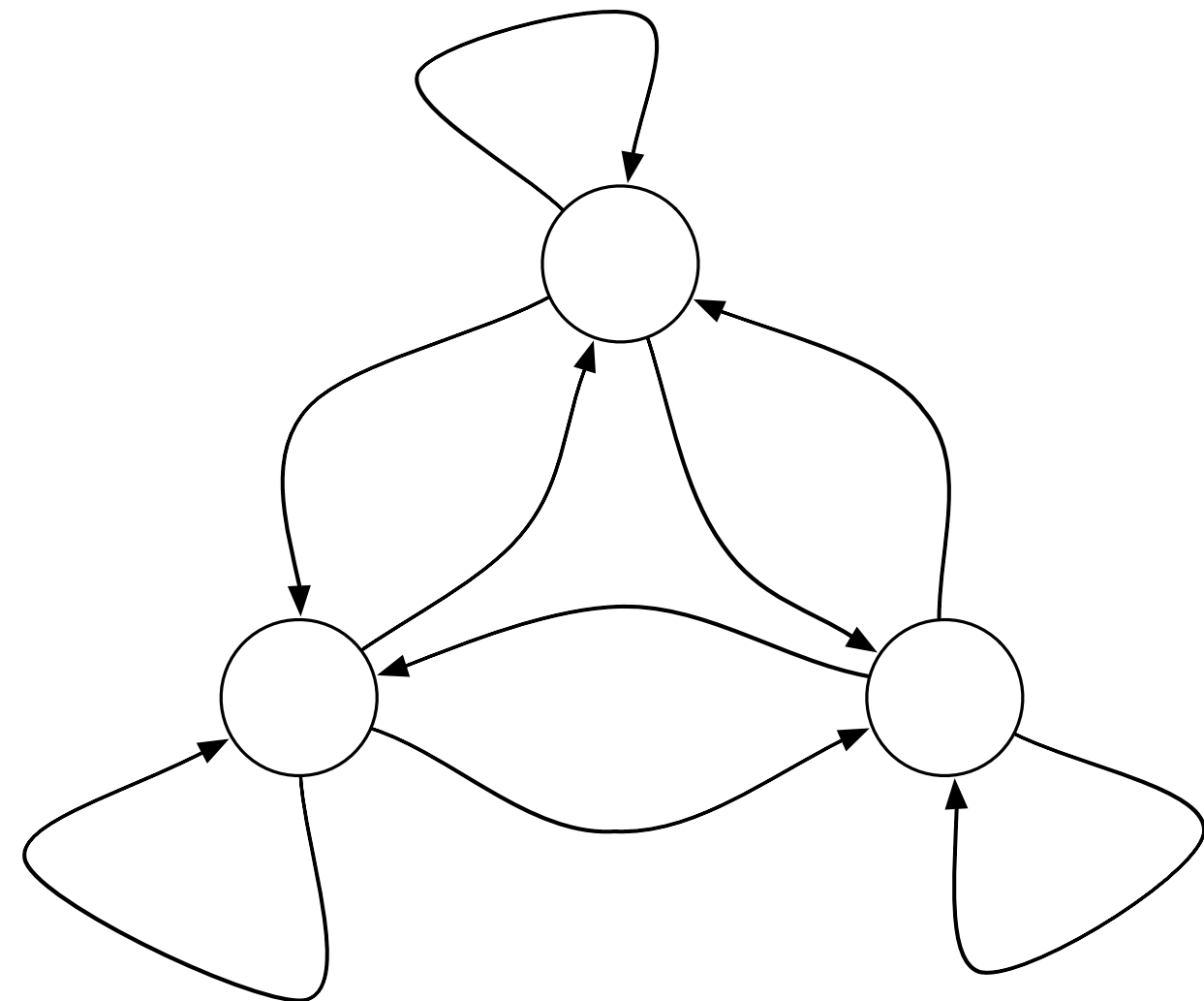
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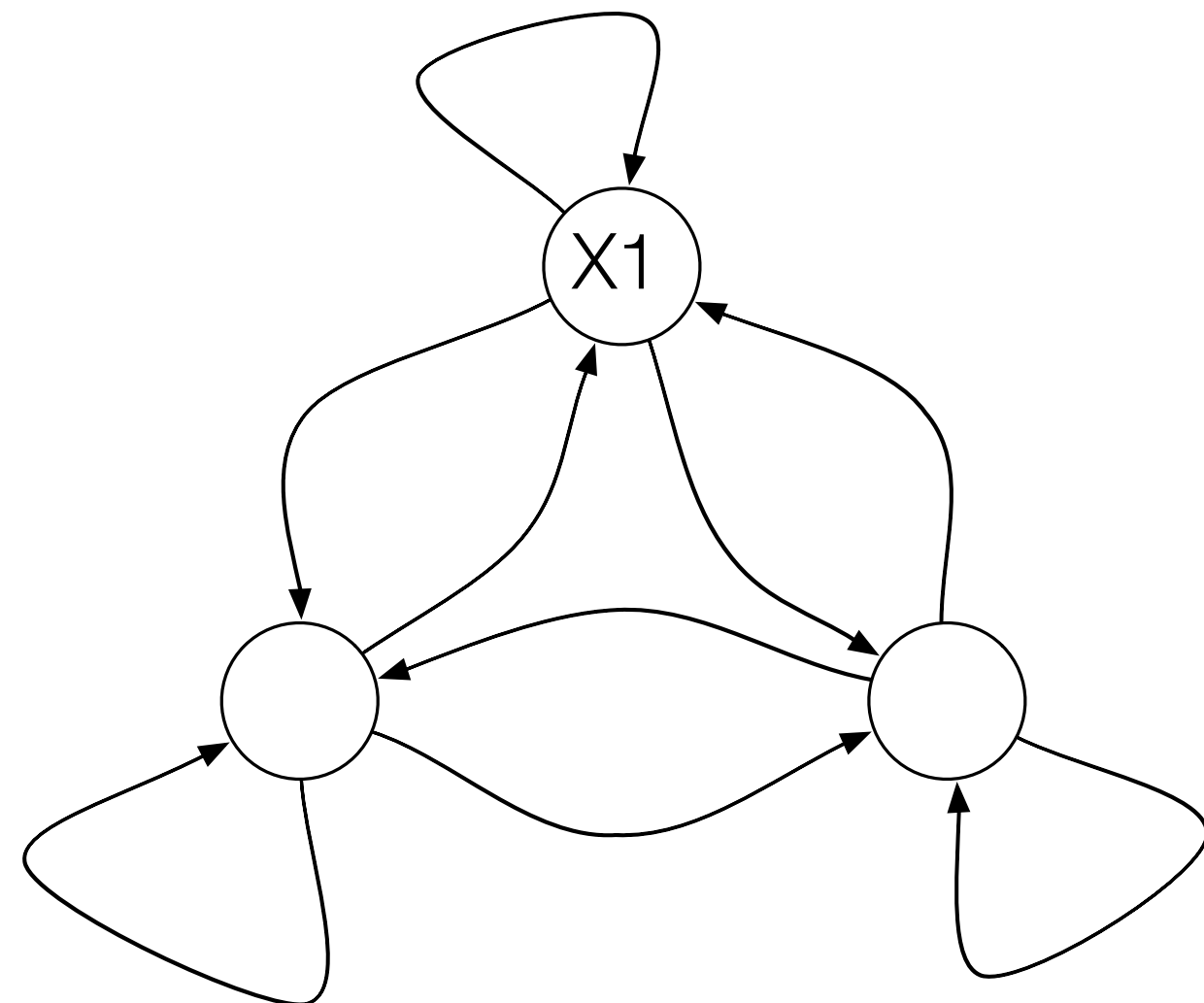
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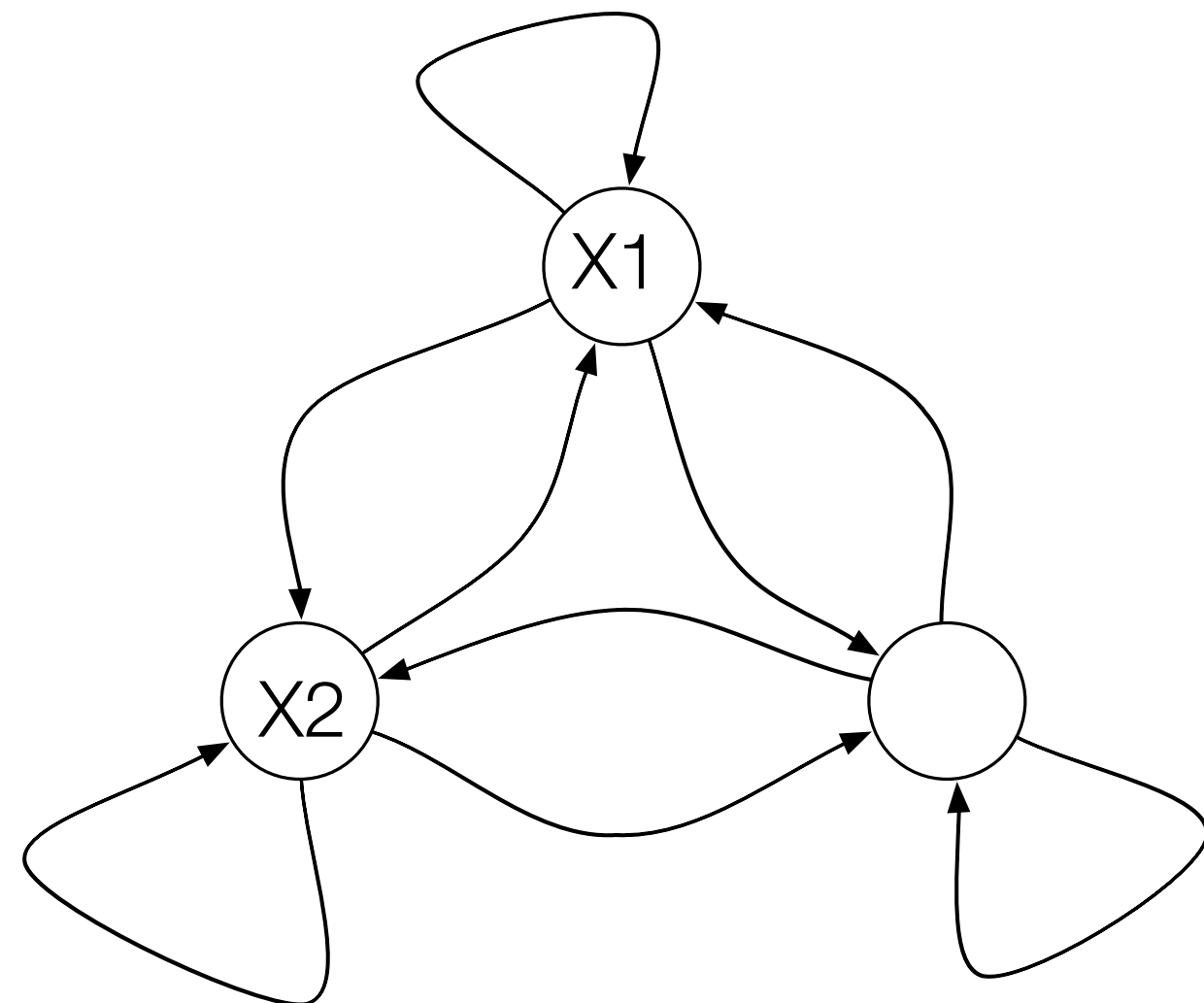
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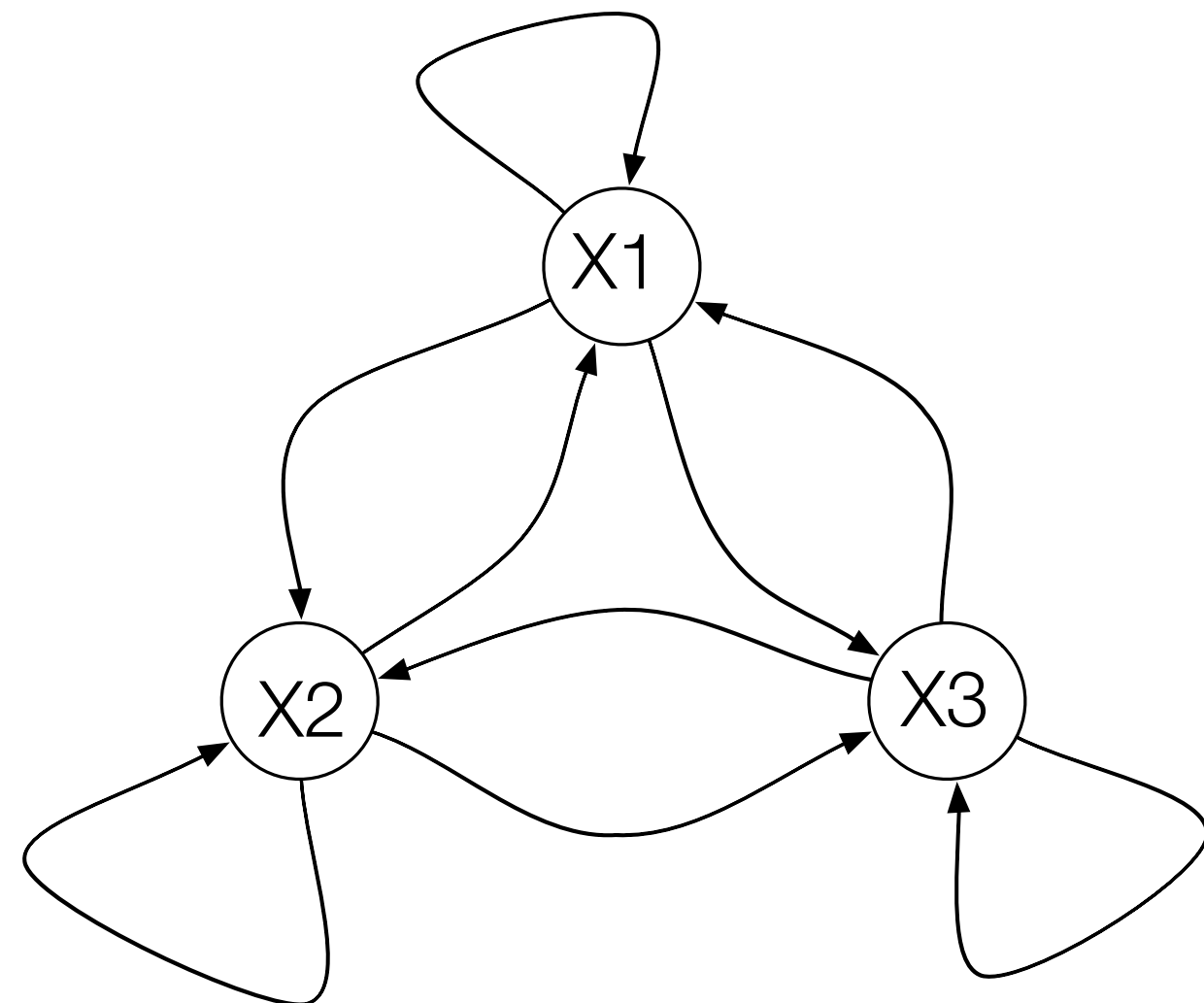
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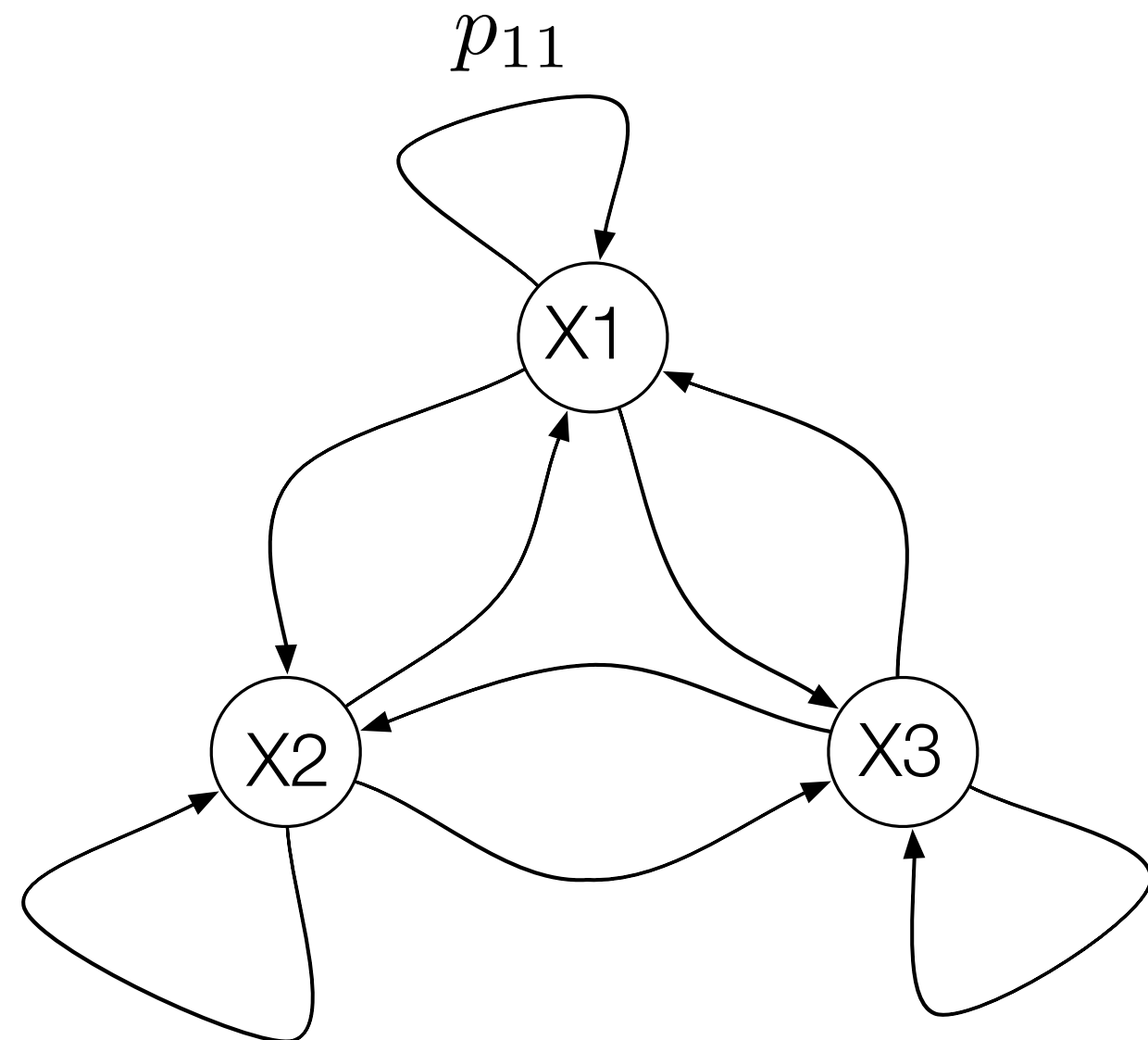
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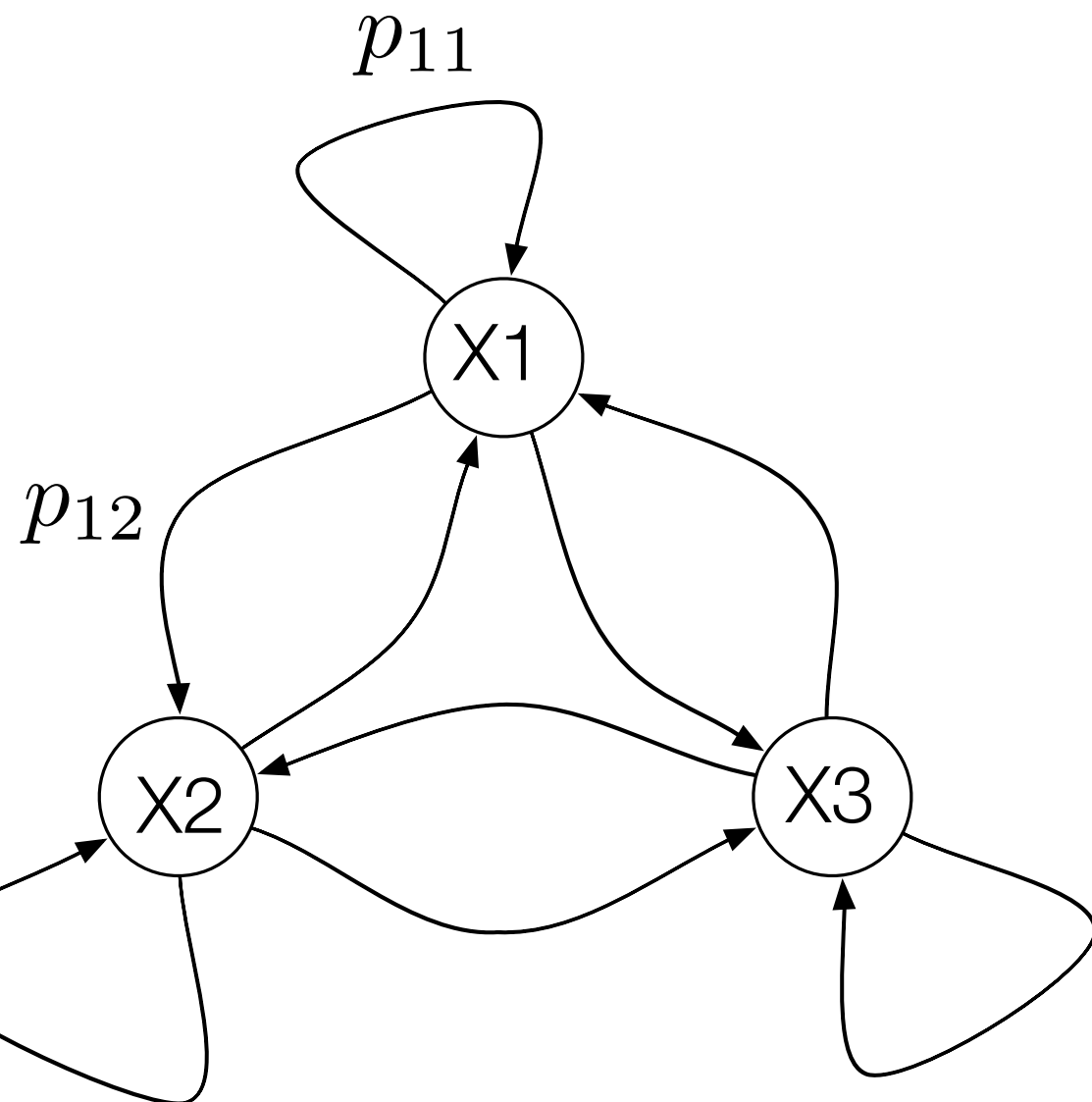
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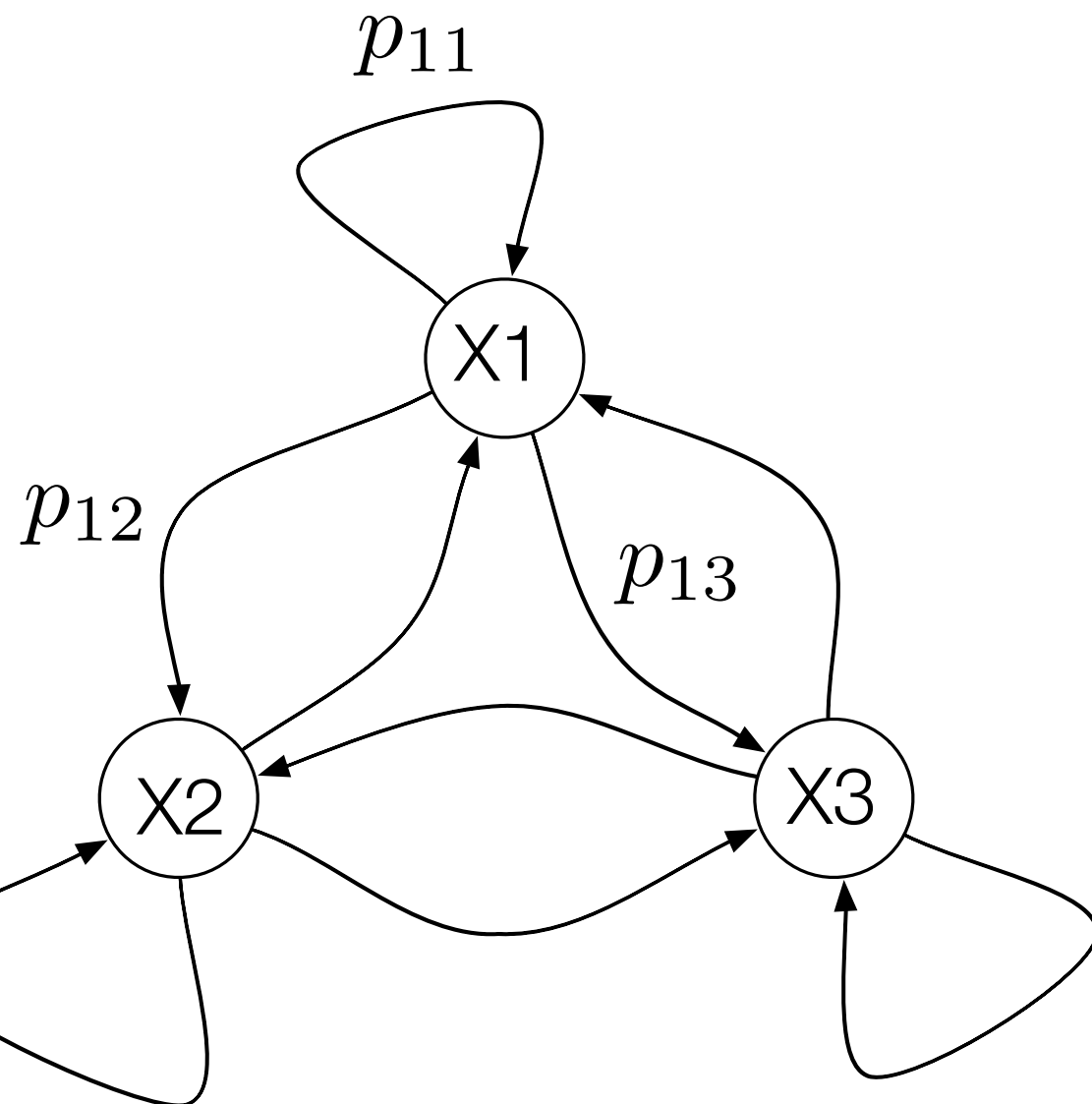
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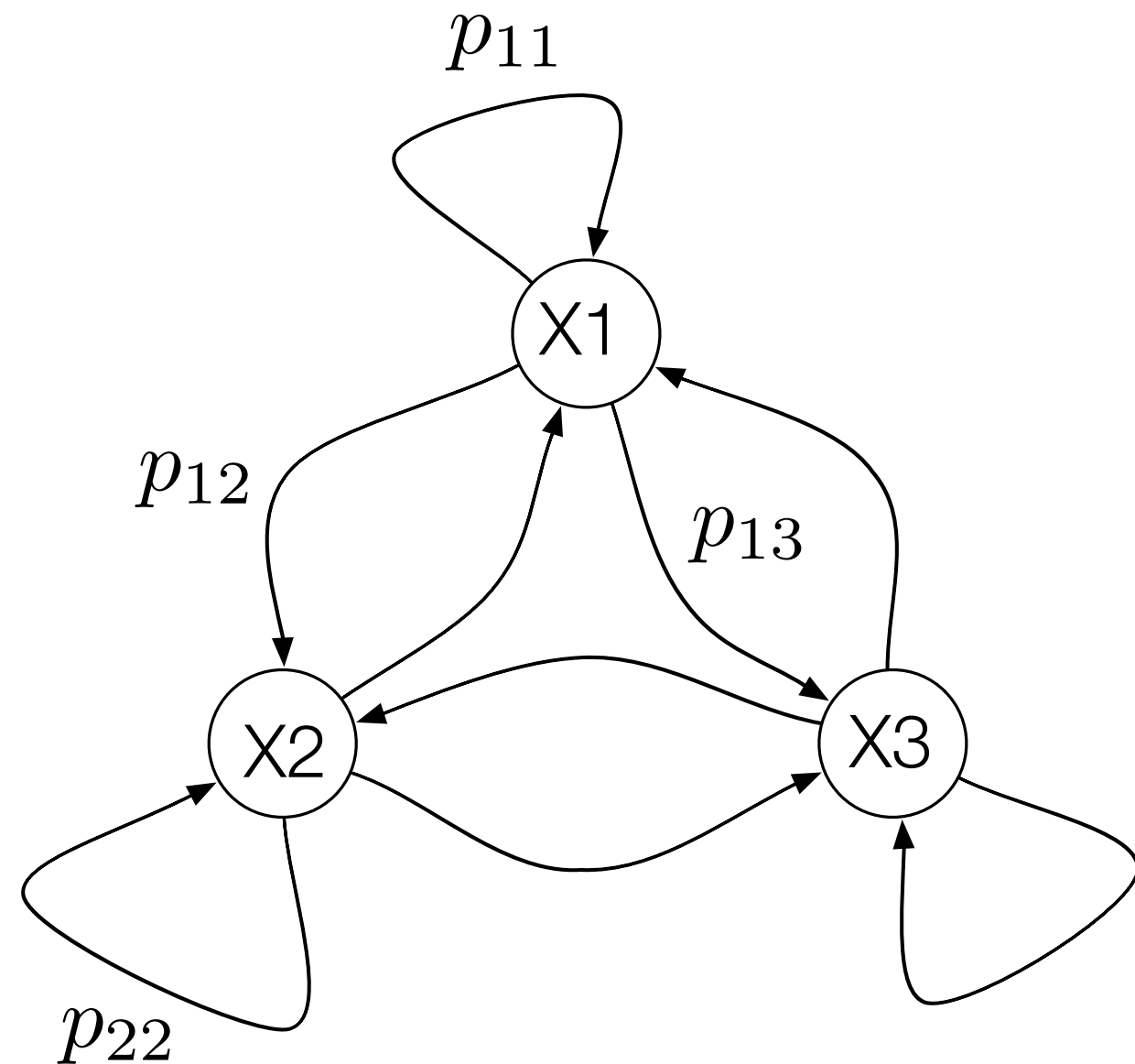
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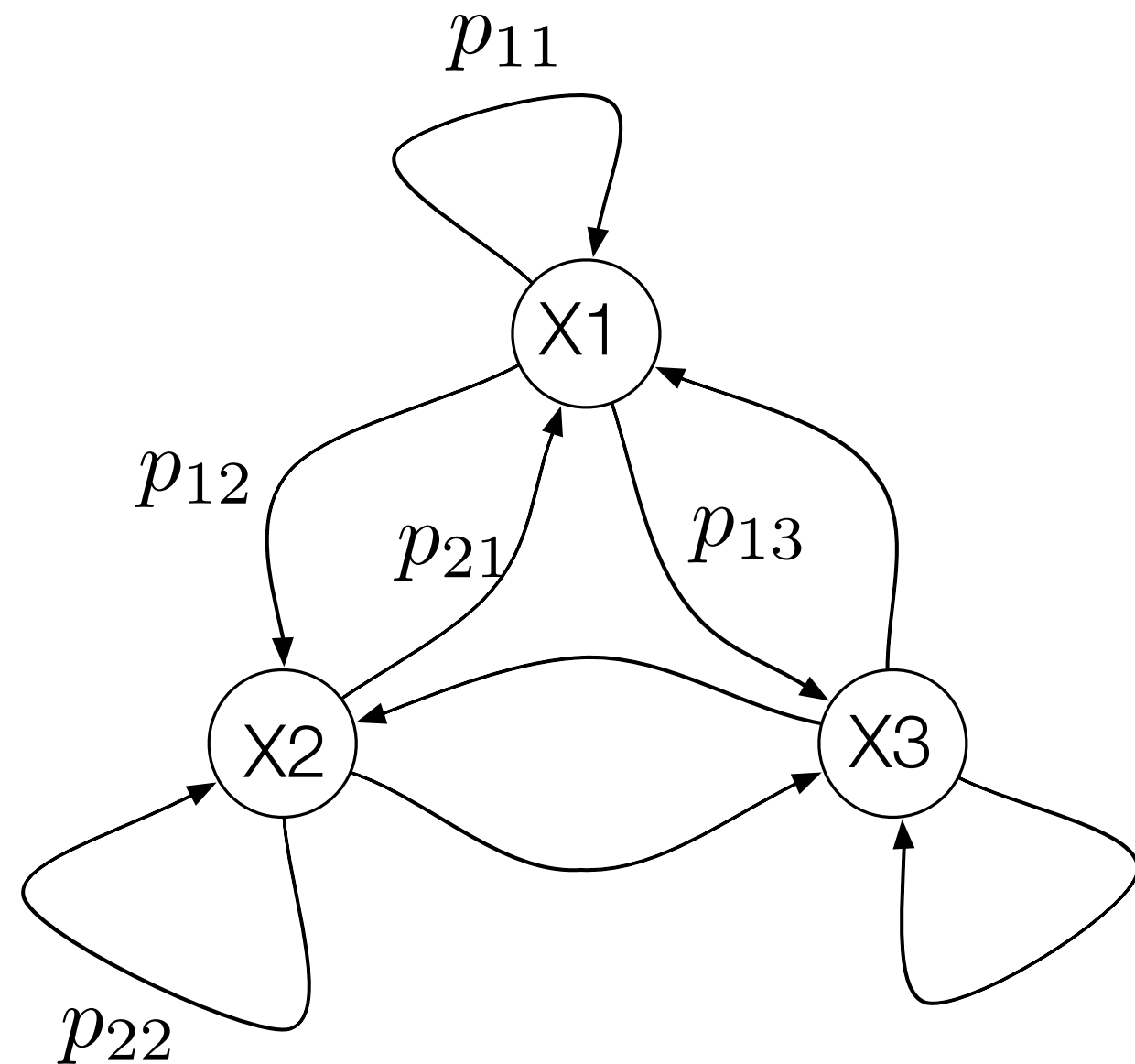
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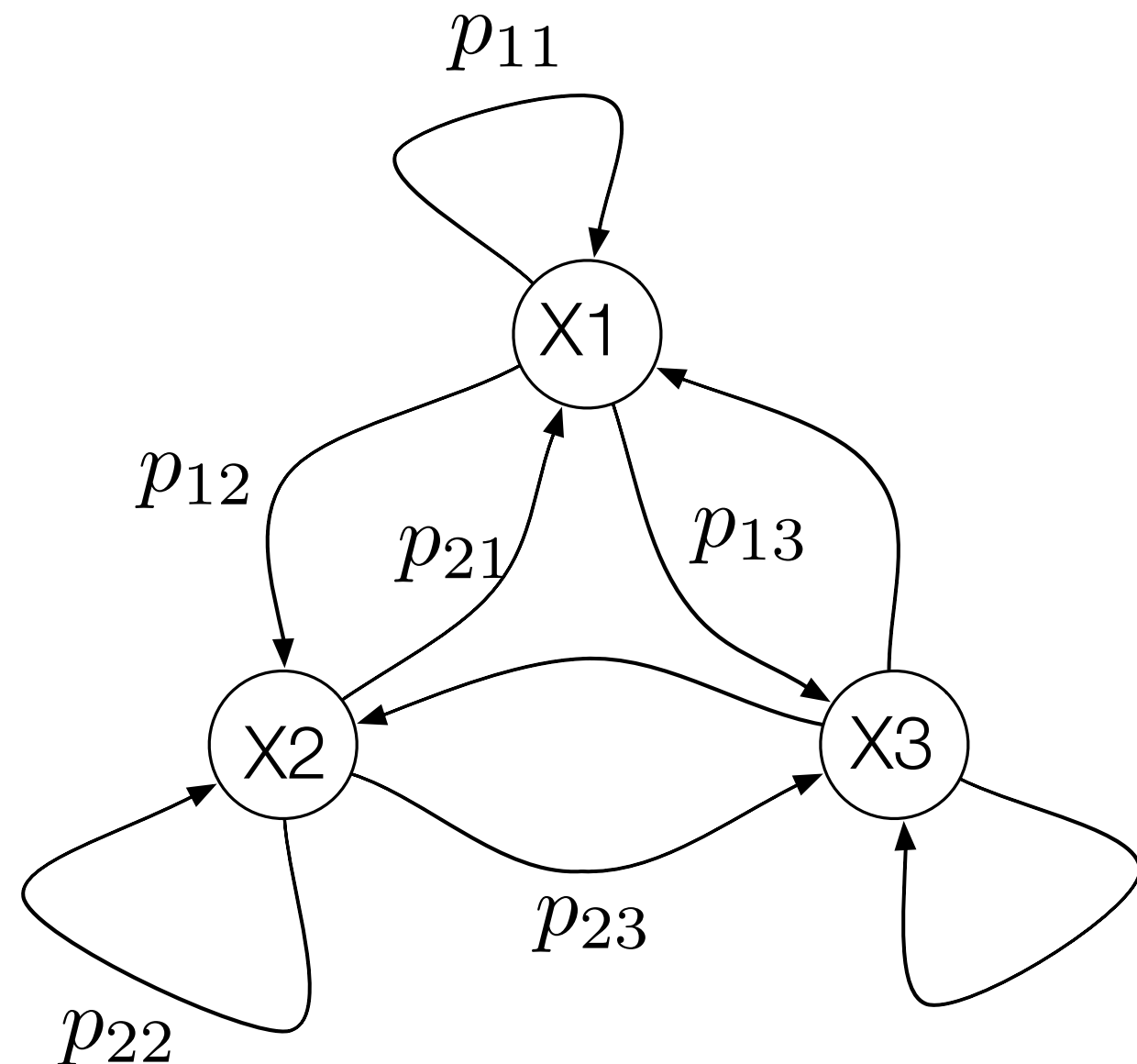
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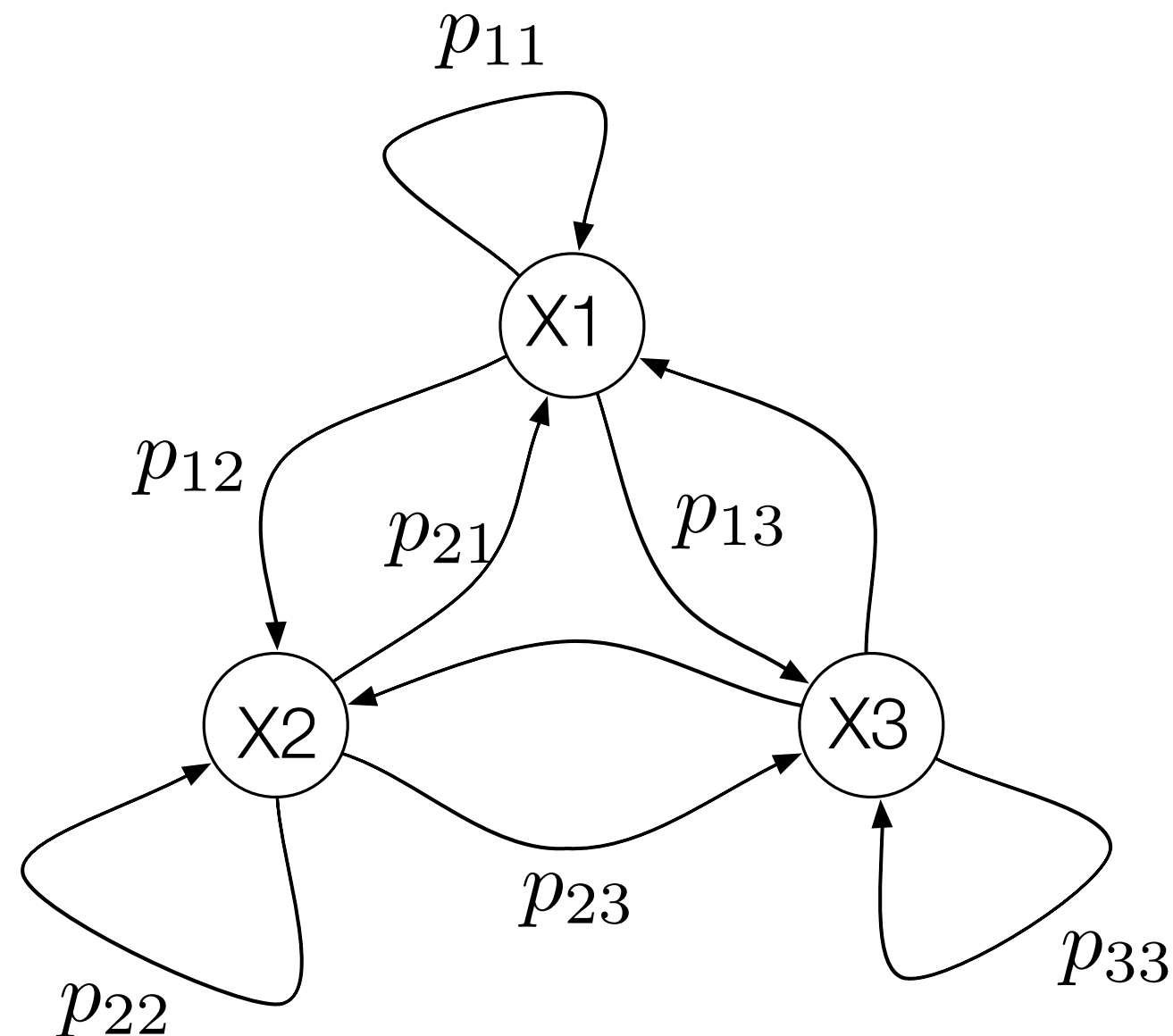
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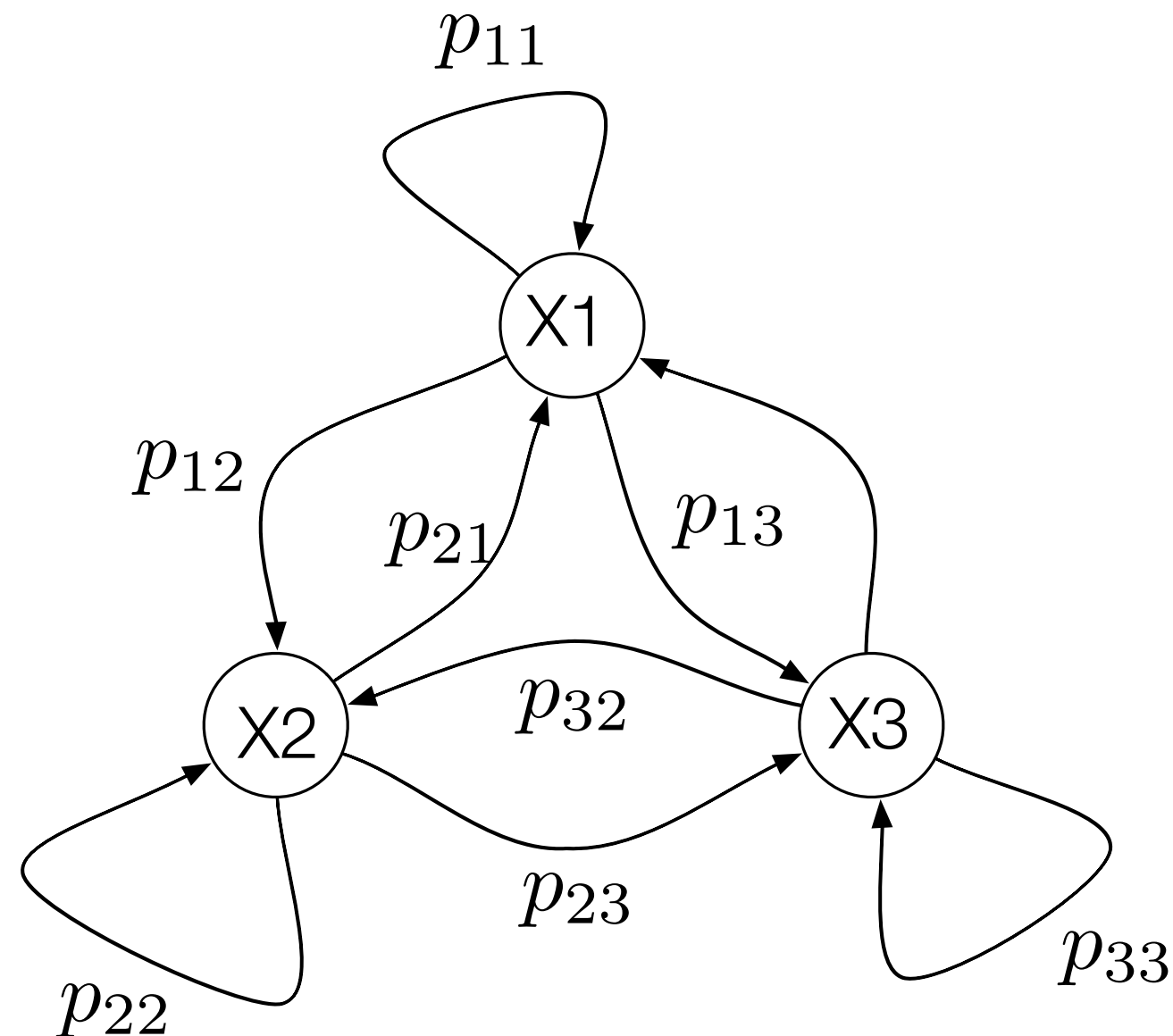
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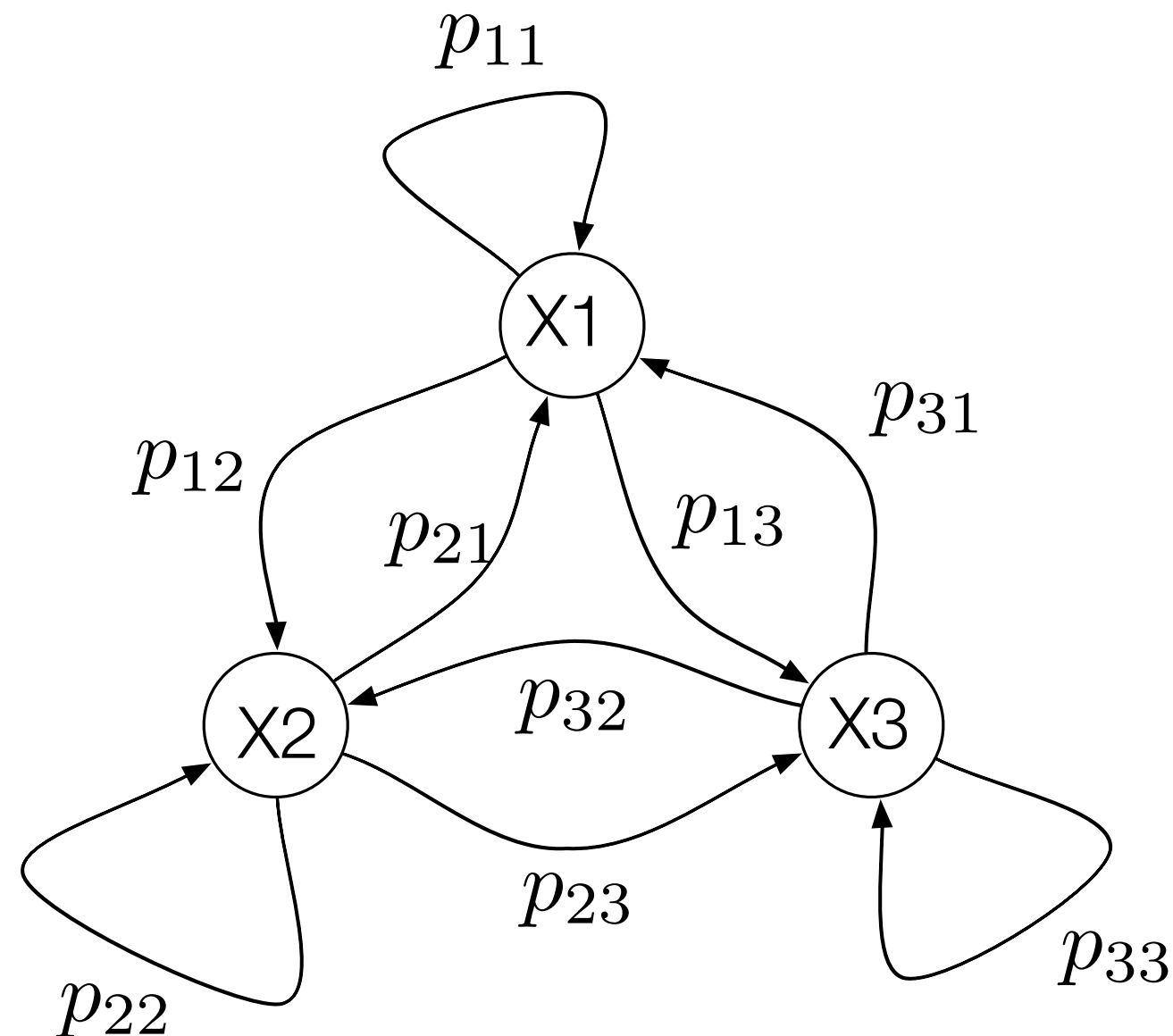
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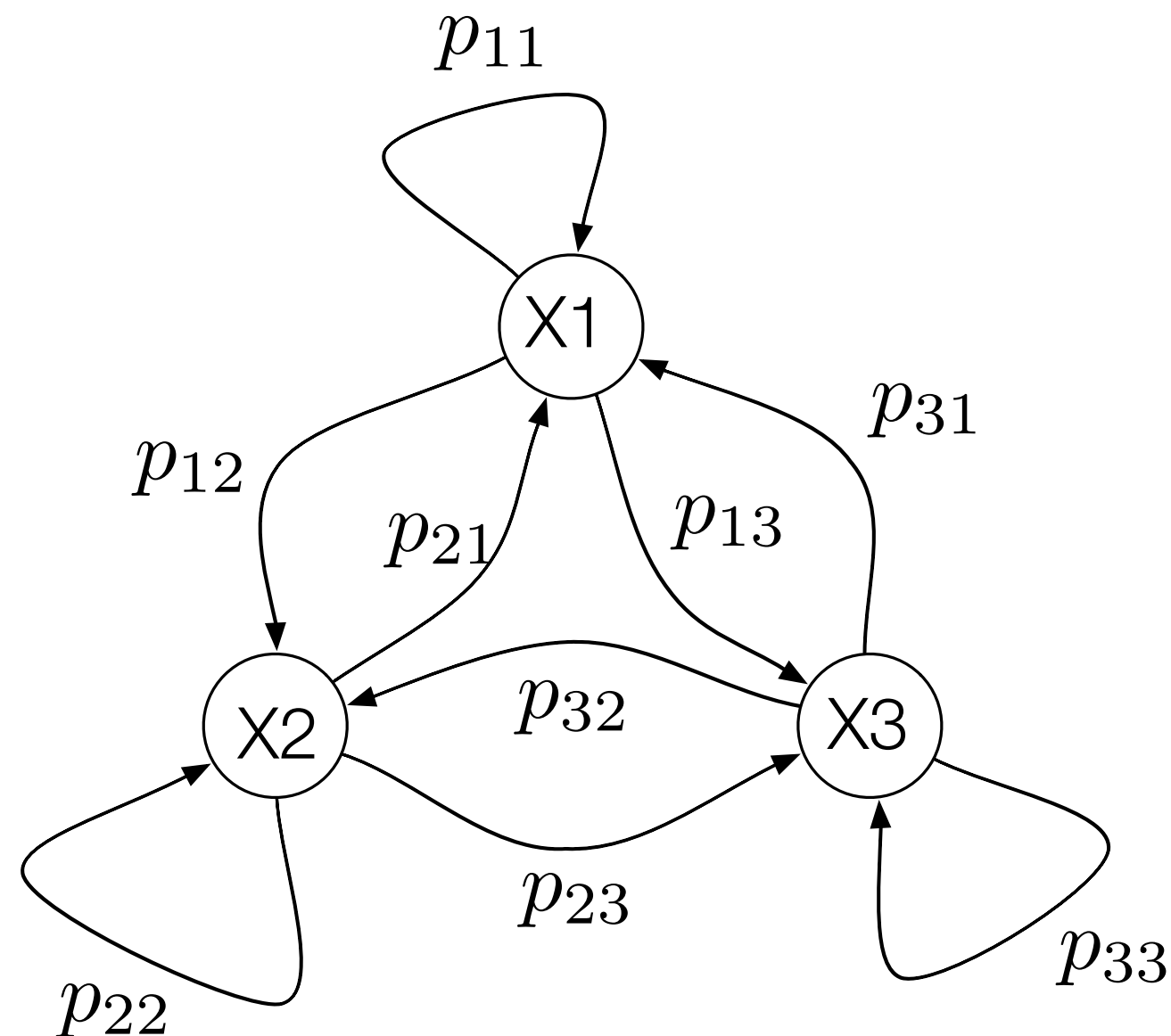
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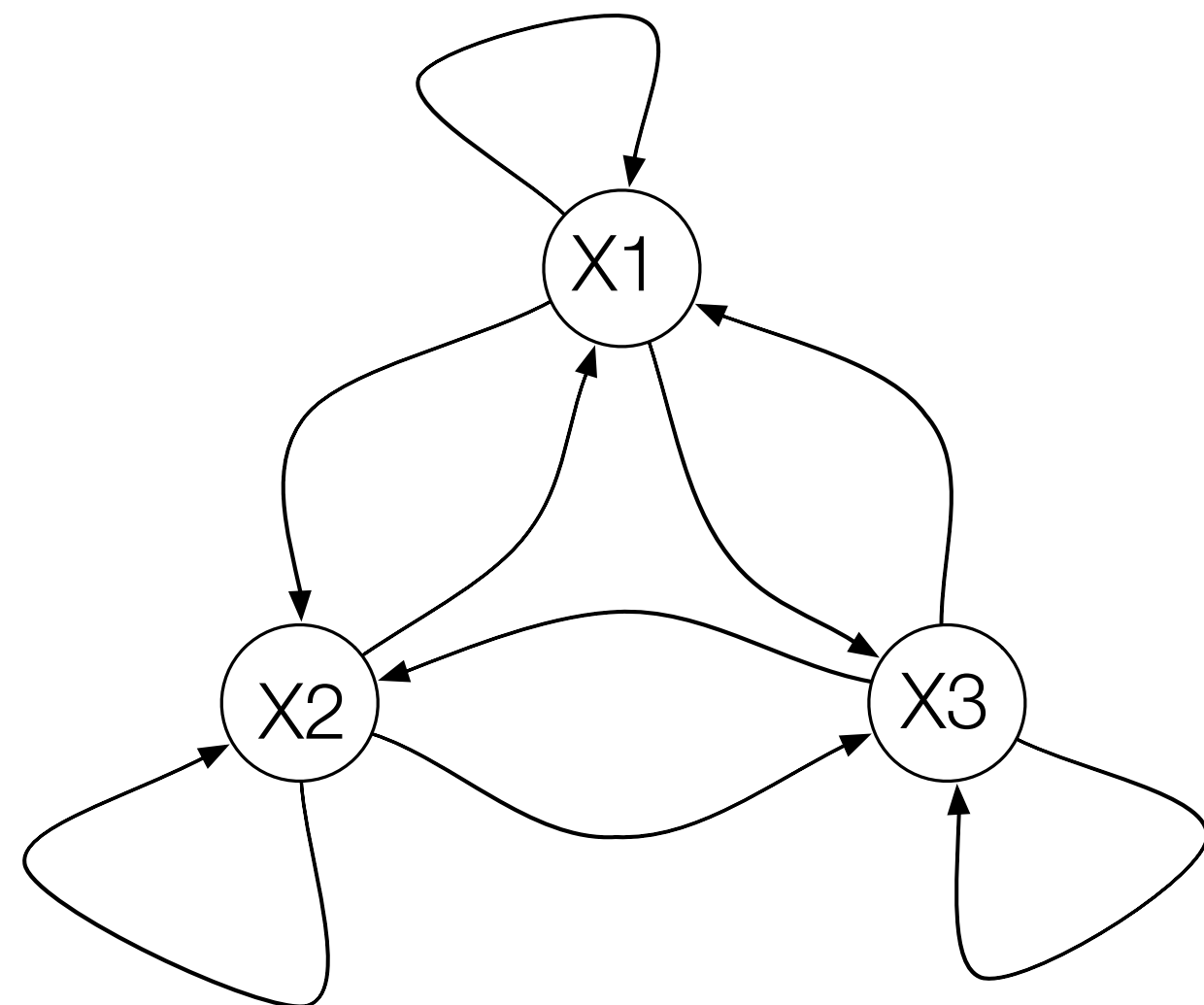
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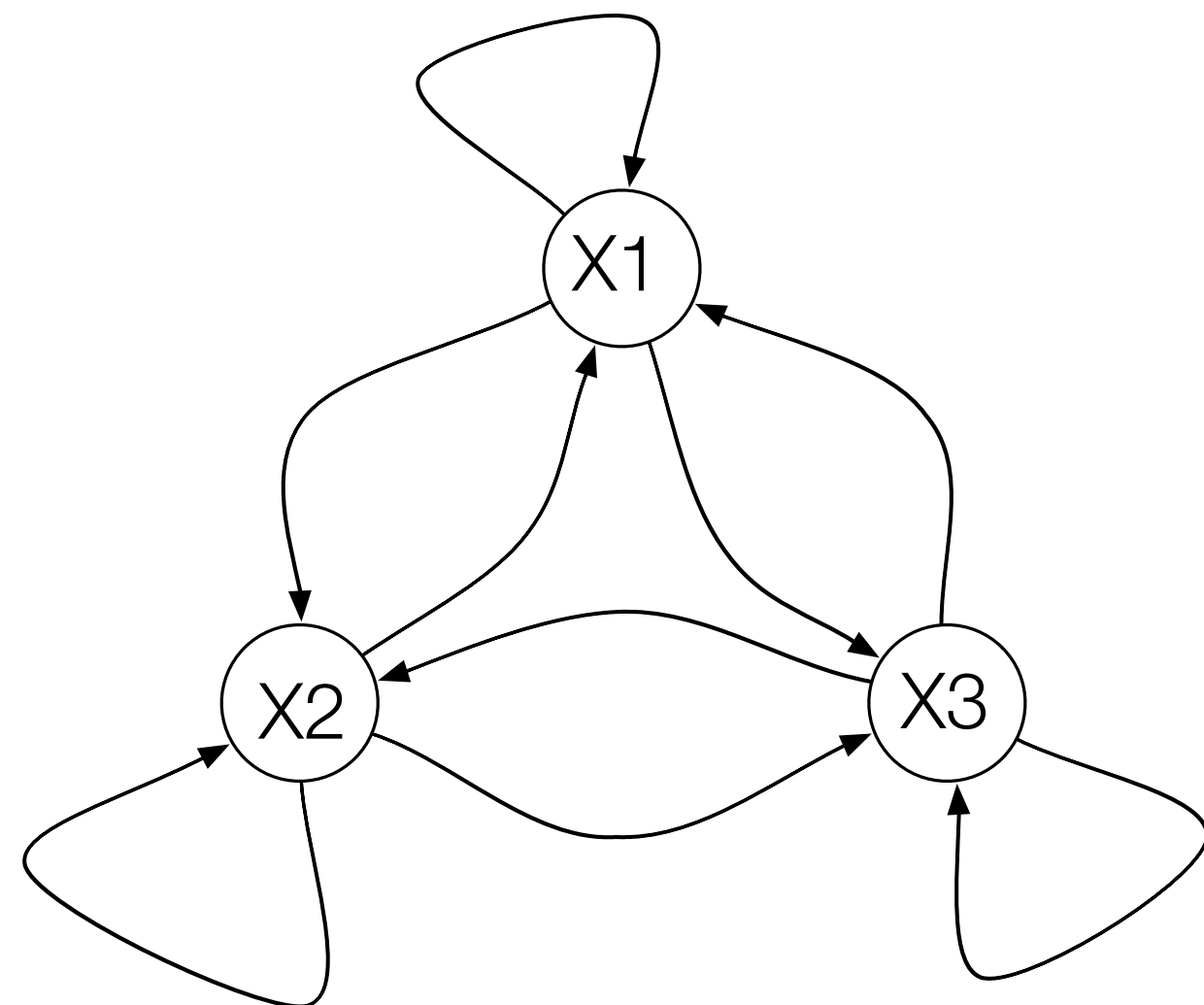
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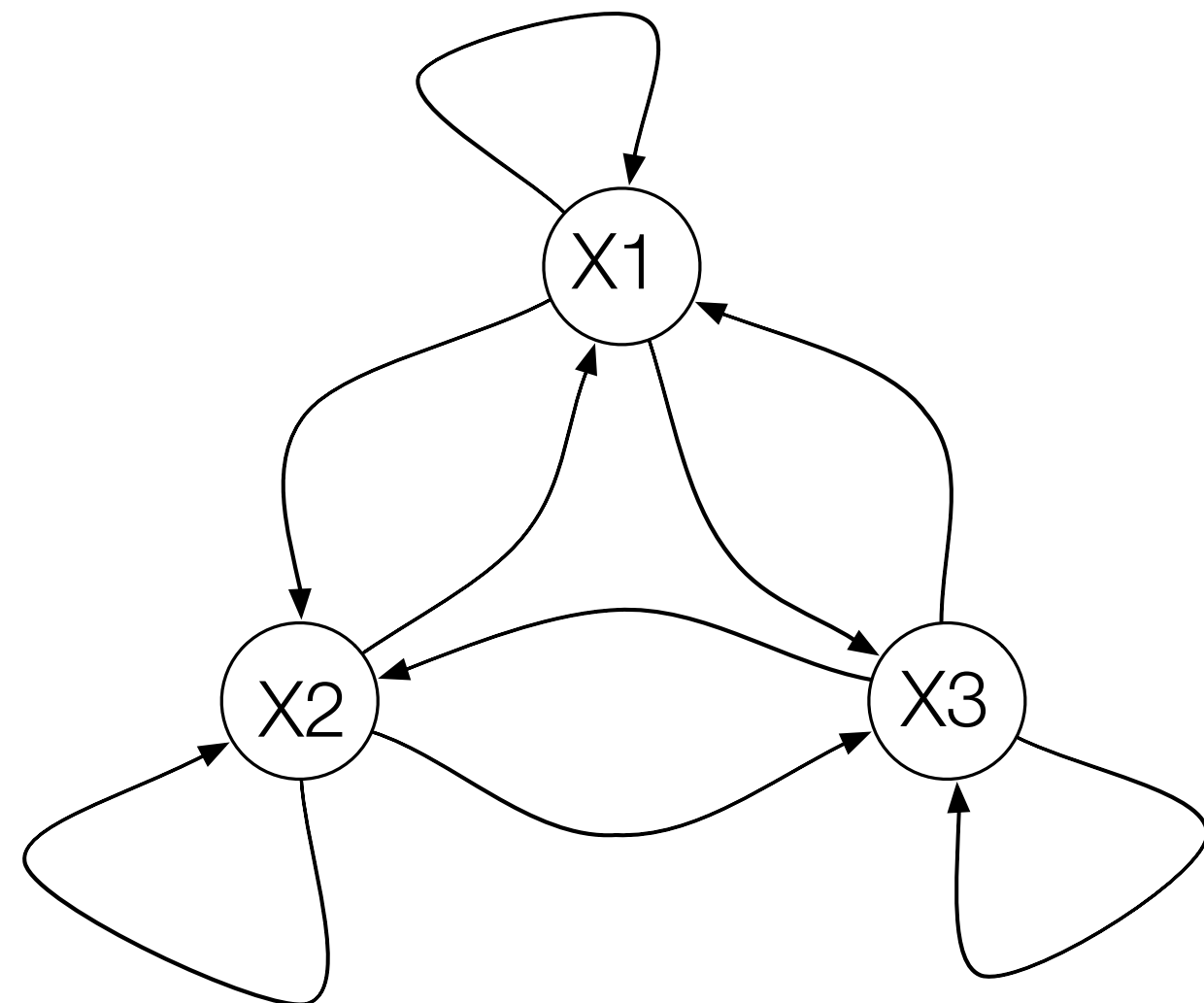
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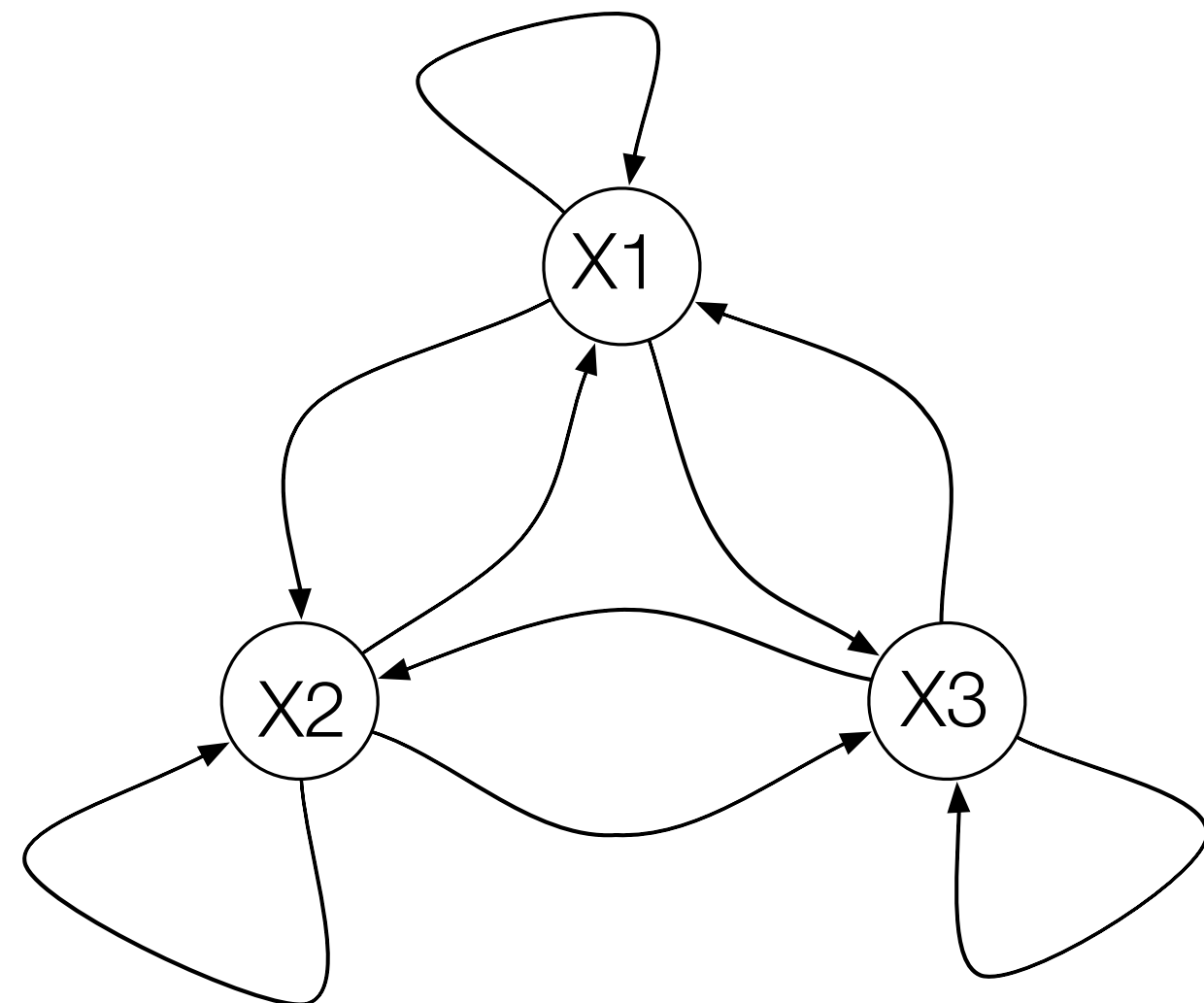


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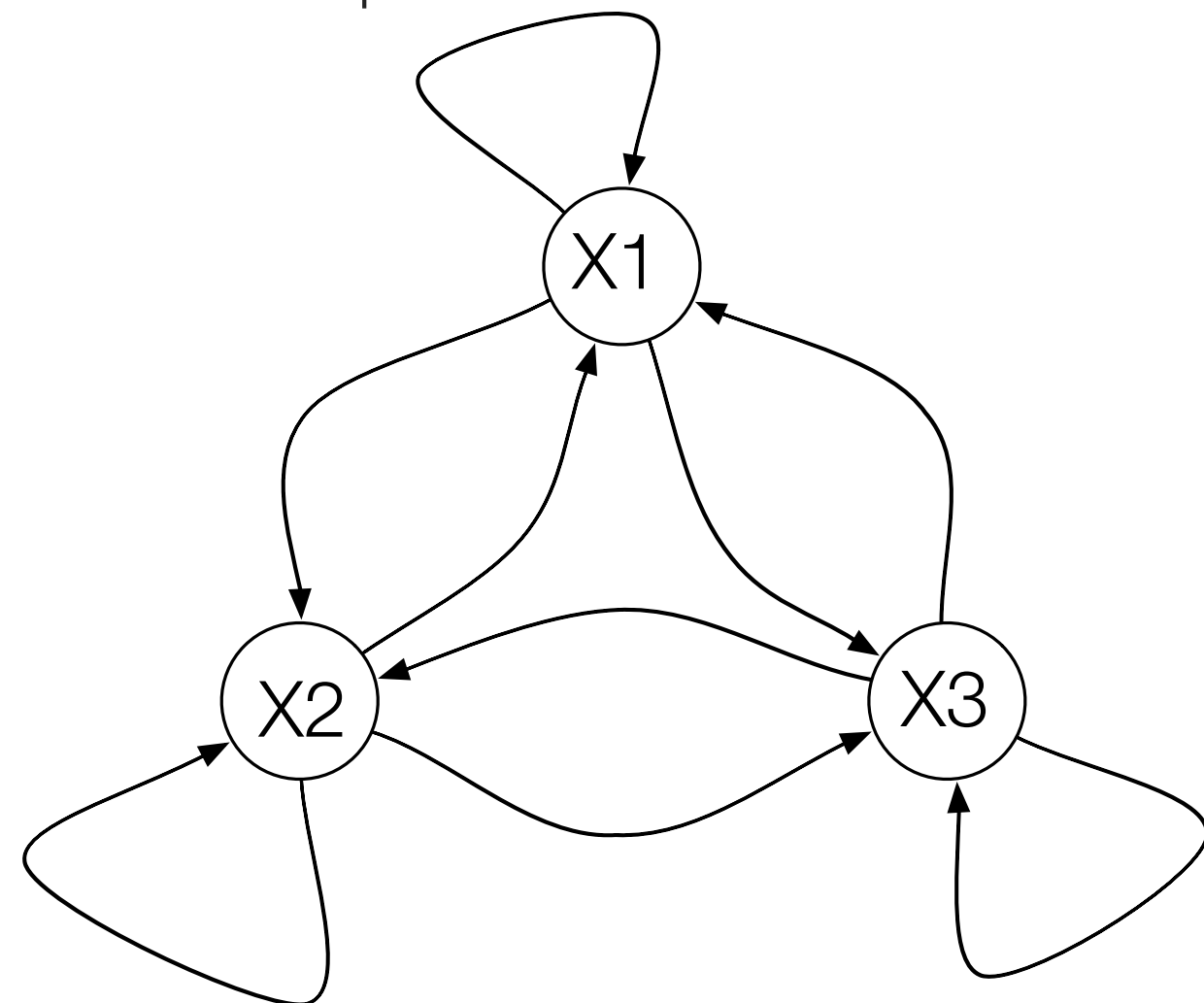
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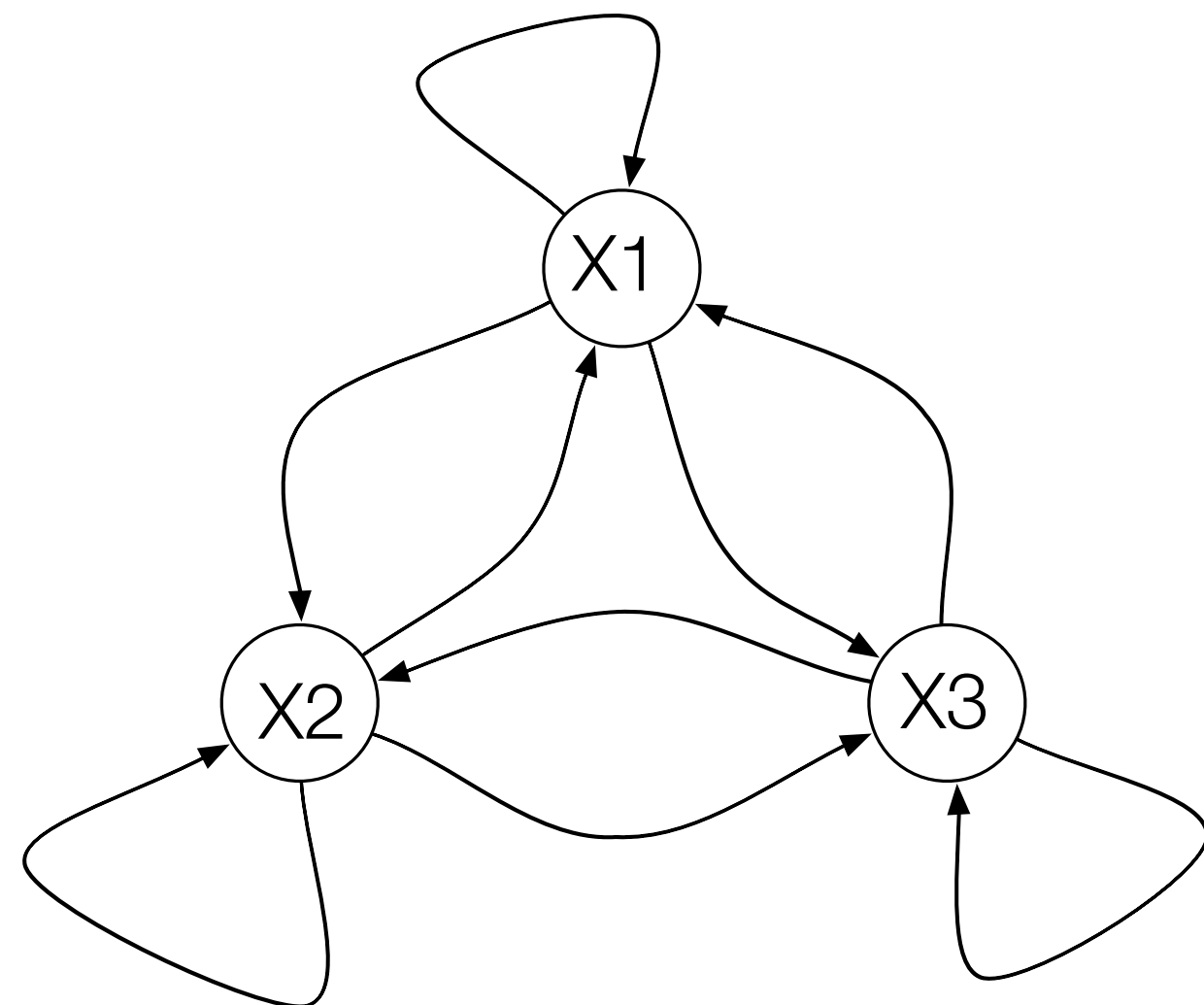
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- A state  $i$  has **period  $m$**  if any return to state  $i$  must occur in multiples of  $m$  time steps.



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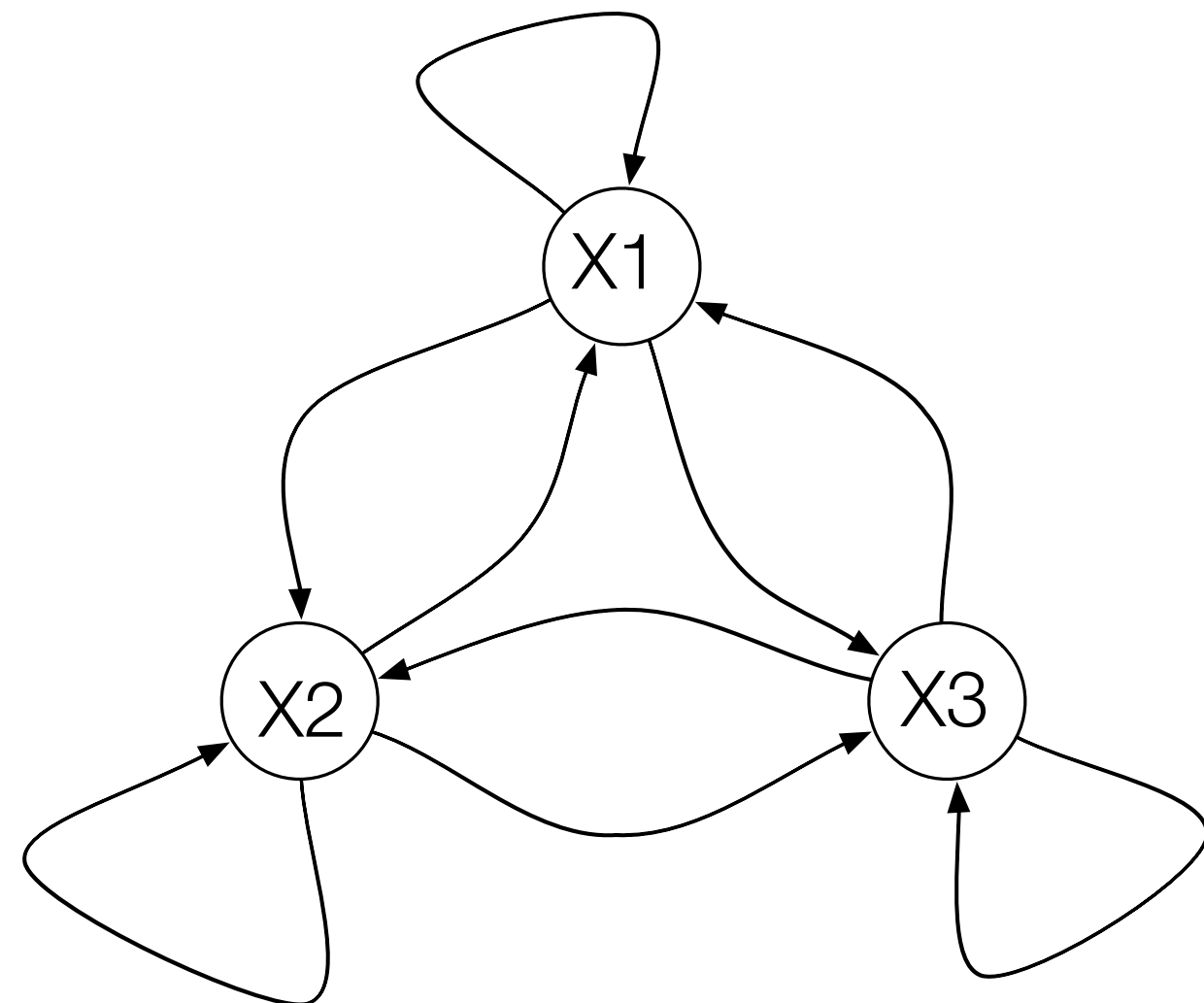
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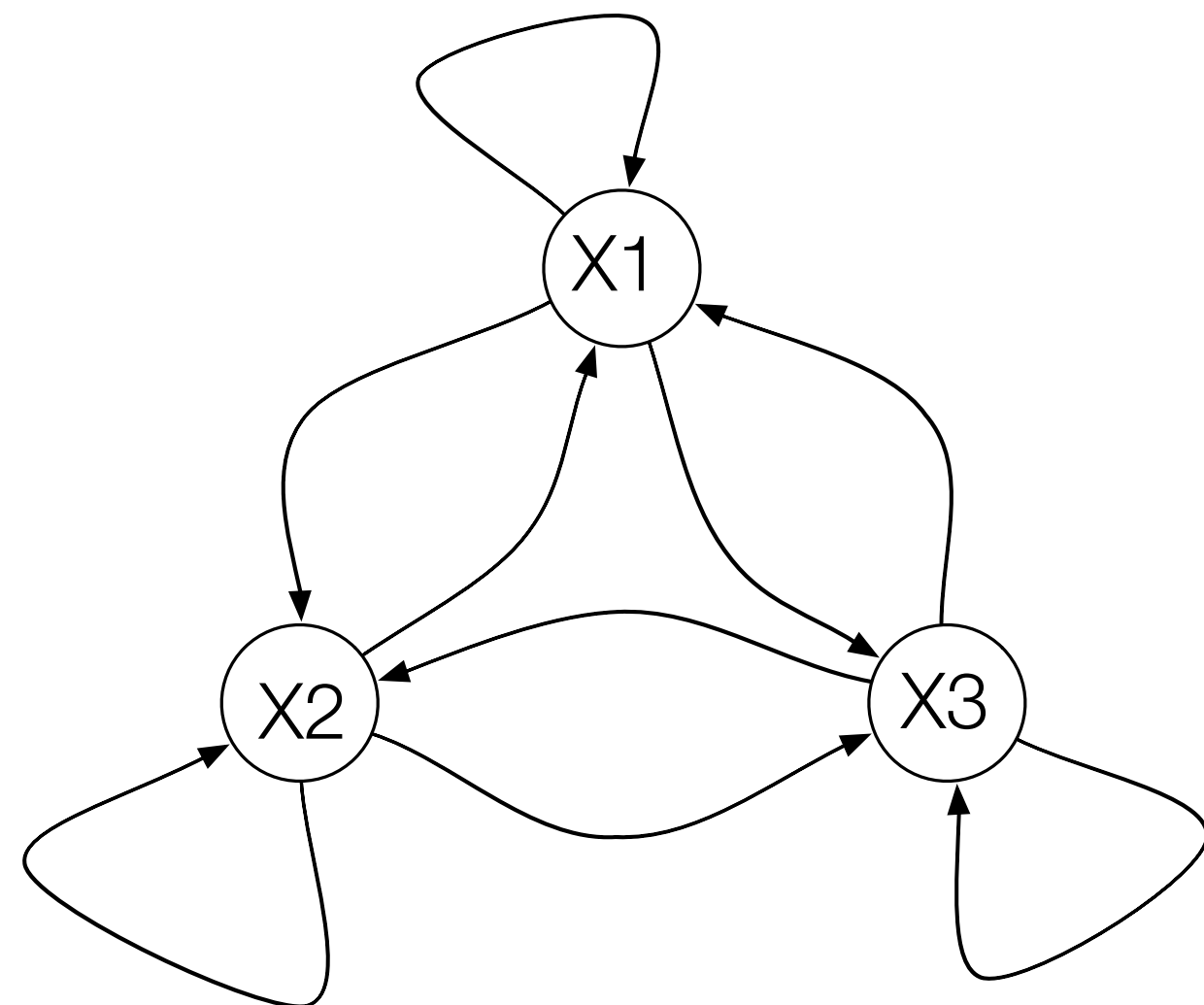
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- A state  $j$  is said to be accessible from state  $i$  if  $p_{ij}^{(n)} > 0$  for some time step  $n$ . If two states are accessible to each other, they are said to communicate with each other.



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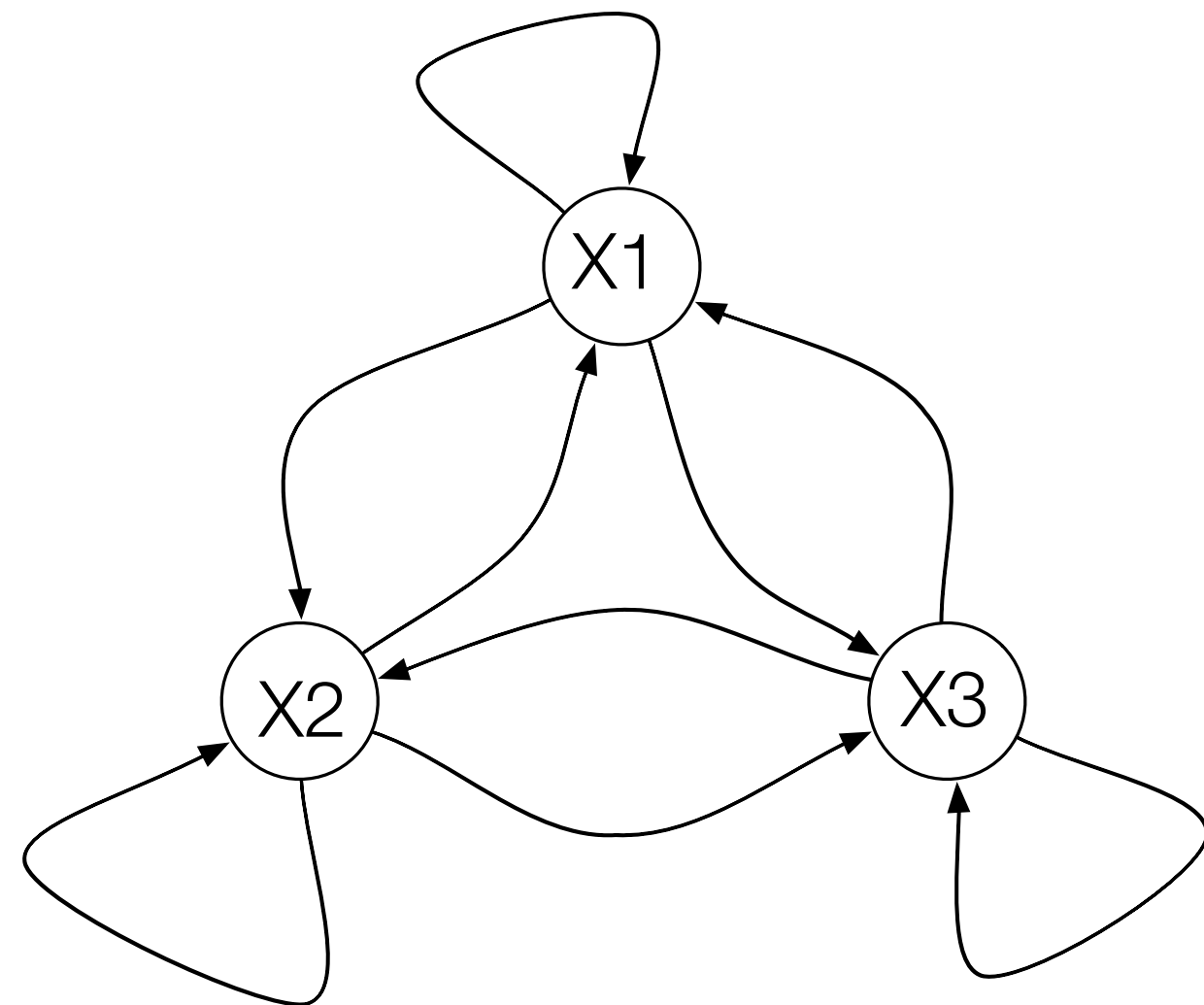
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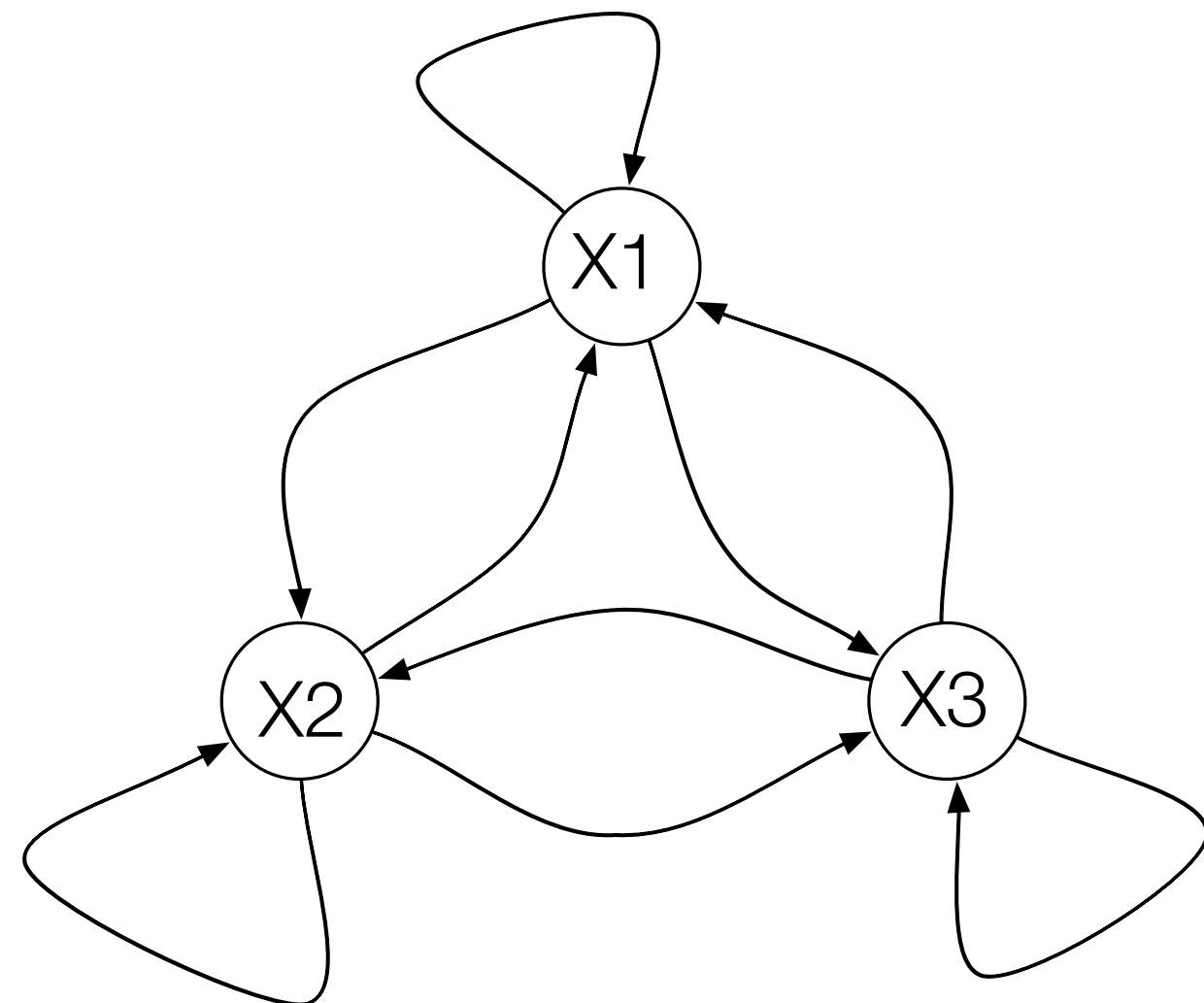
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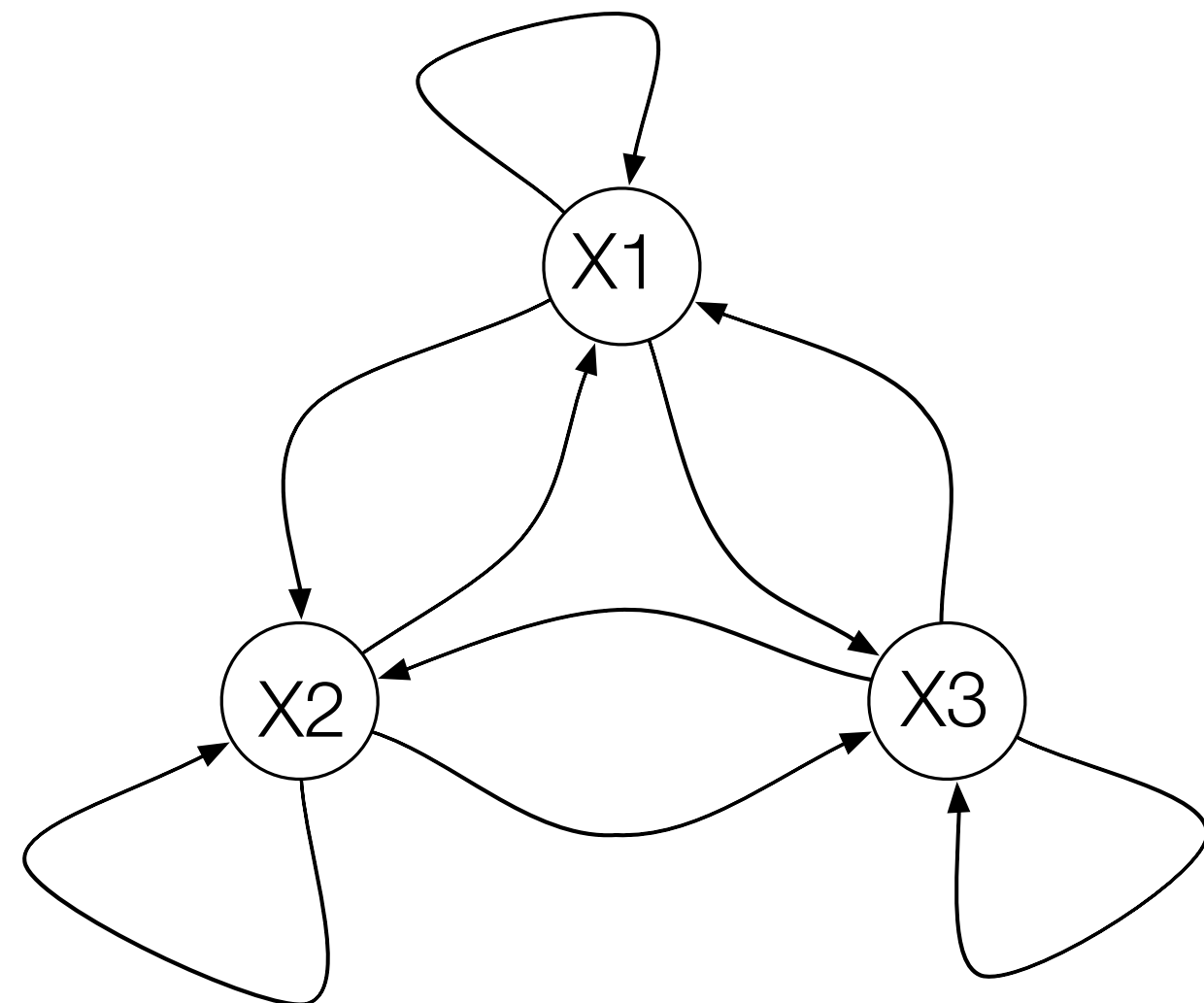
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- $\Rightarrow$  standard theorems of theory of Markov chains hold
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- If a consensus is reached, then it is unique!

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$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{matrix} \text{no} \\ \text{consensus} \end{matrix}$$

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \quad \frac{1}{3}F_1 + \frac{2}{3}F_2$$

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

# Example

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

$$\frac{1}{3}F_1 + \frac{2}{3}F_2$$

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{matrix} \text{no} \\ \text{consensus} \end{matrix}$$

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

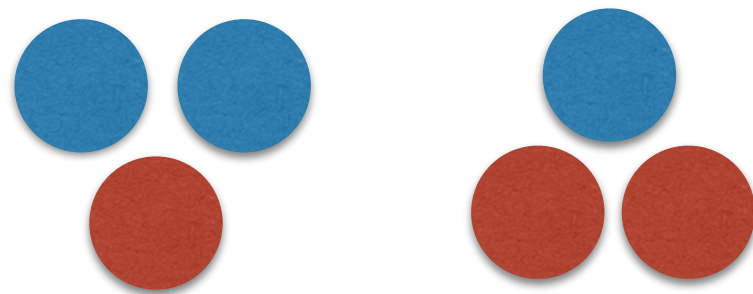
$$\frac{1}{3}F_1 + \frac{2}{3}F_2$$

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

no

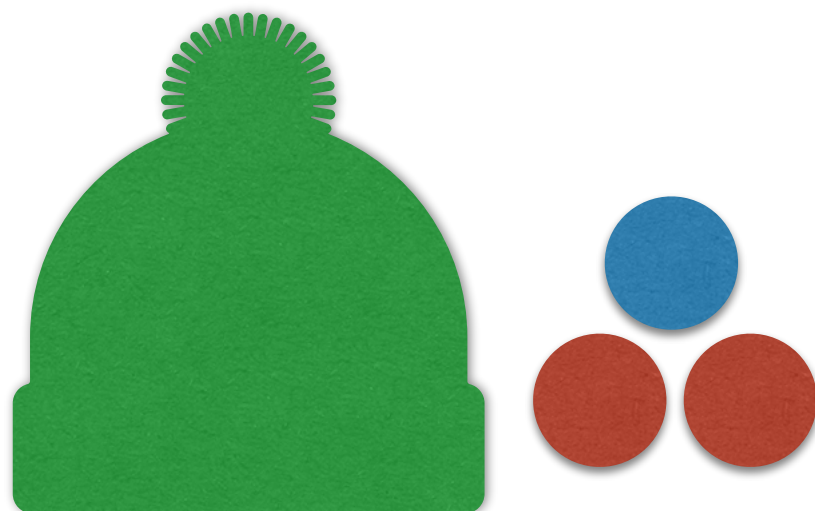
consensus

# Informational cascades

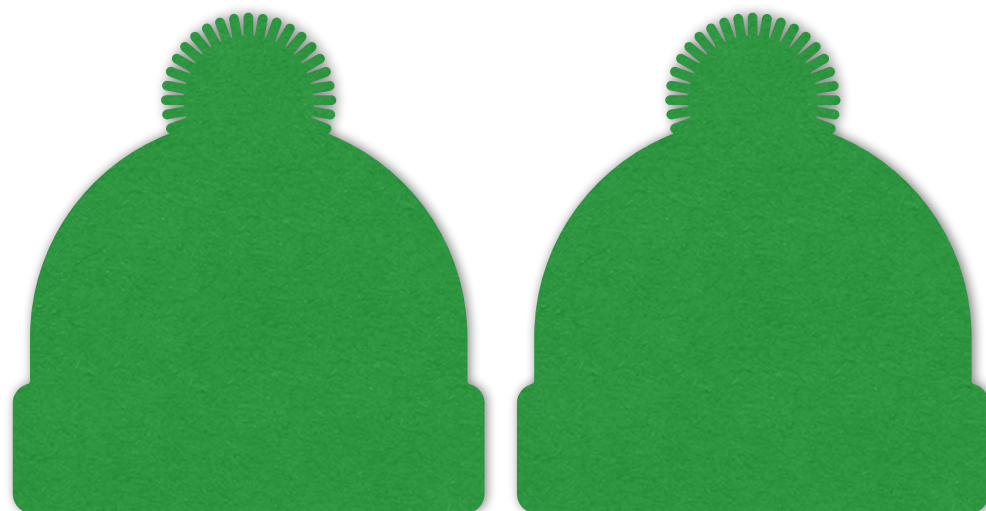




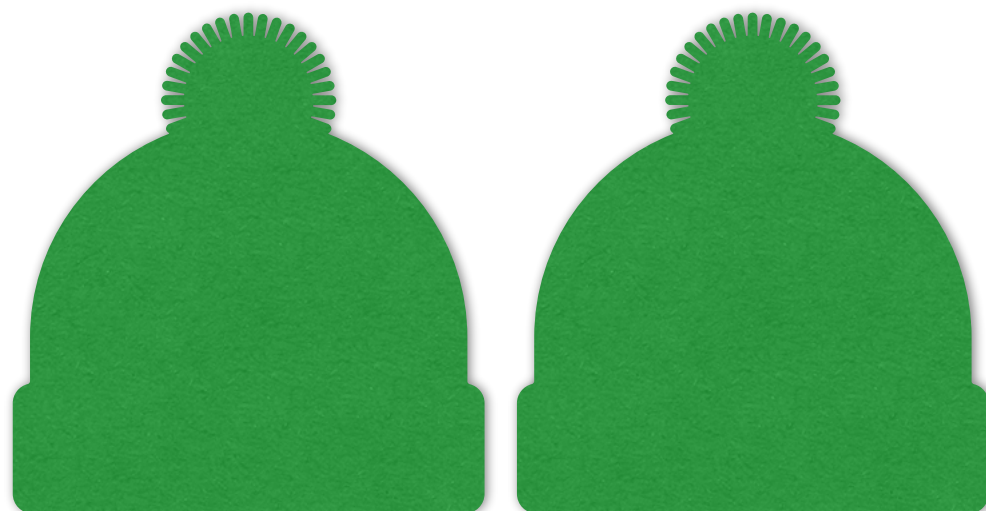
# Informational cascades



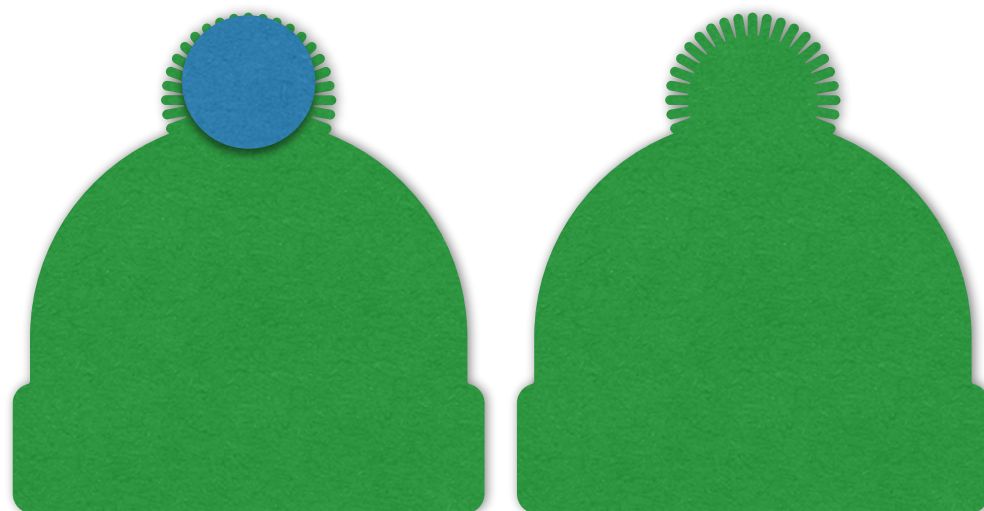
# Informational cascades



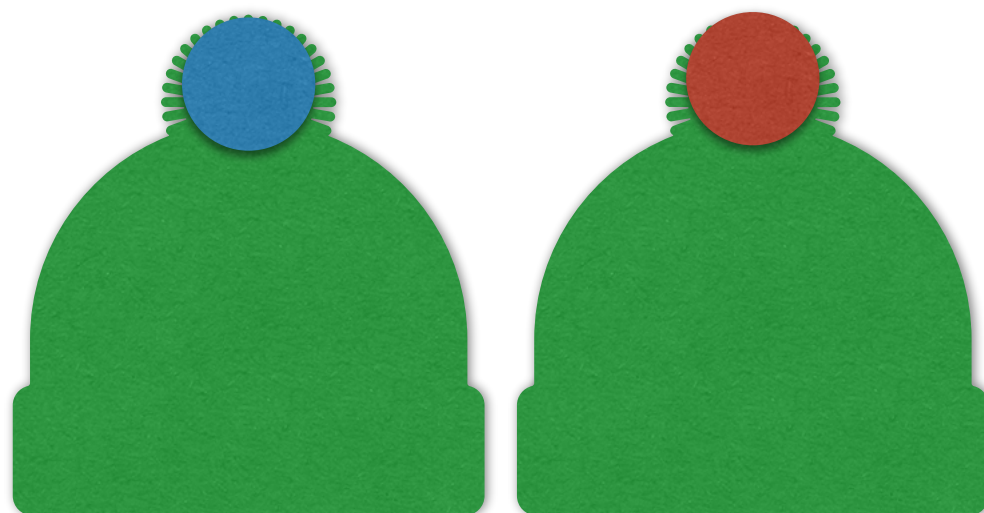
# Informational cascades



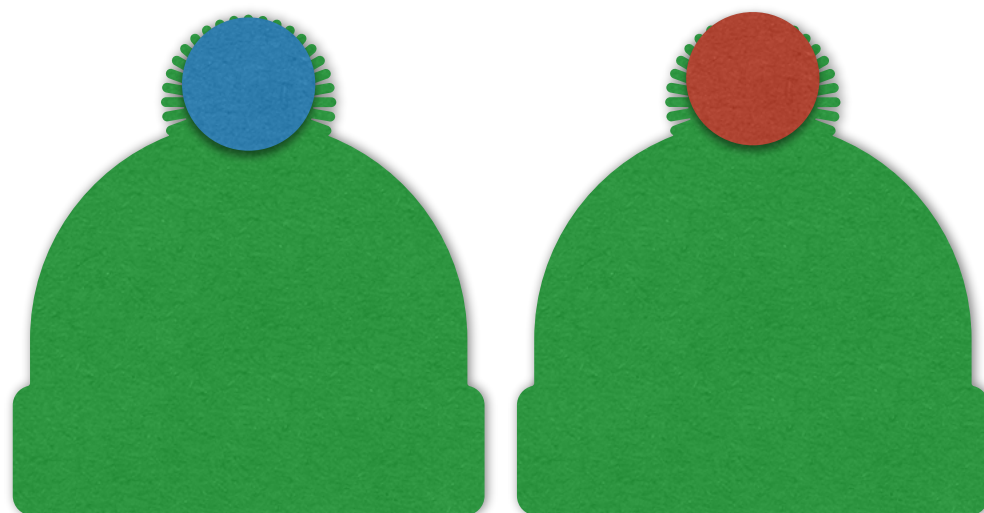
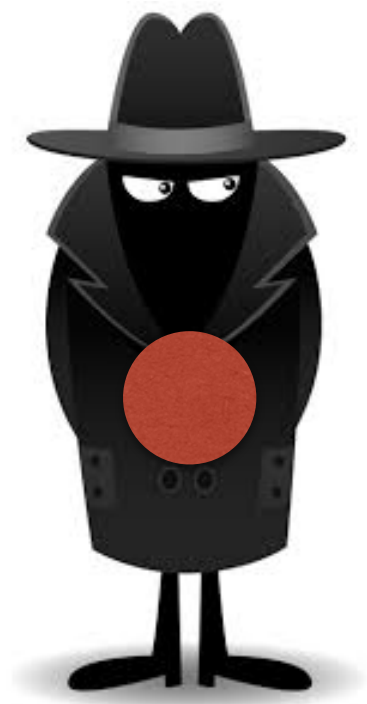
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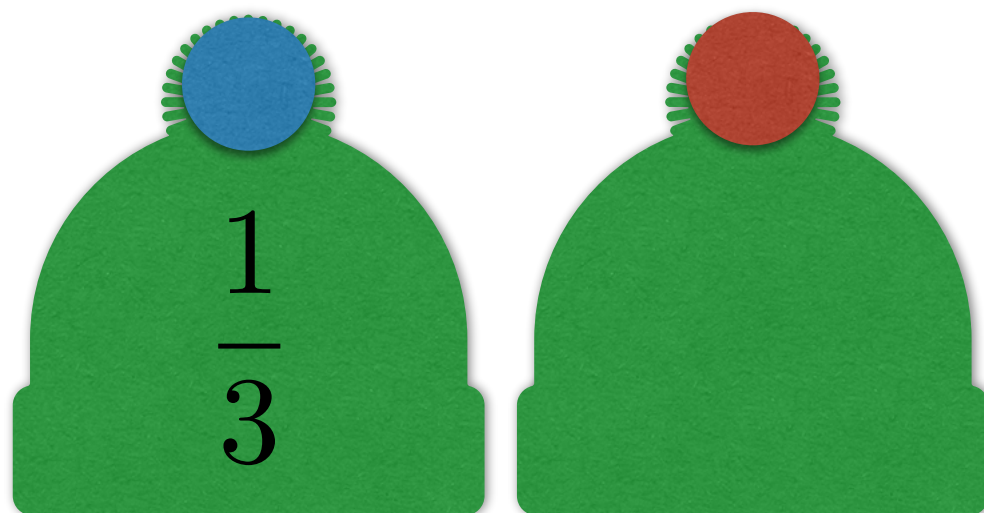
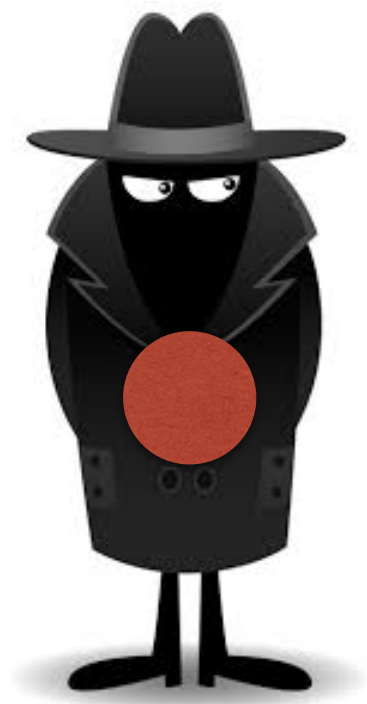
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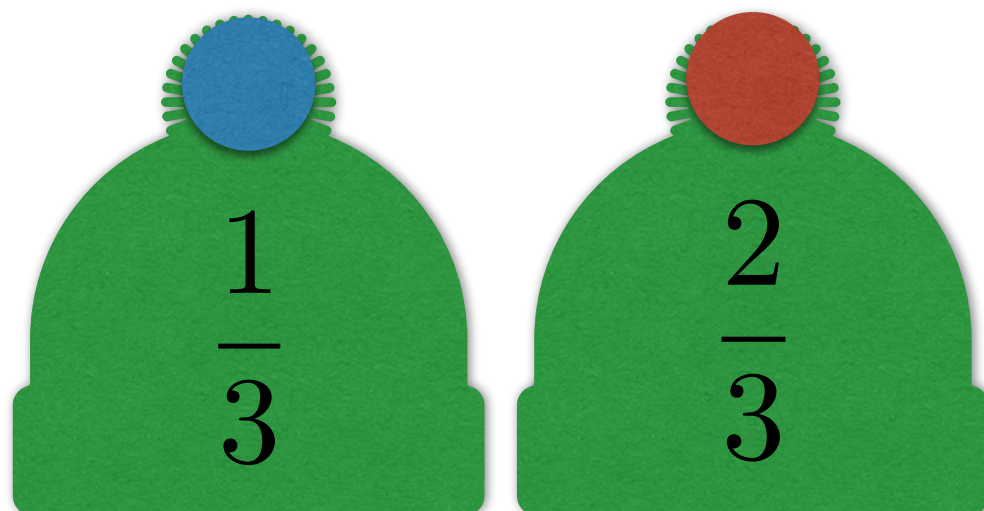
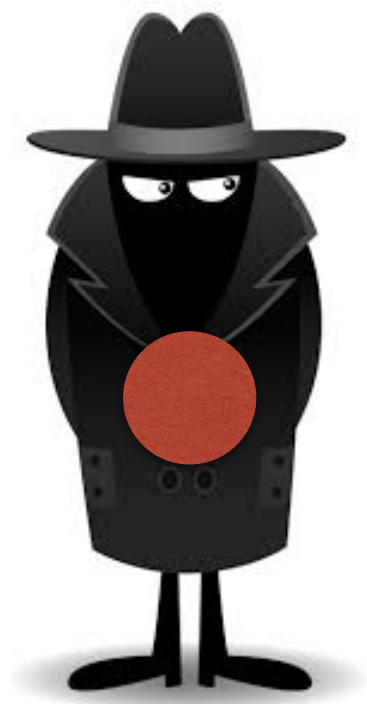
# Informational cascades



# Informational cascades

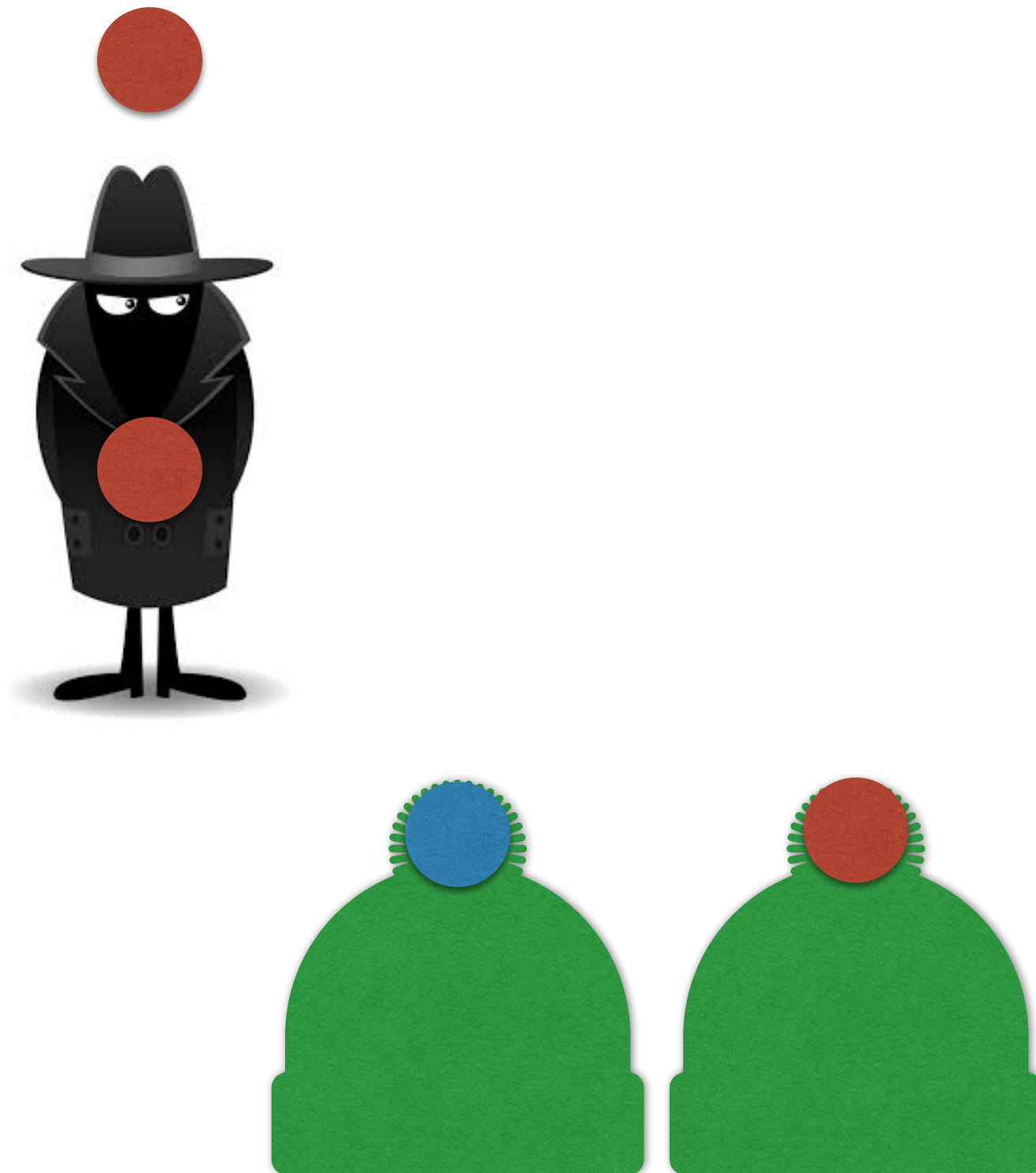


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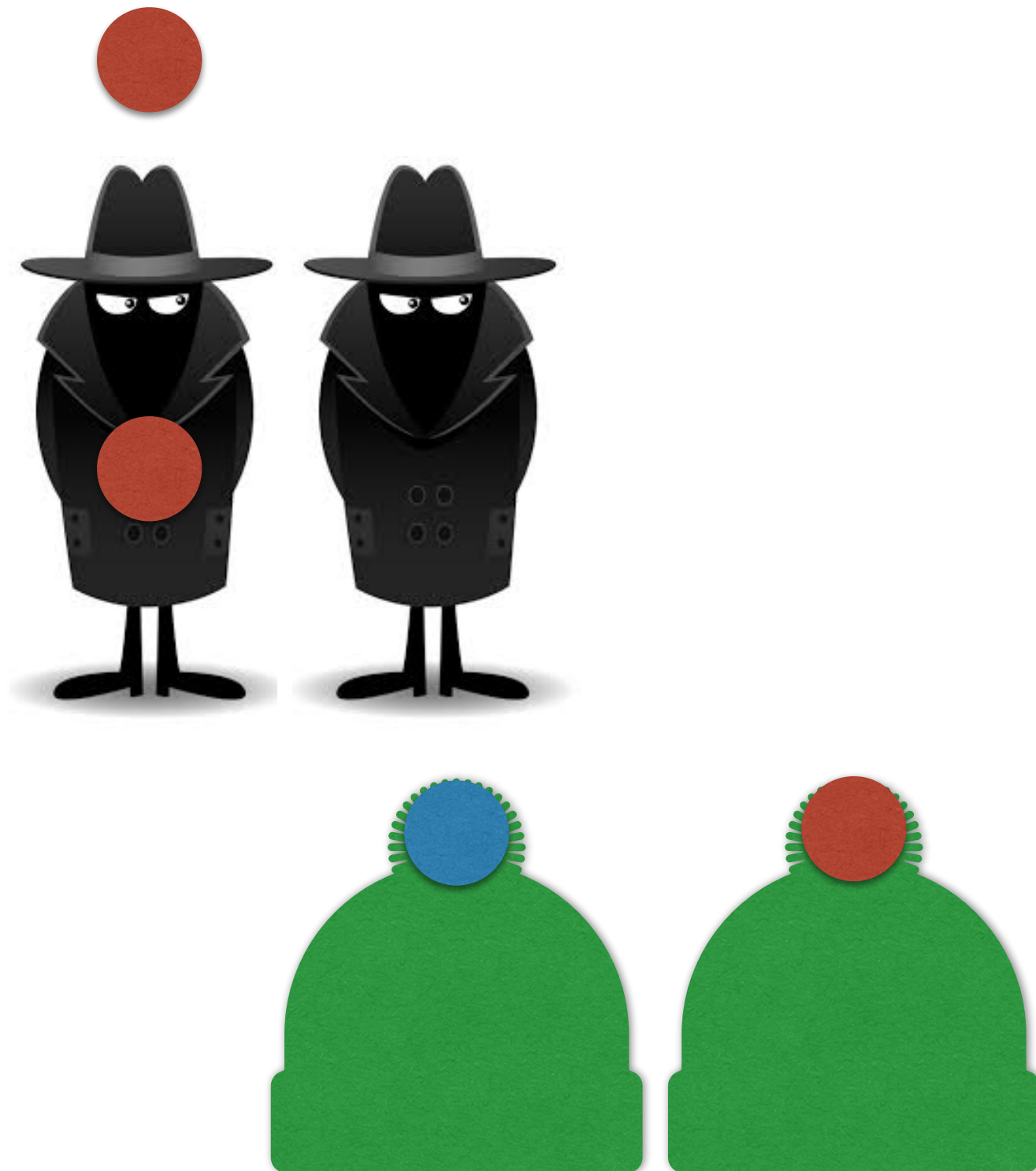




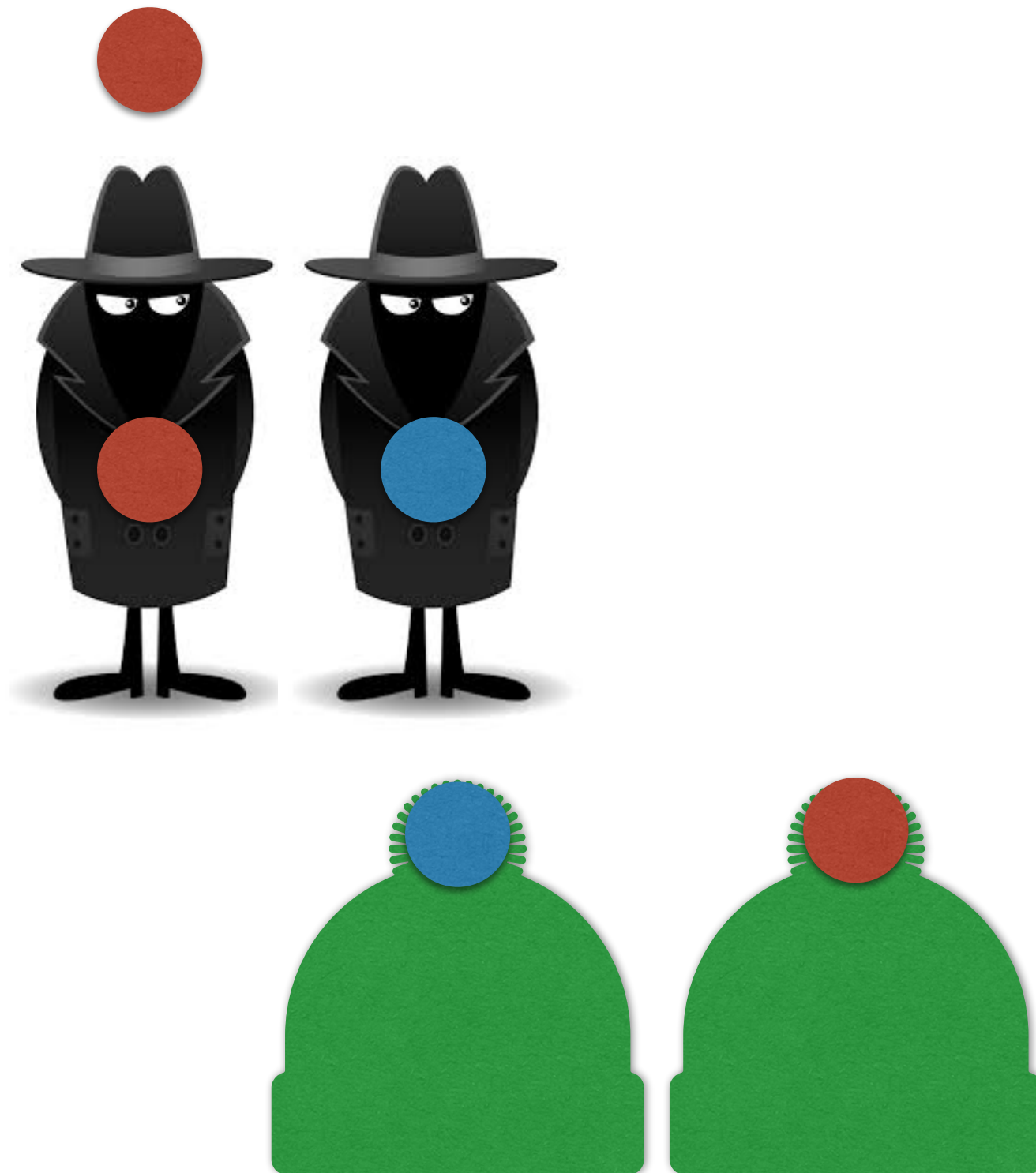
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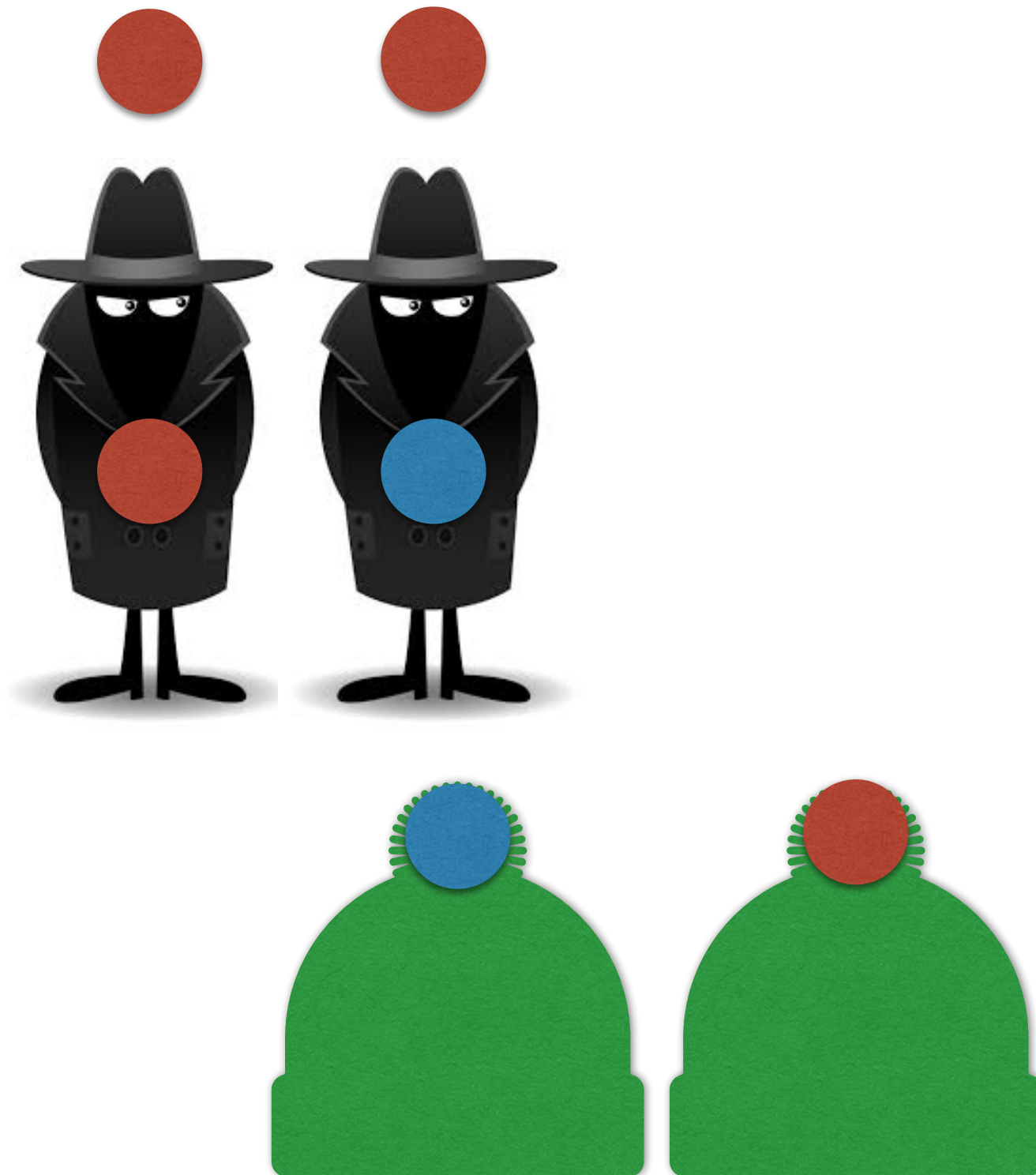
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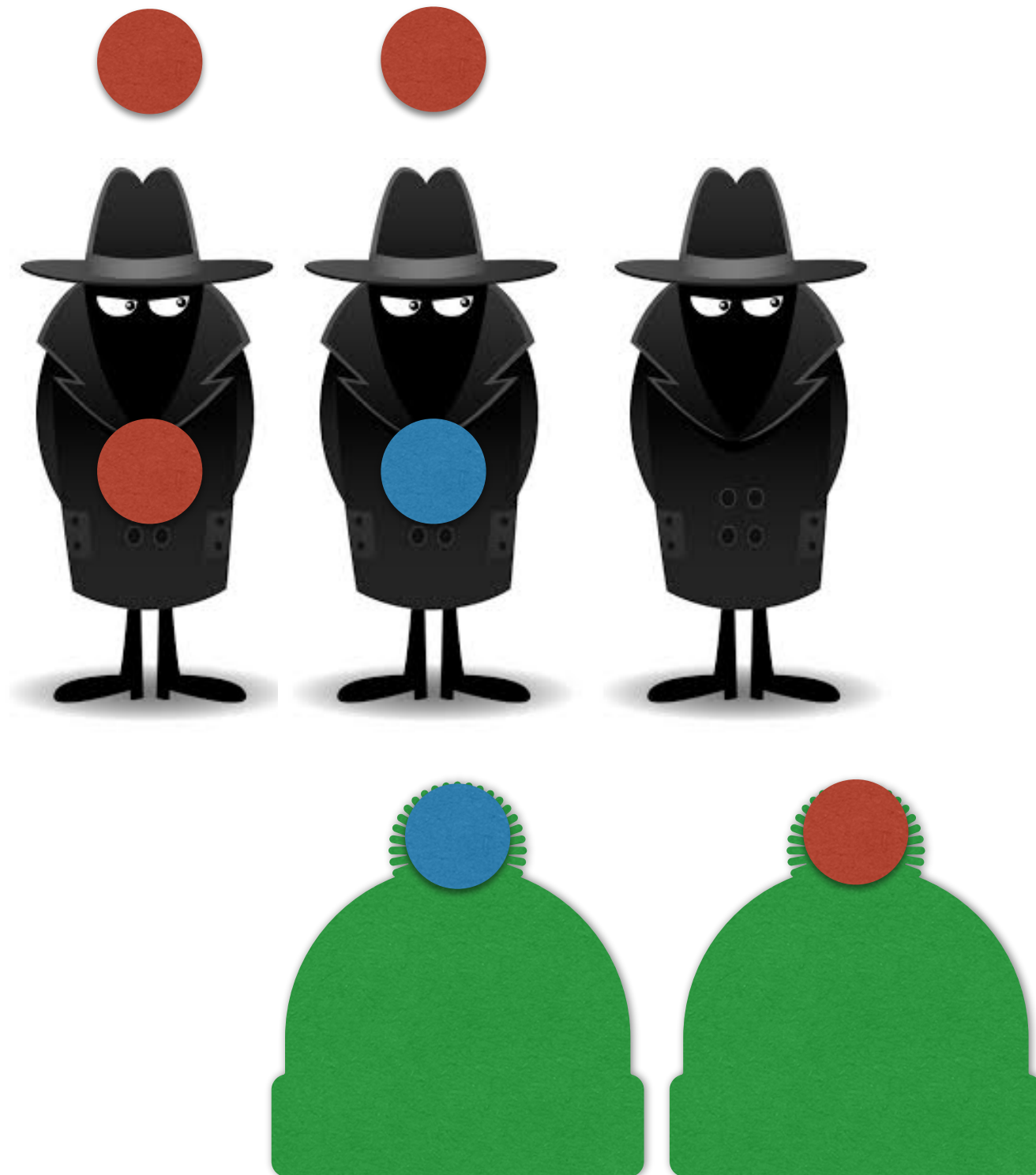
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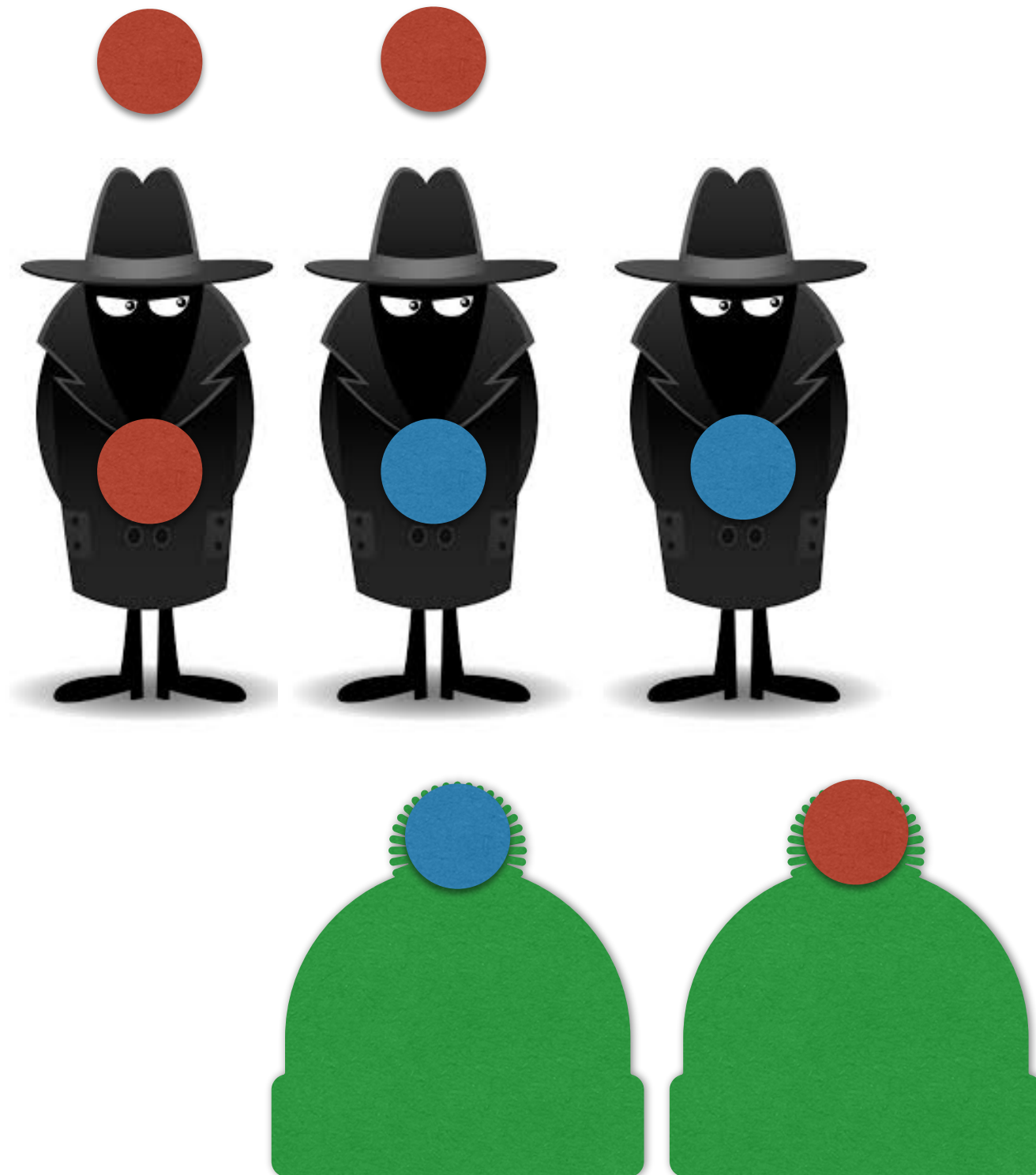
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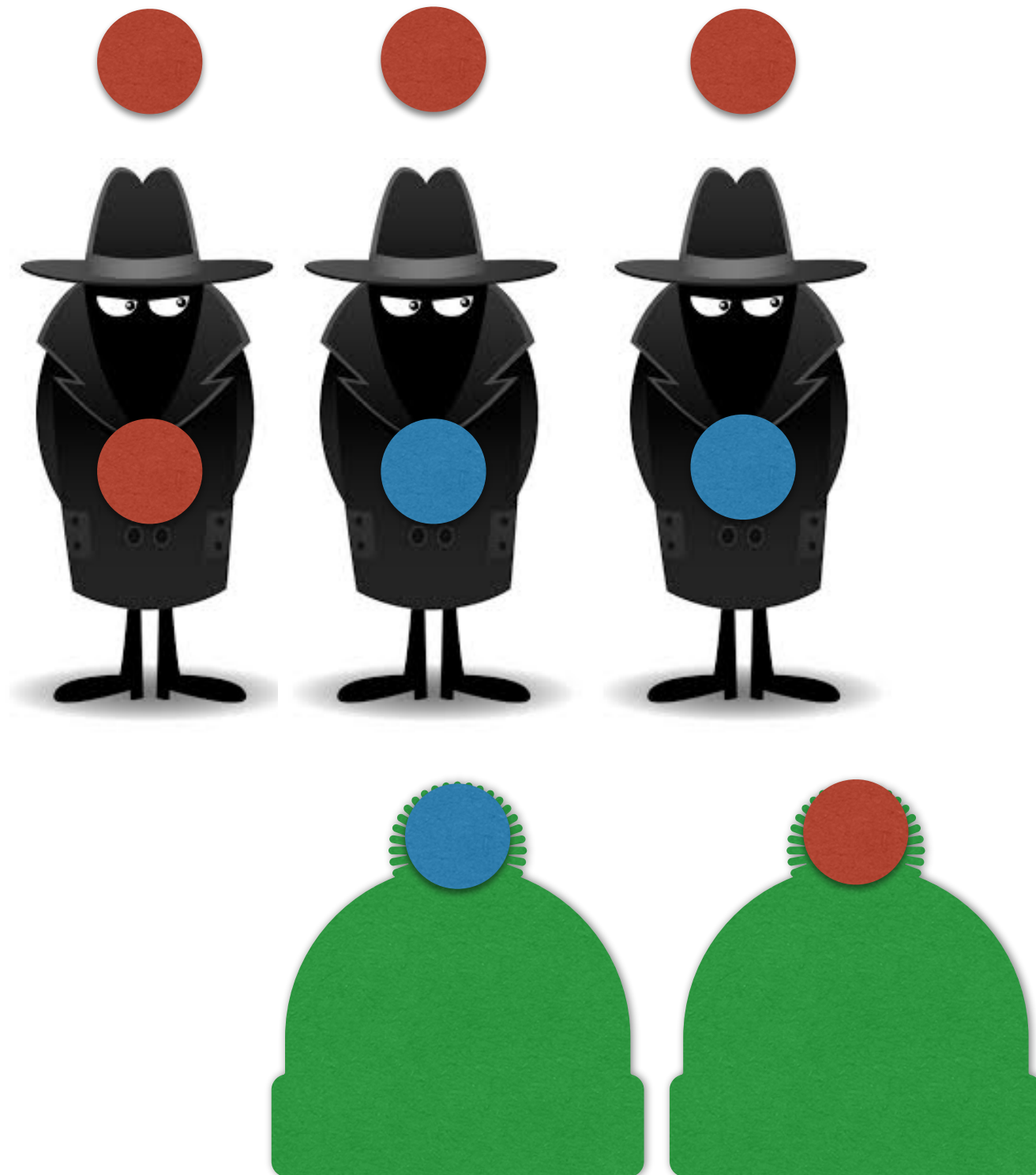
# Informational cascades



# Informational cascades

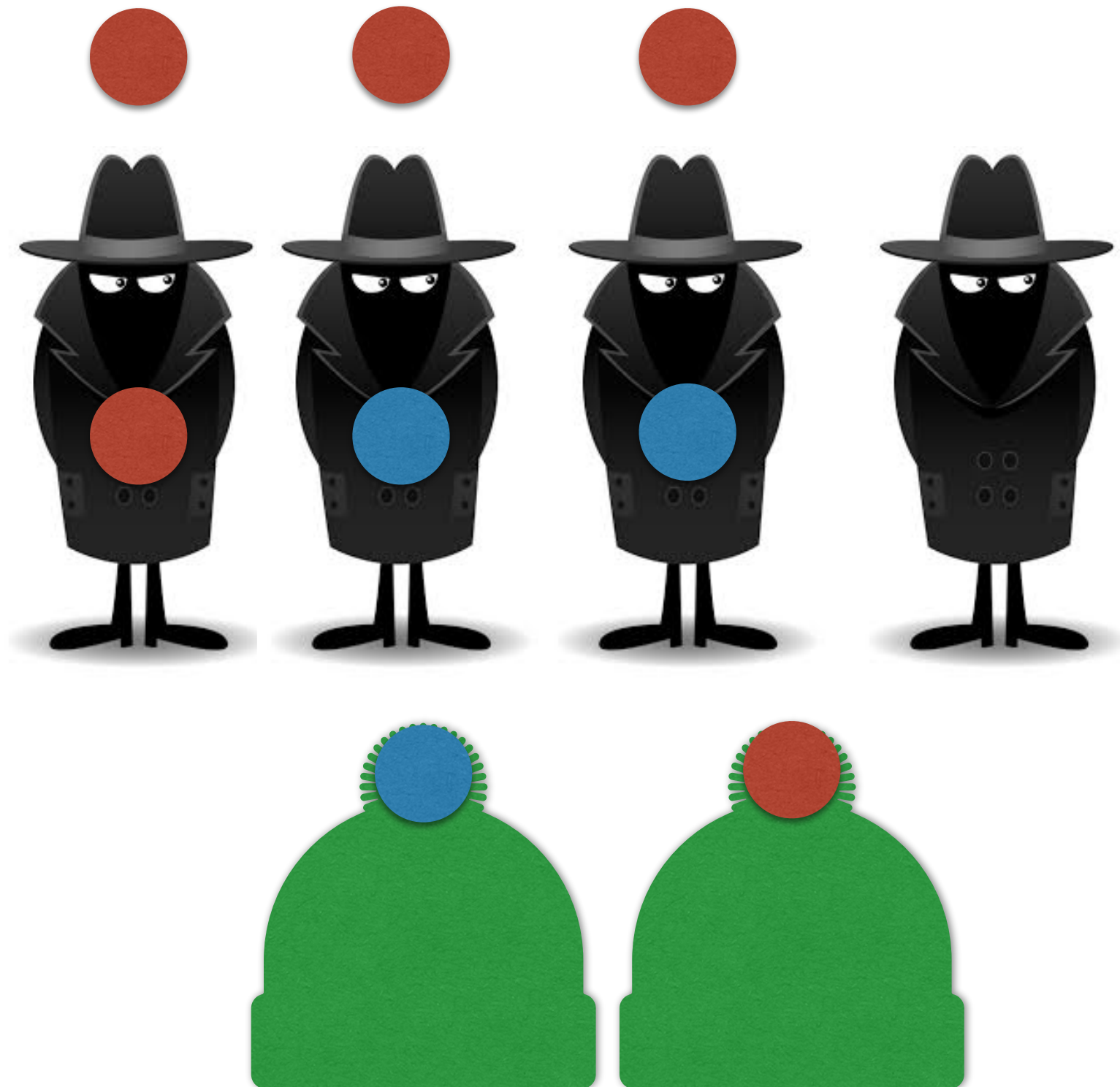


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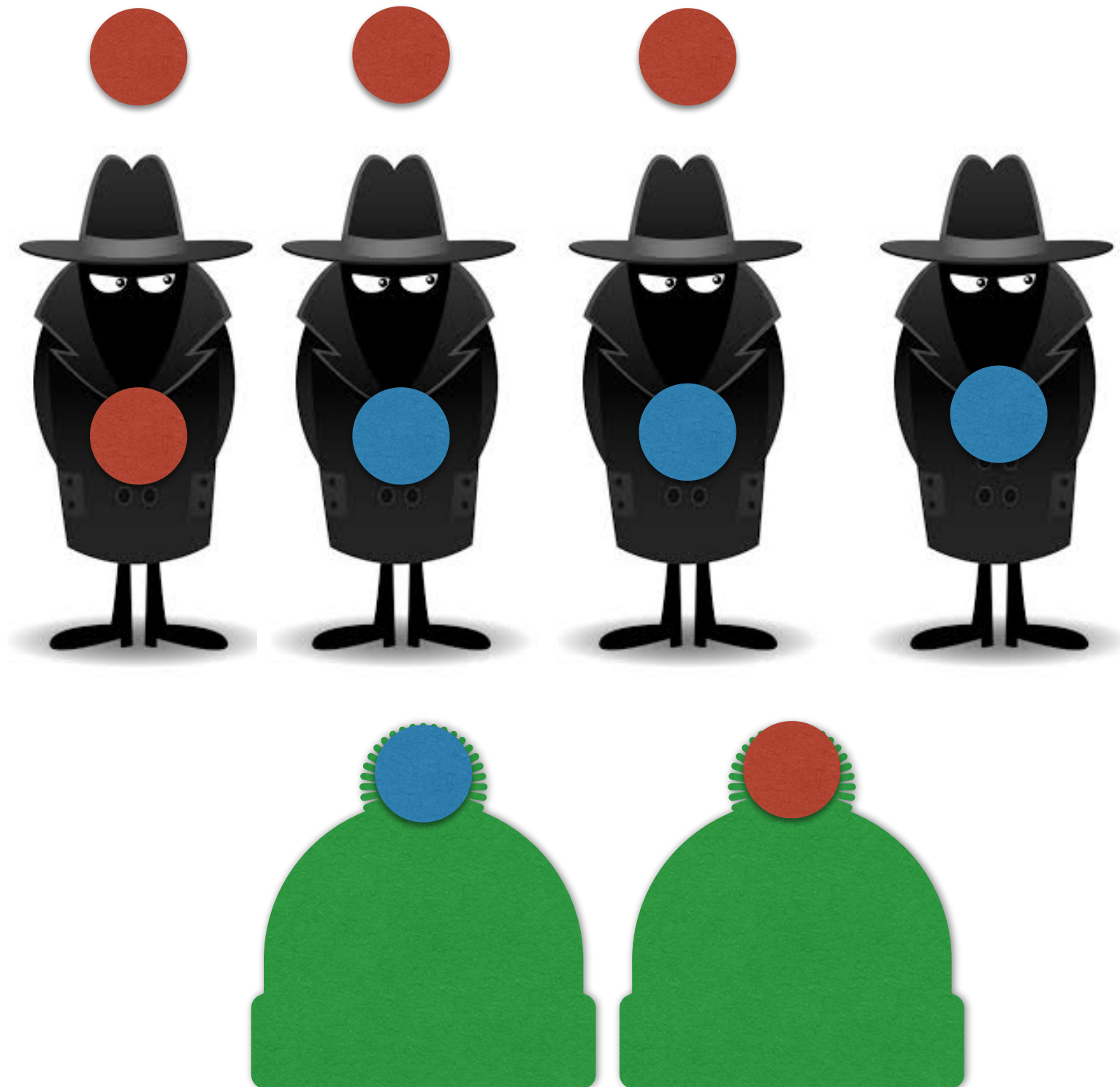


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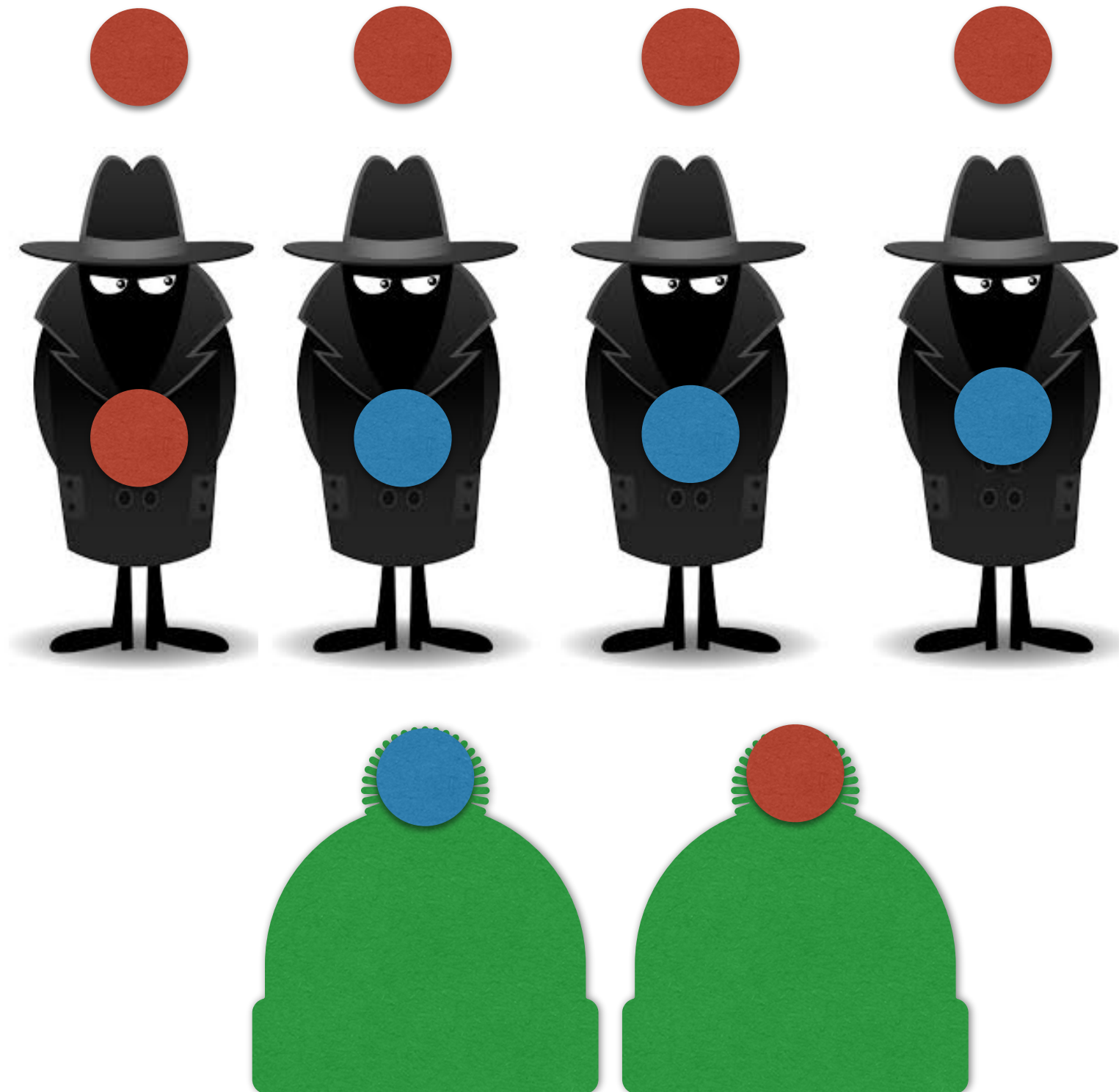




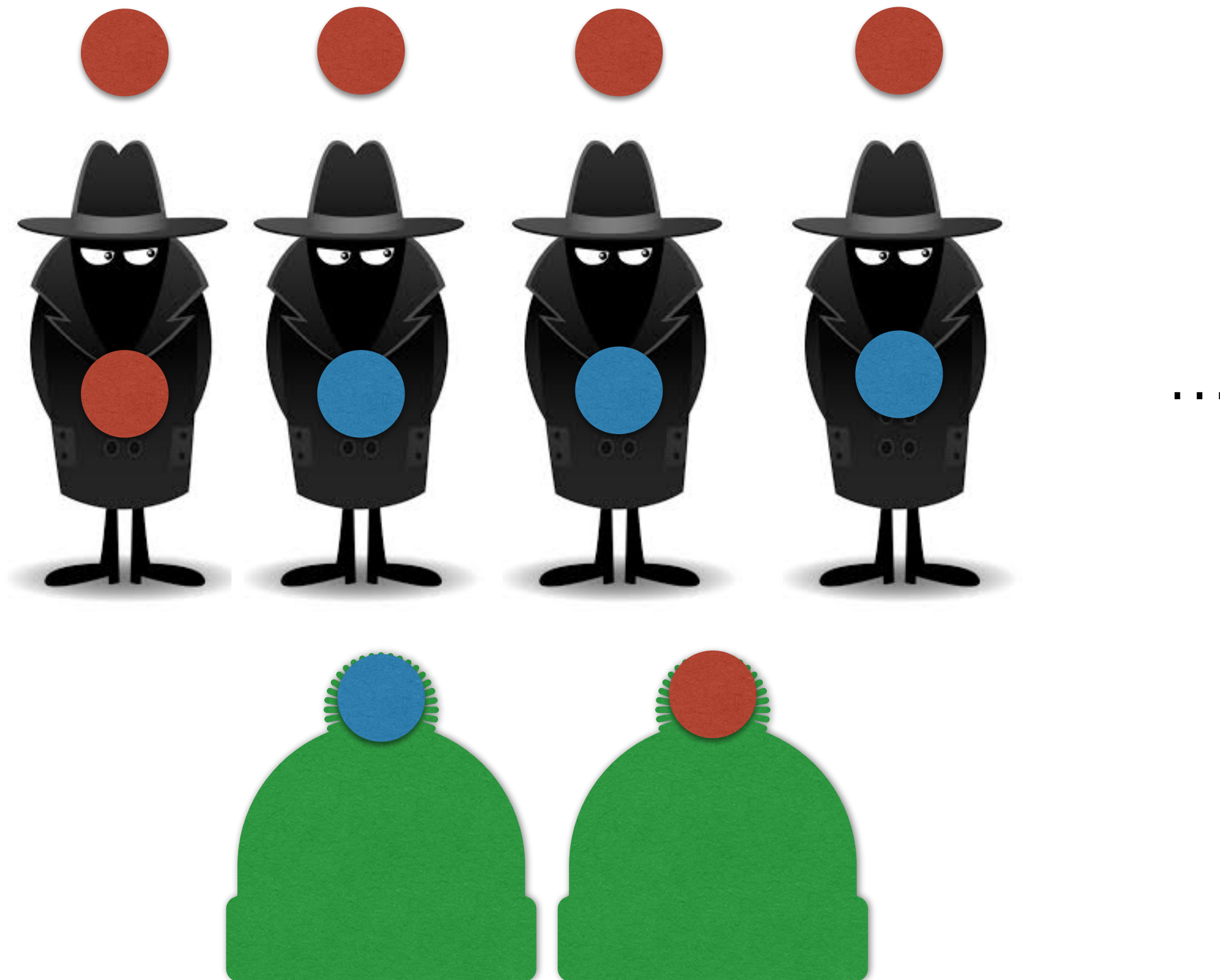
# Informational cascades



# Informational cascades



# Informational cascades





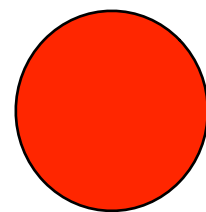
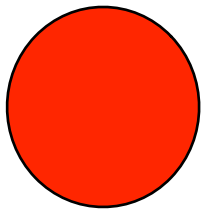
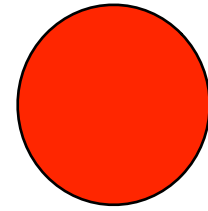
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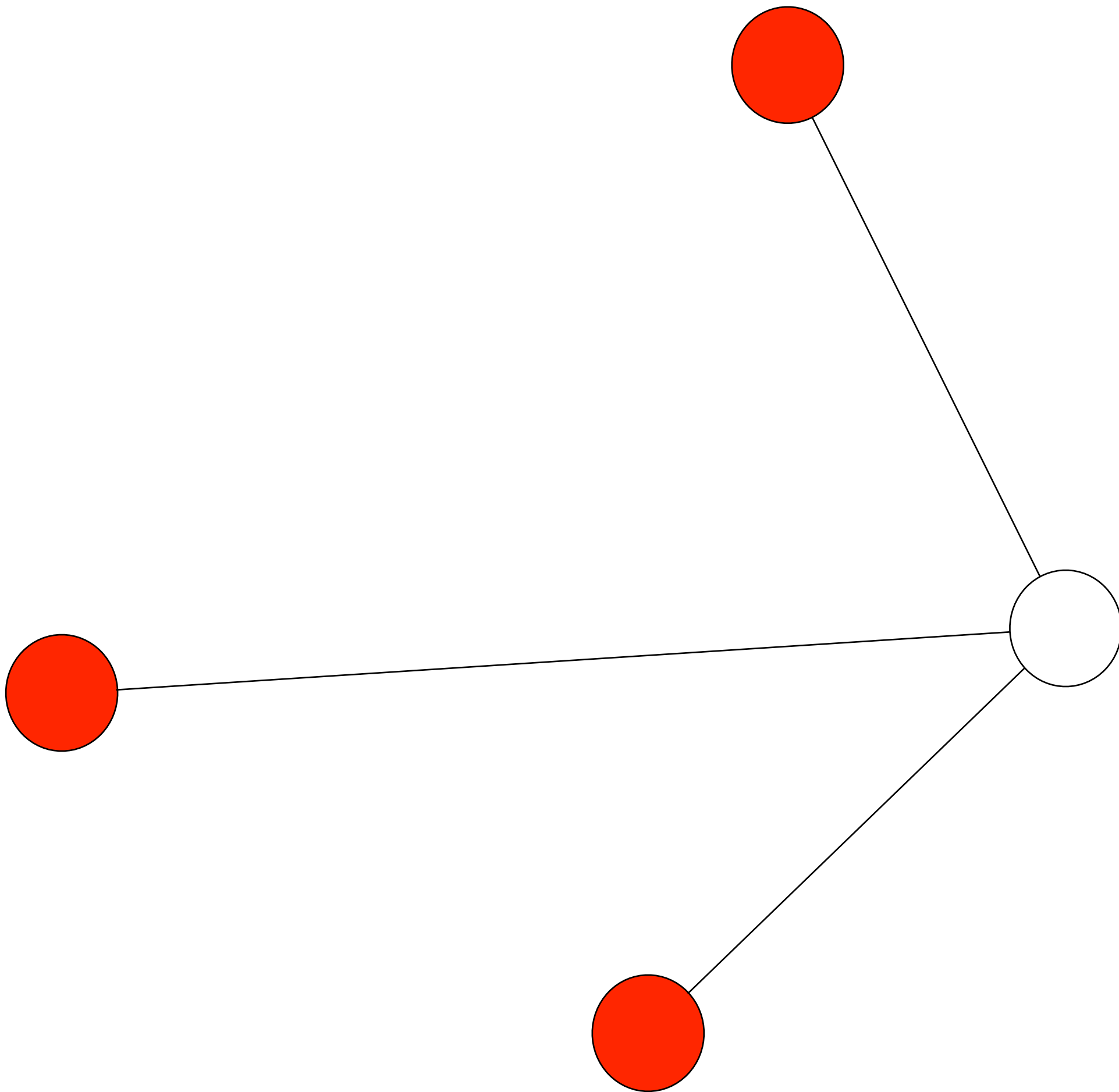
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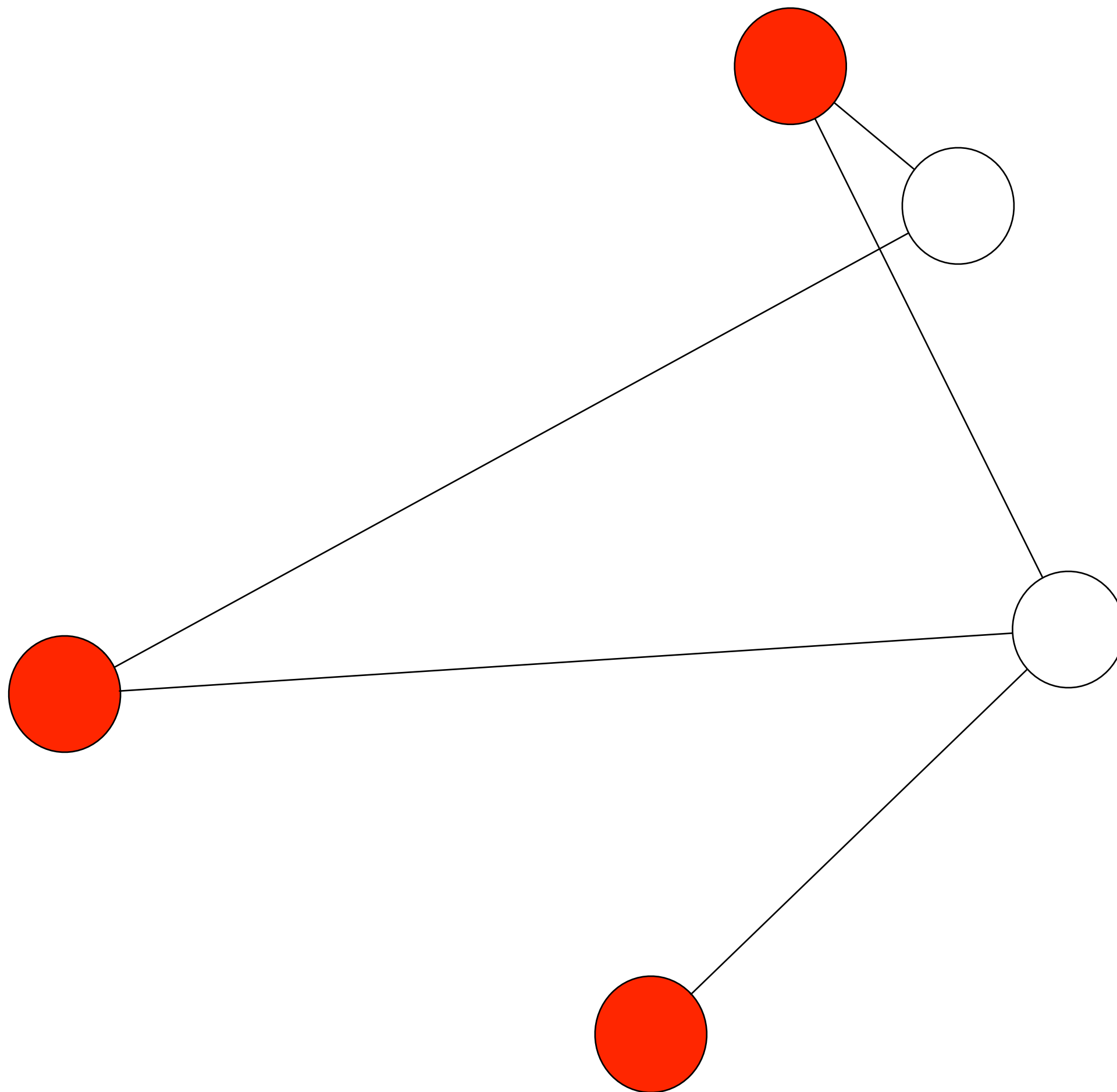
# The "Majority Illusion" in Social Networks

Kristina Lerman , Xiaoran Yan, Xin-Zeng Wu

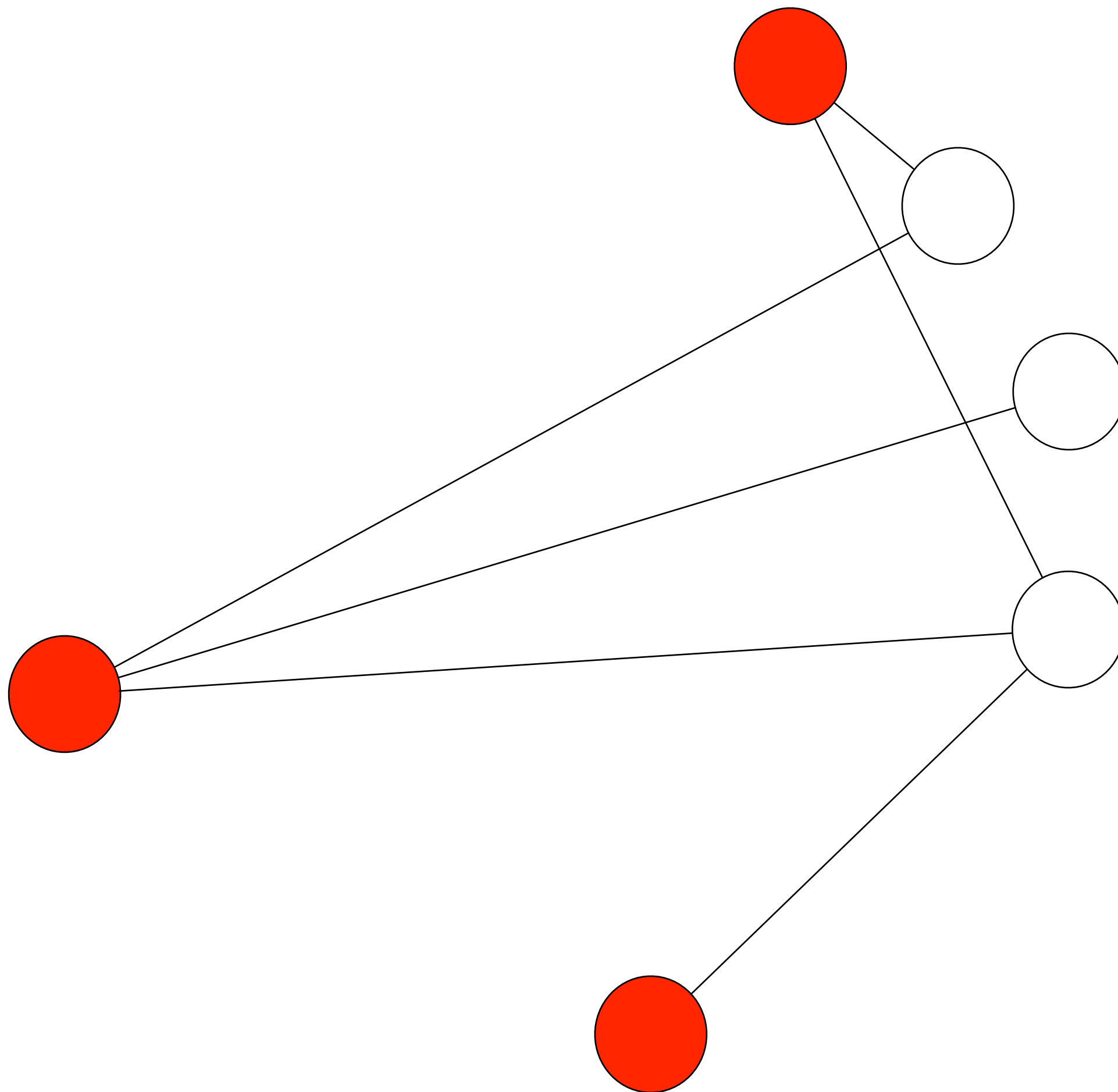
Published: February 17, 2016 • <https://doi.org/10.1371/journal.pone.0147617>

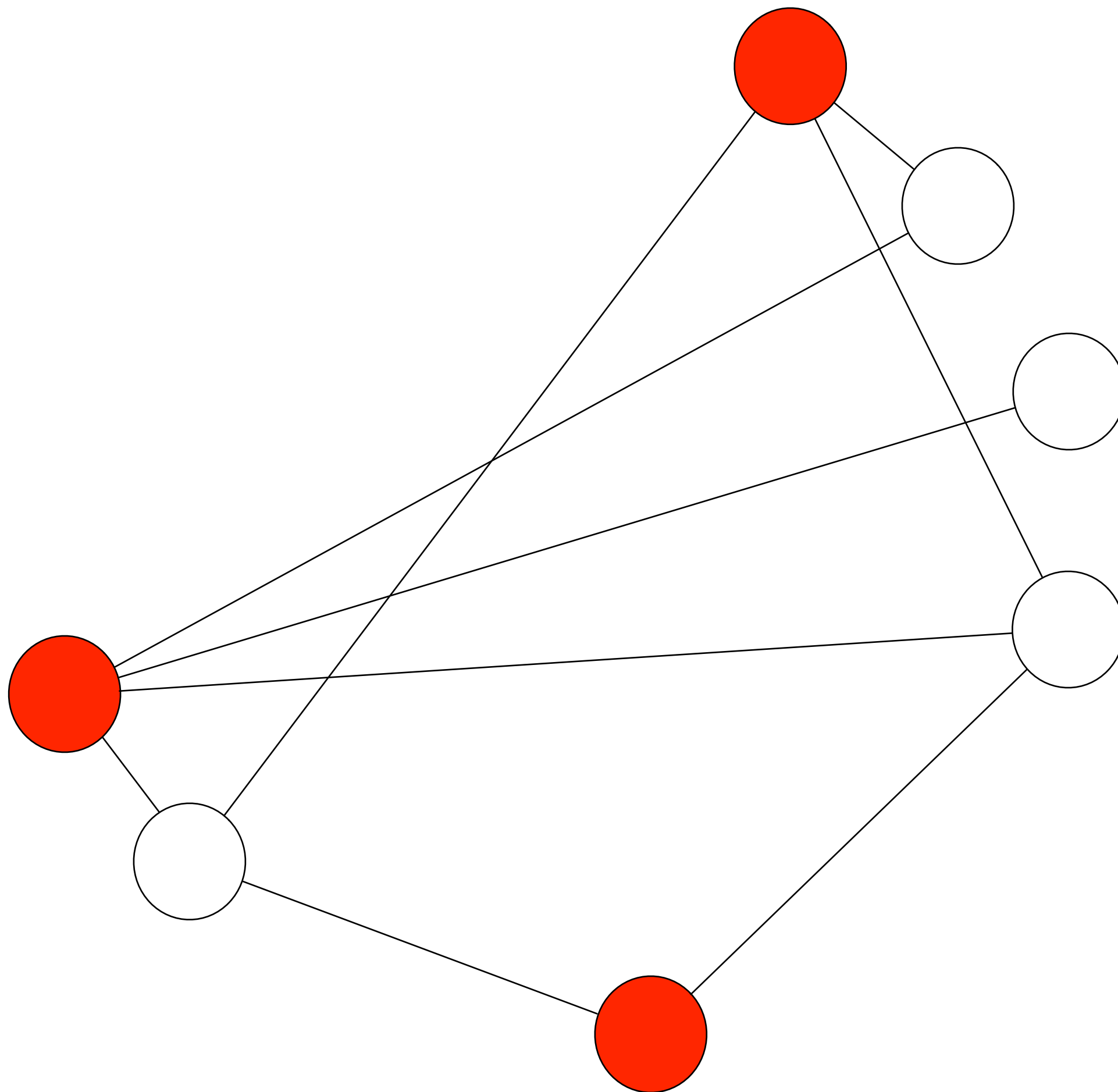


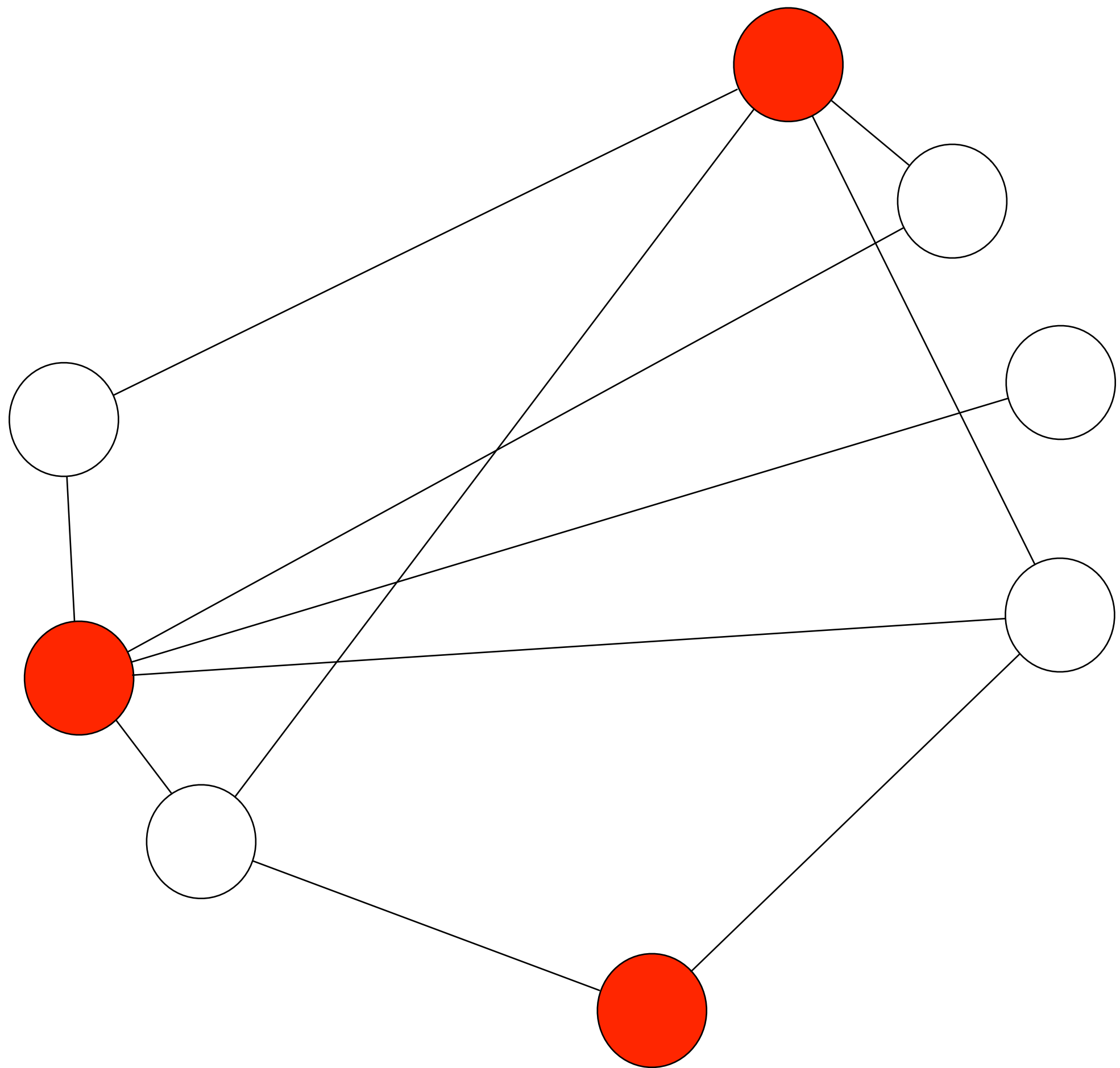


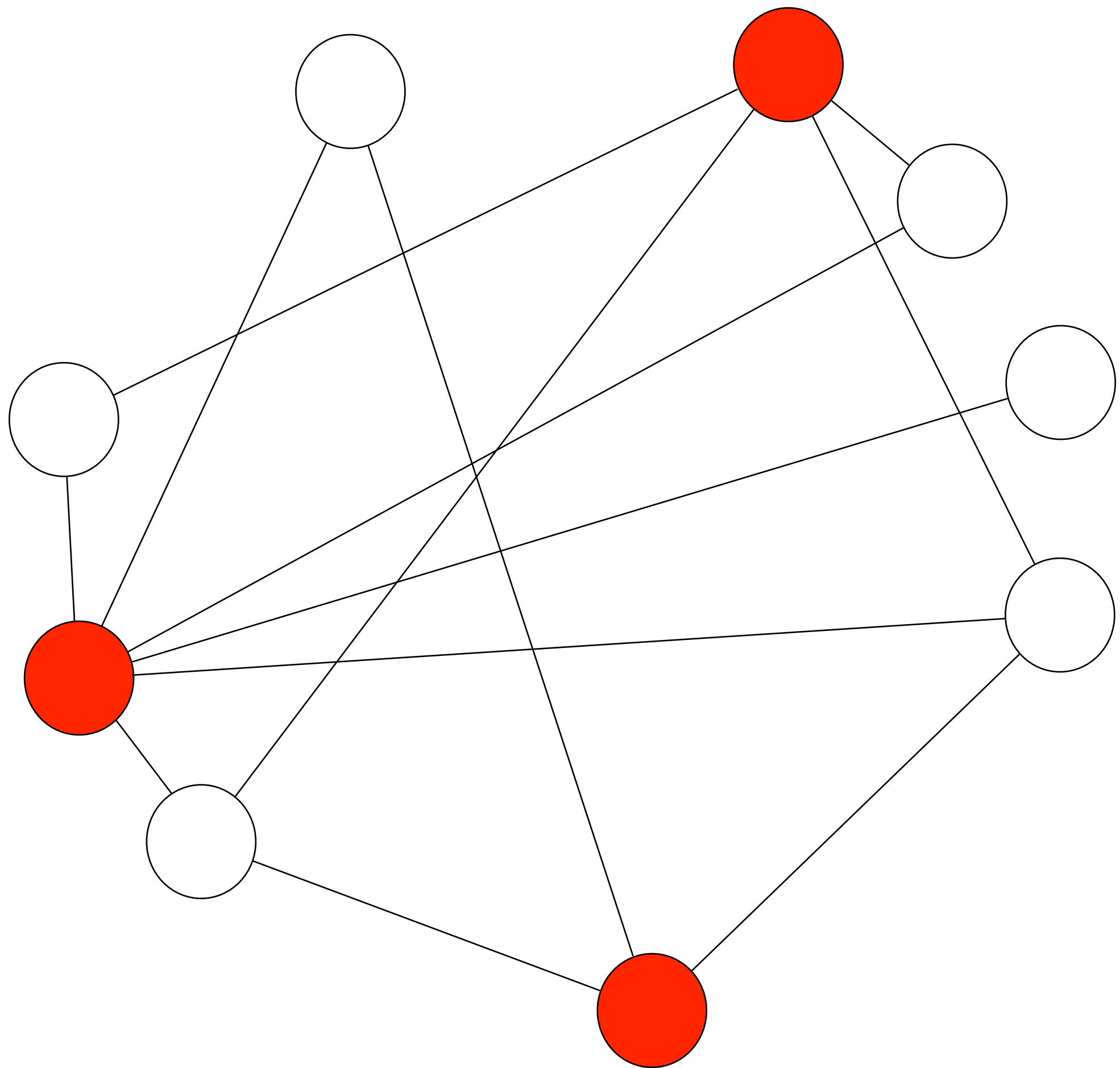


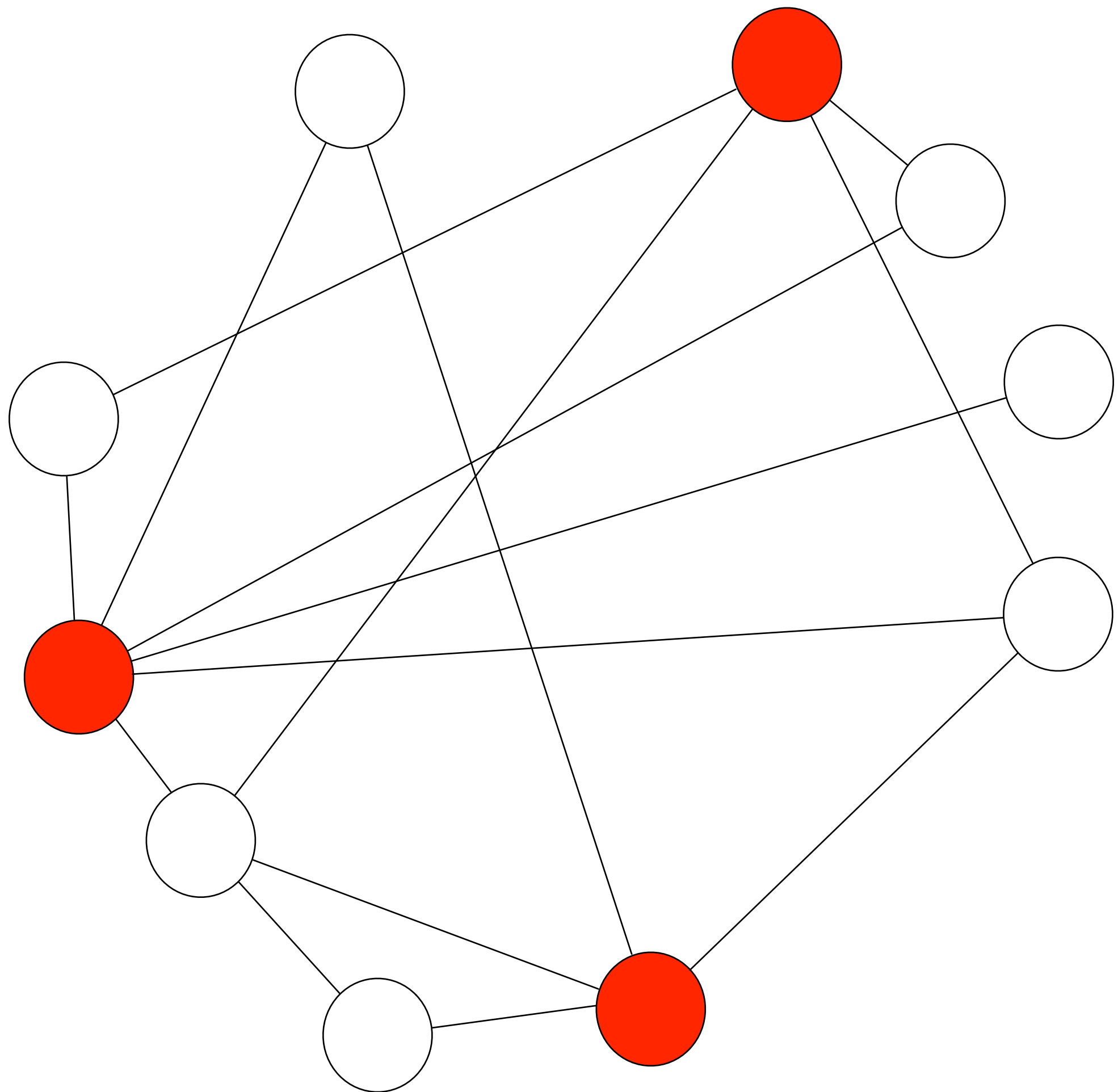


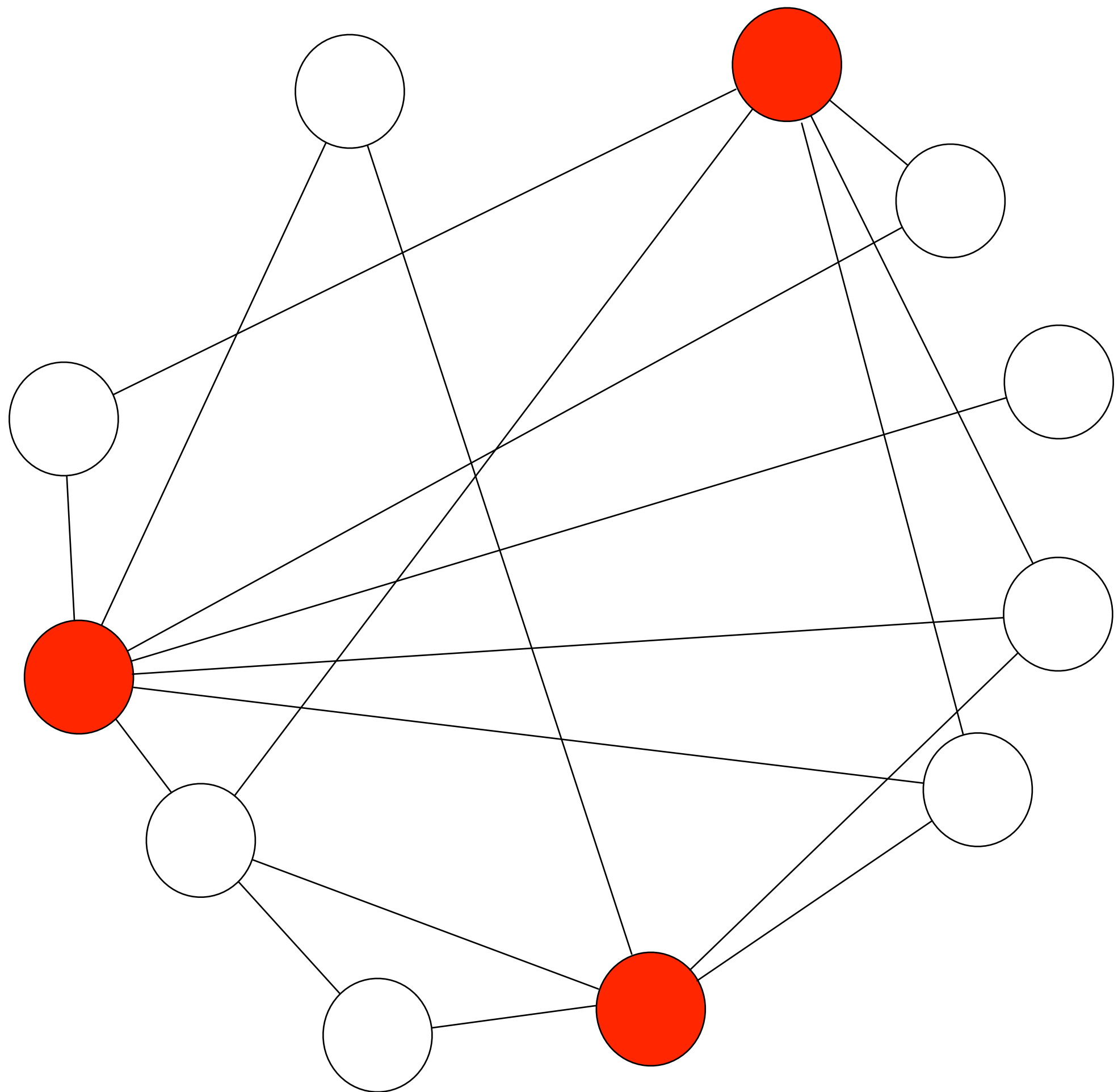


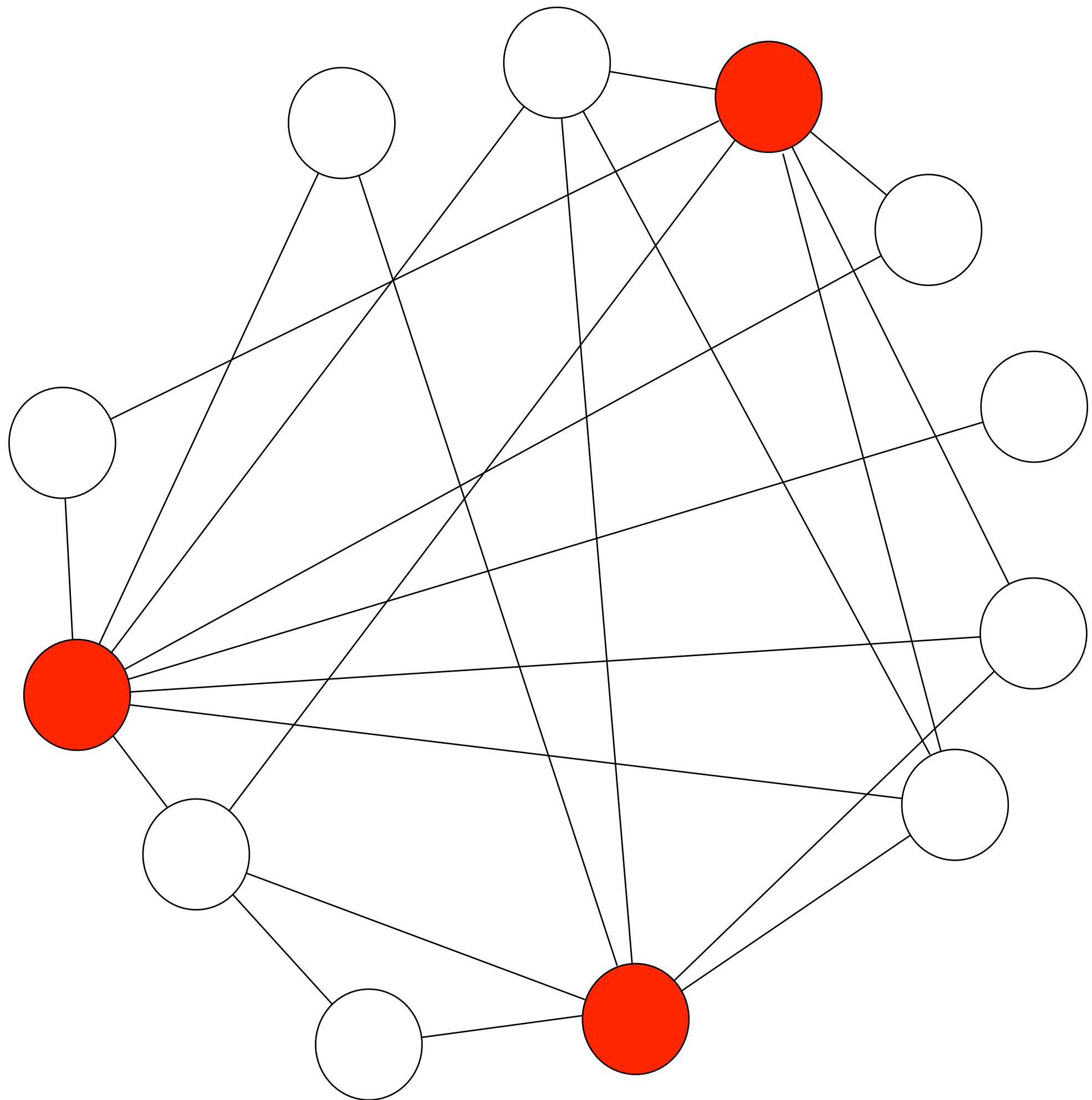


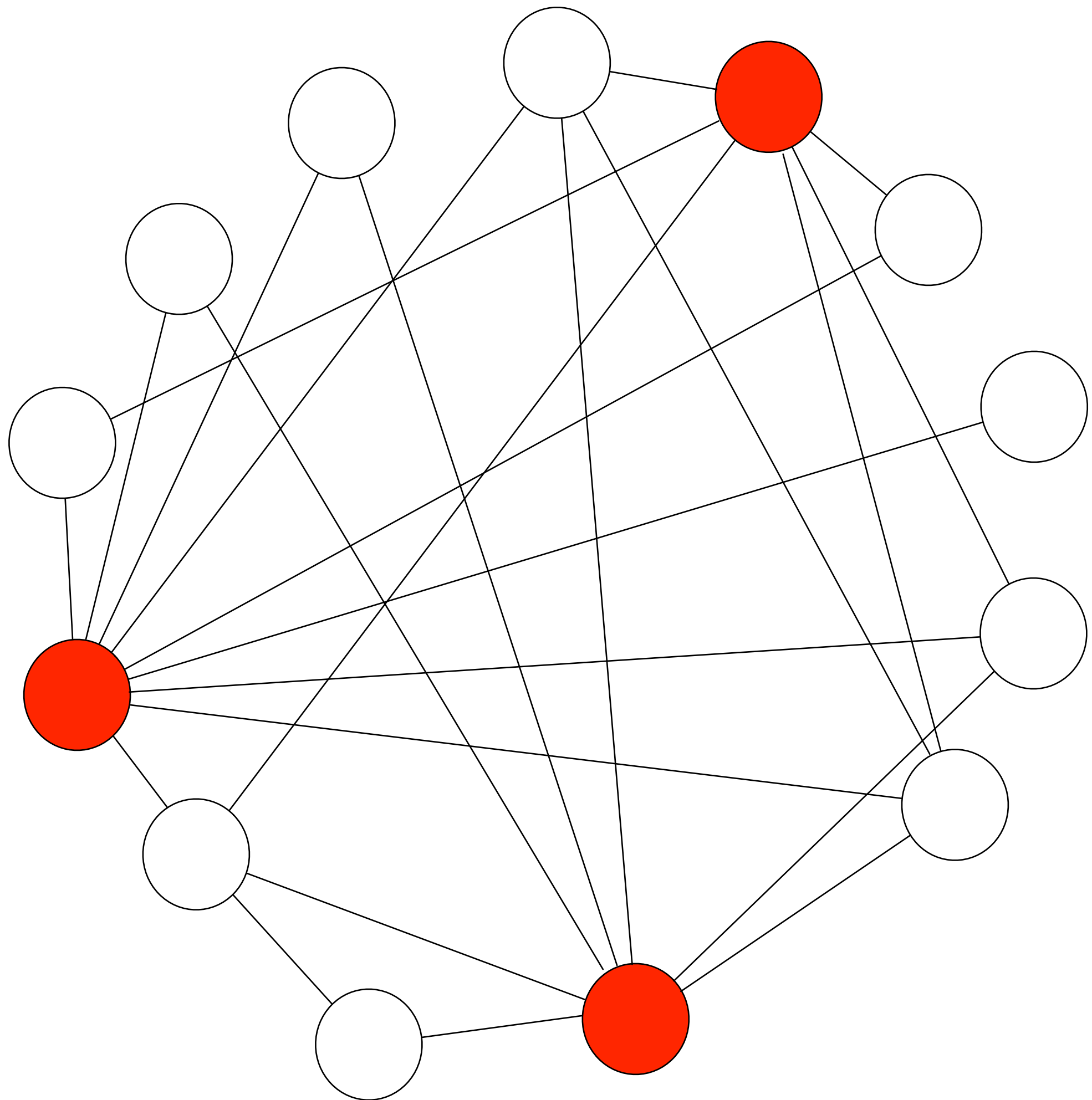




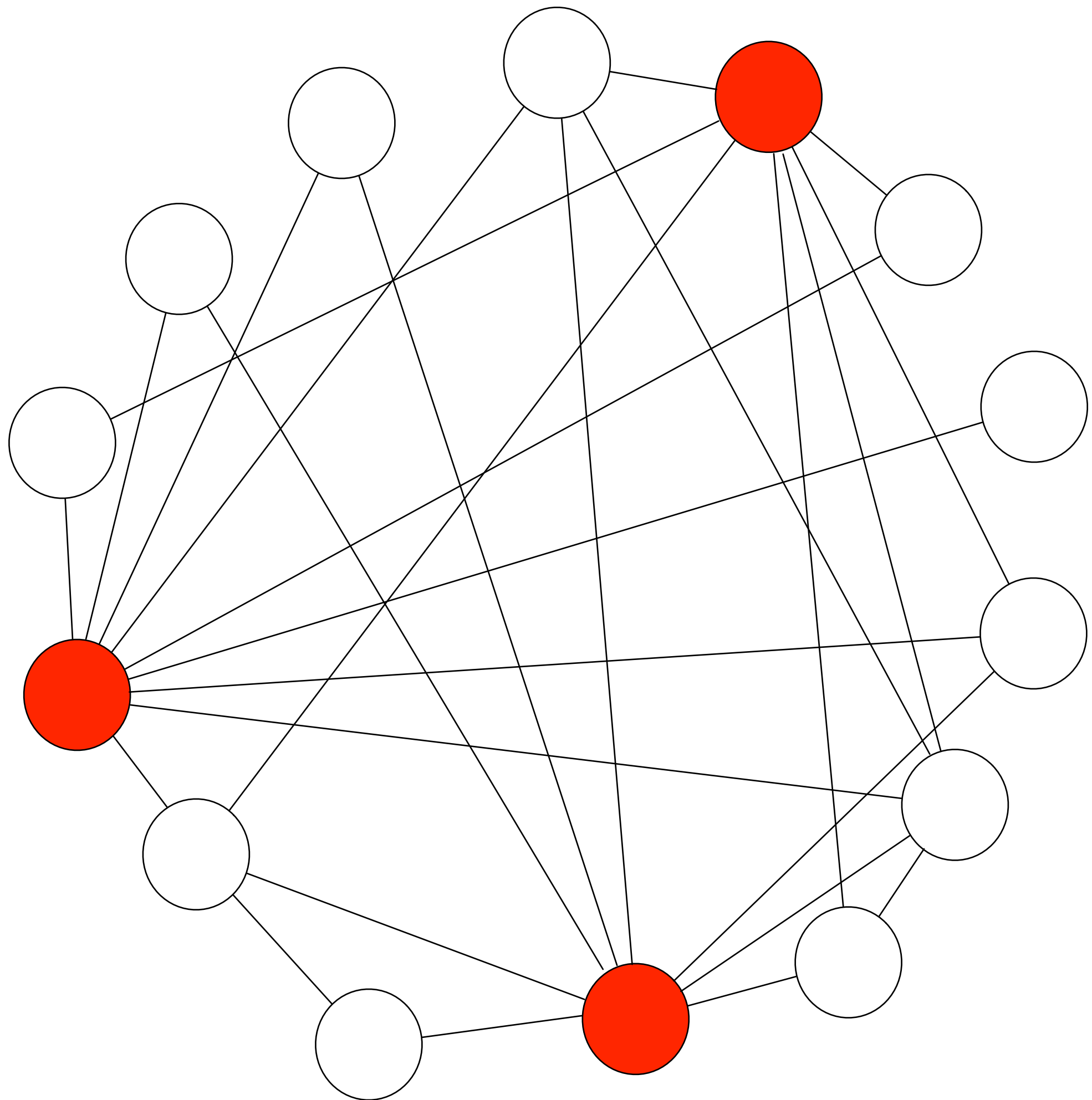


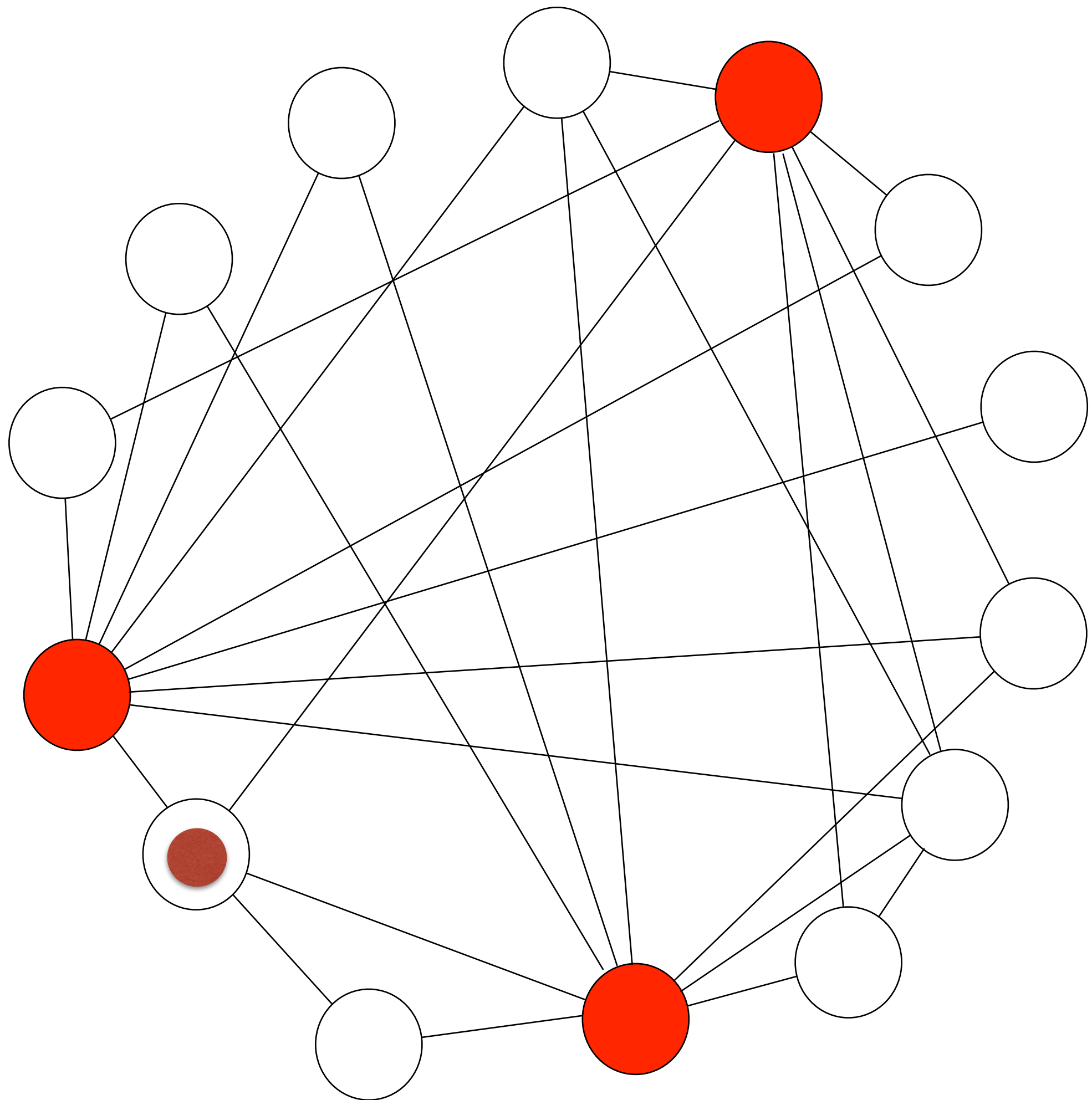


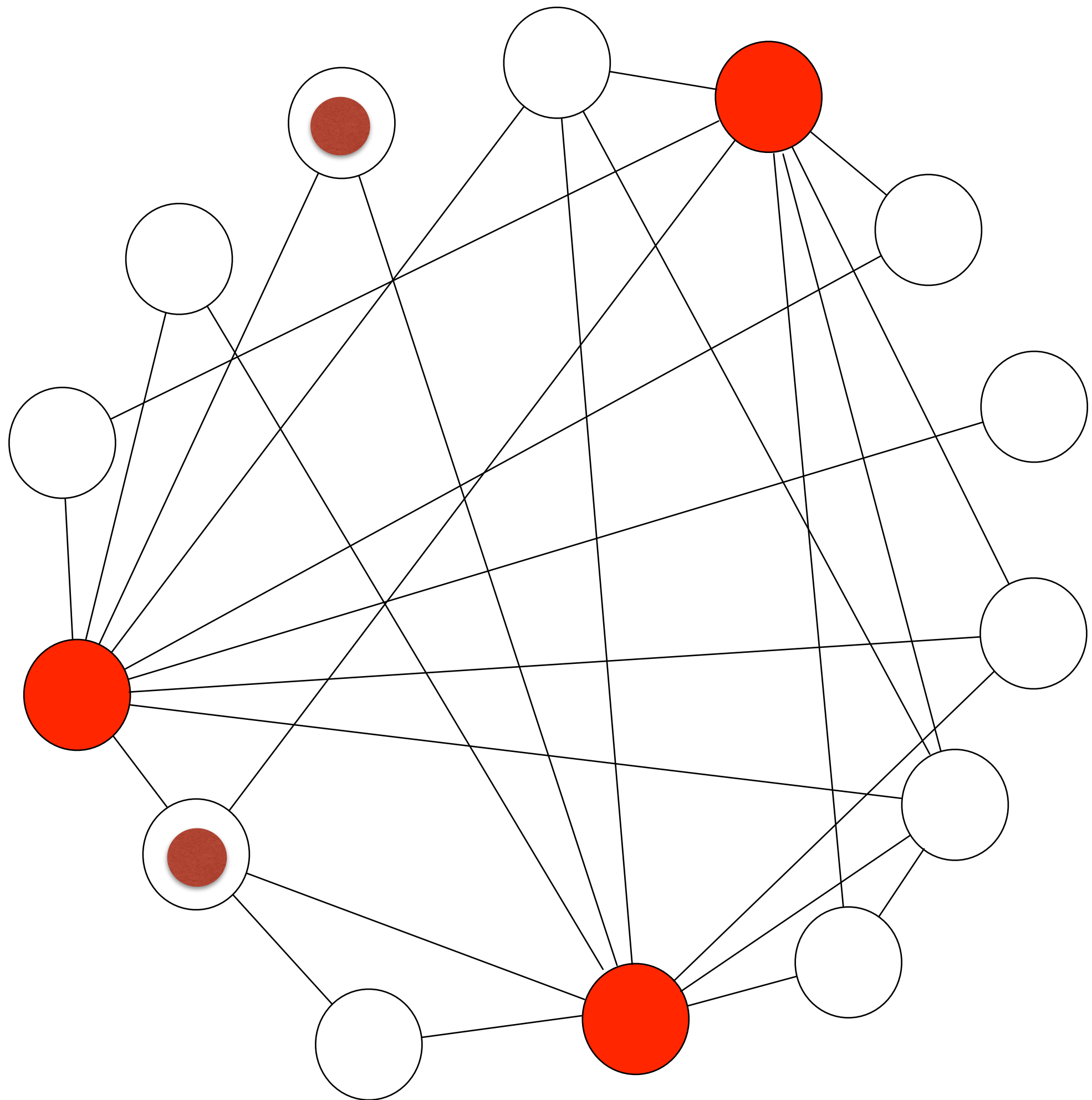


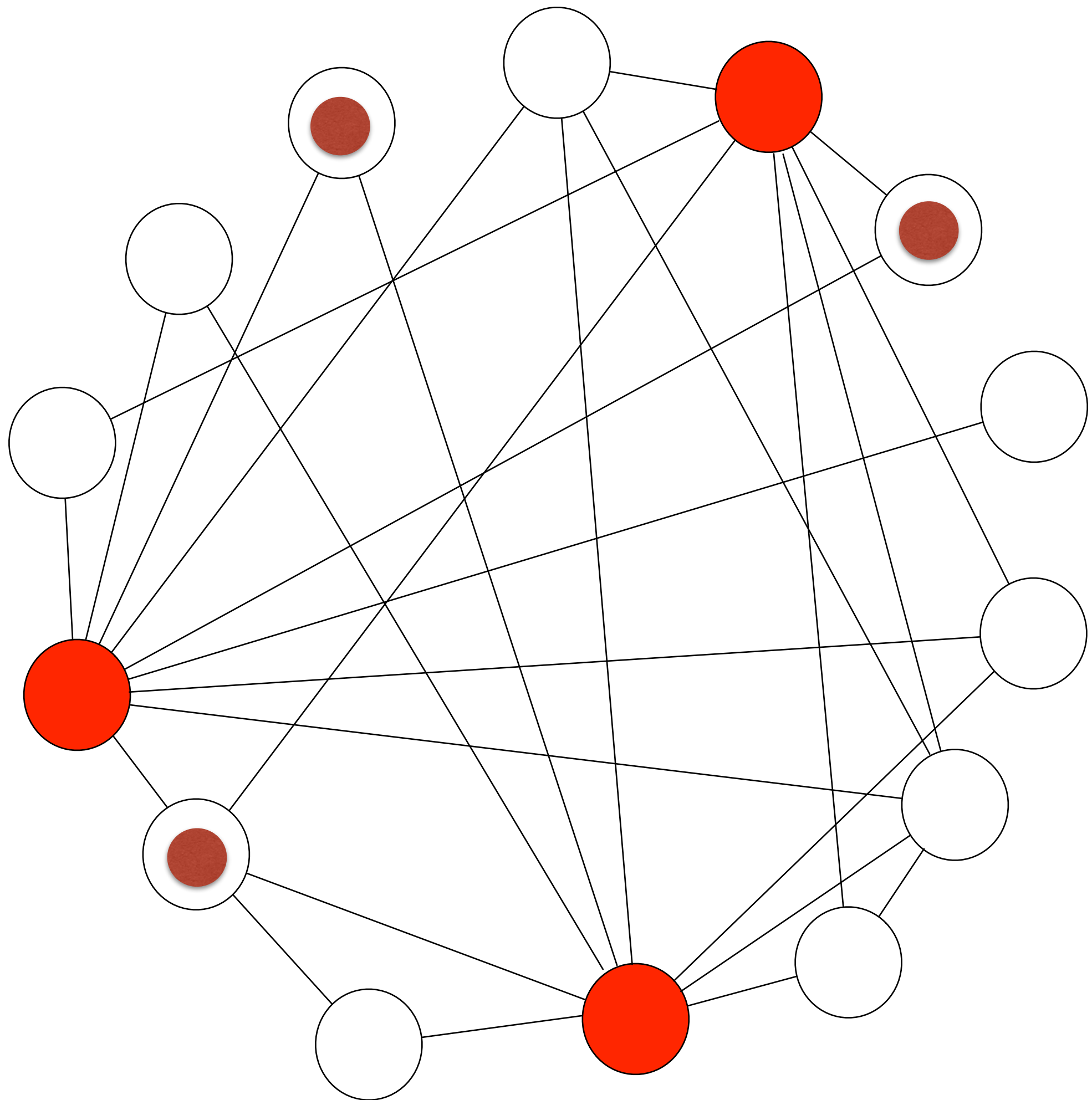


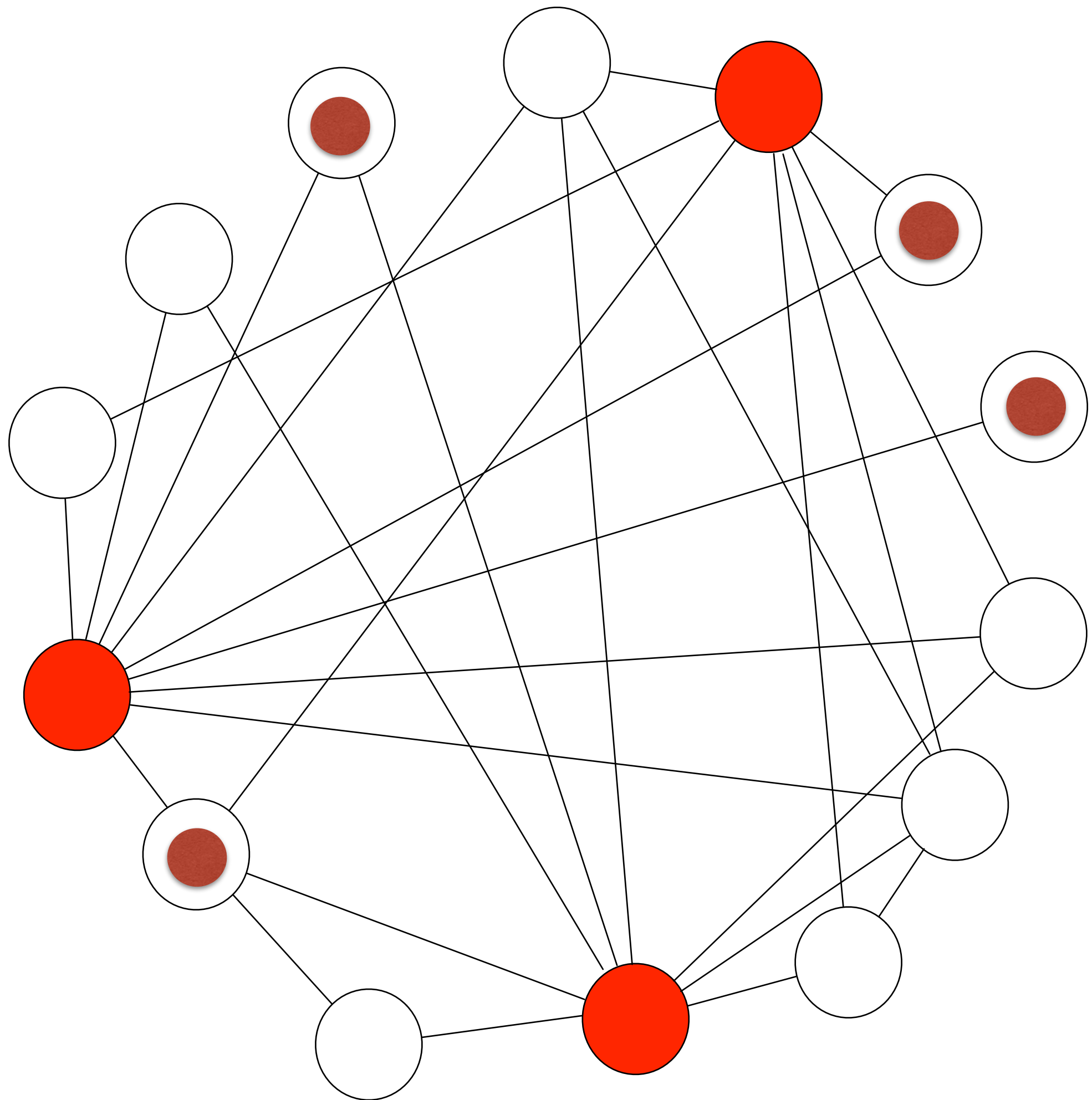


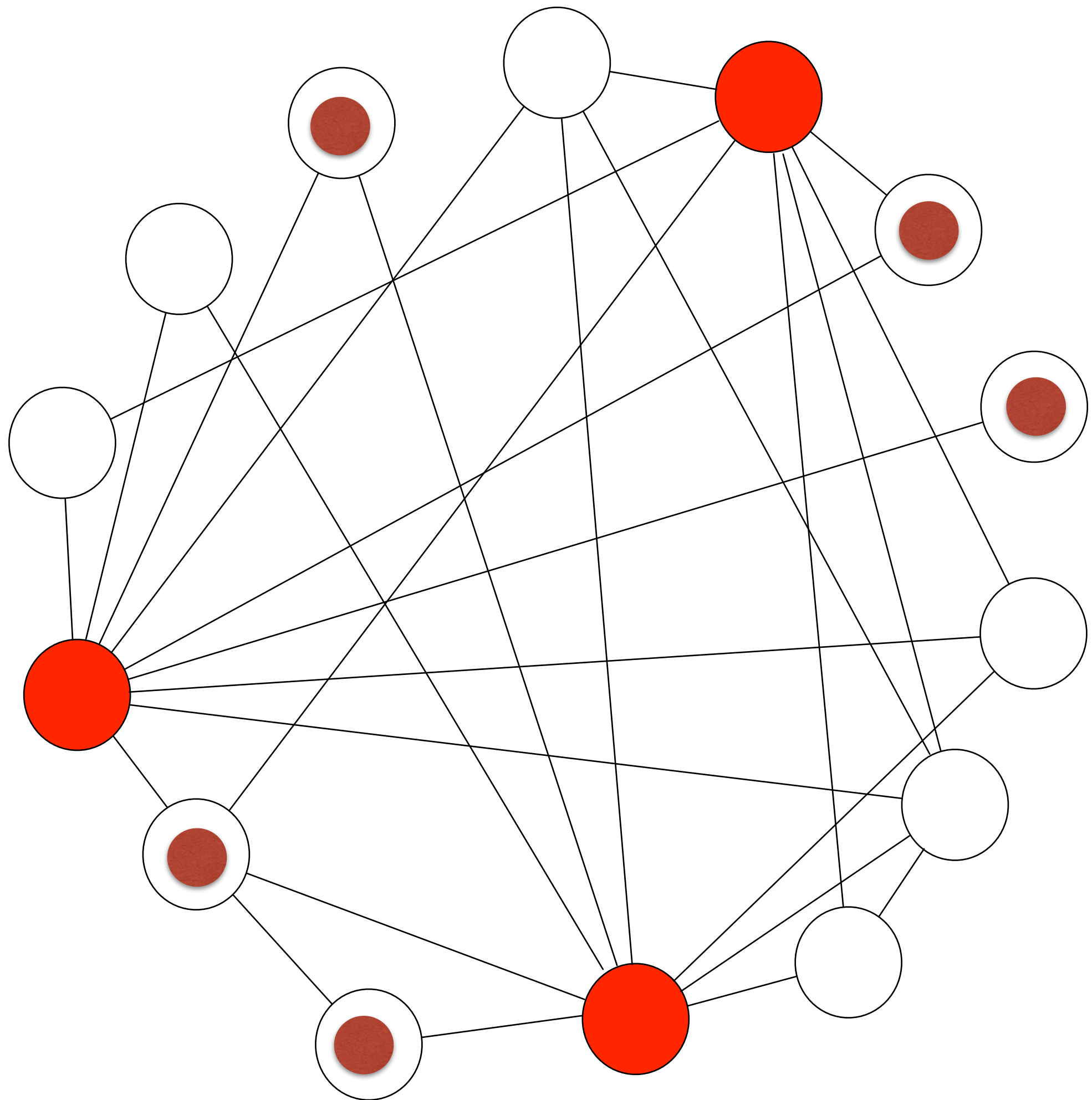


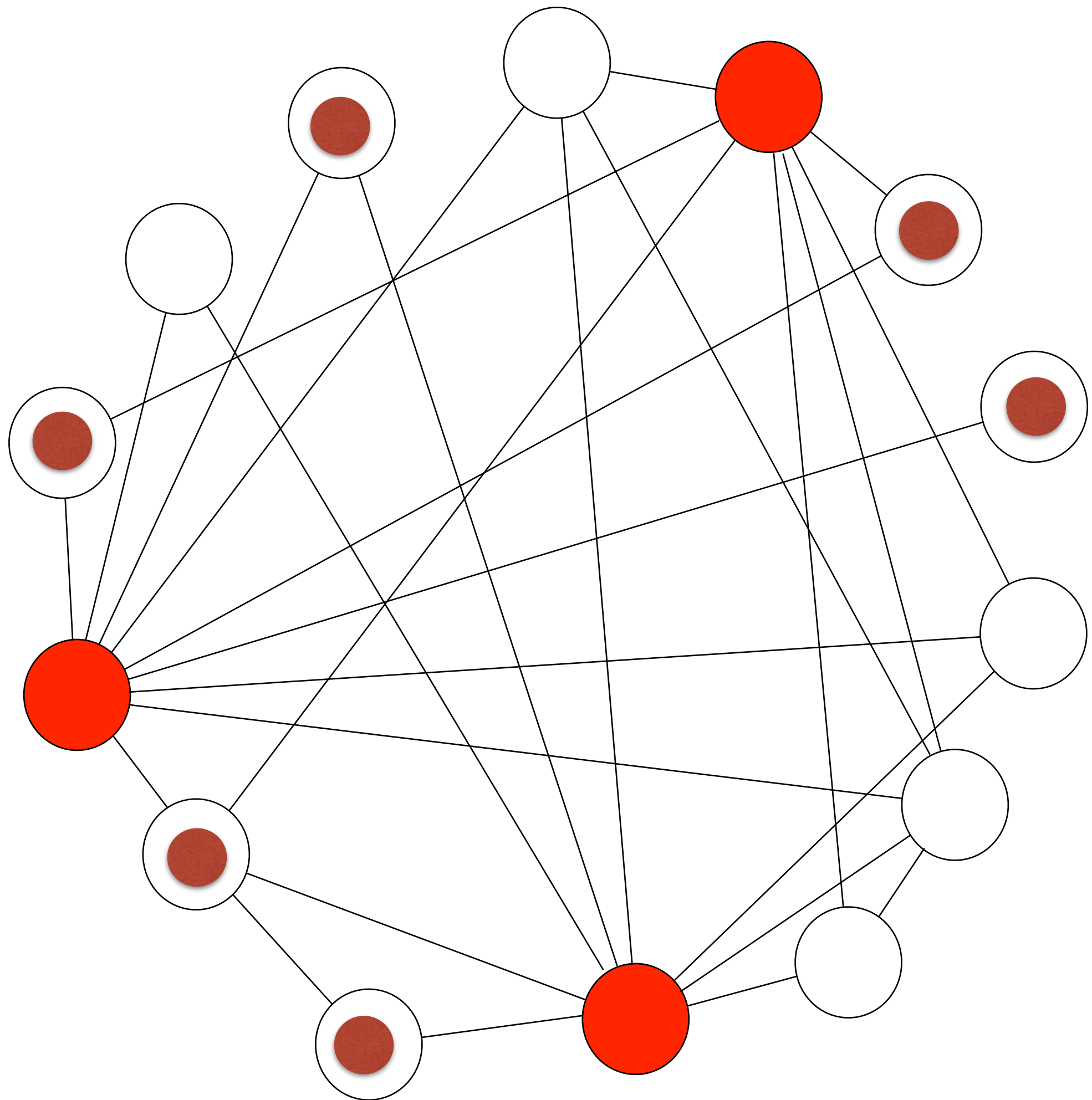


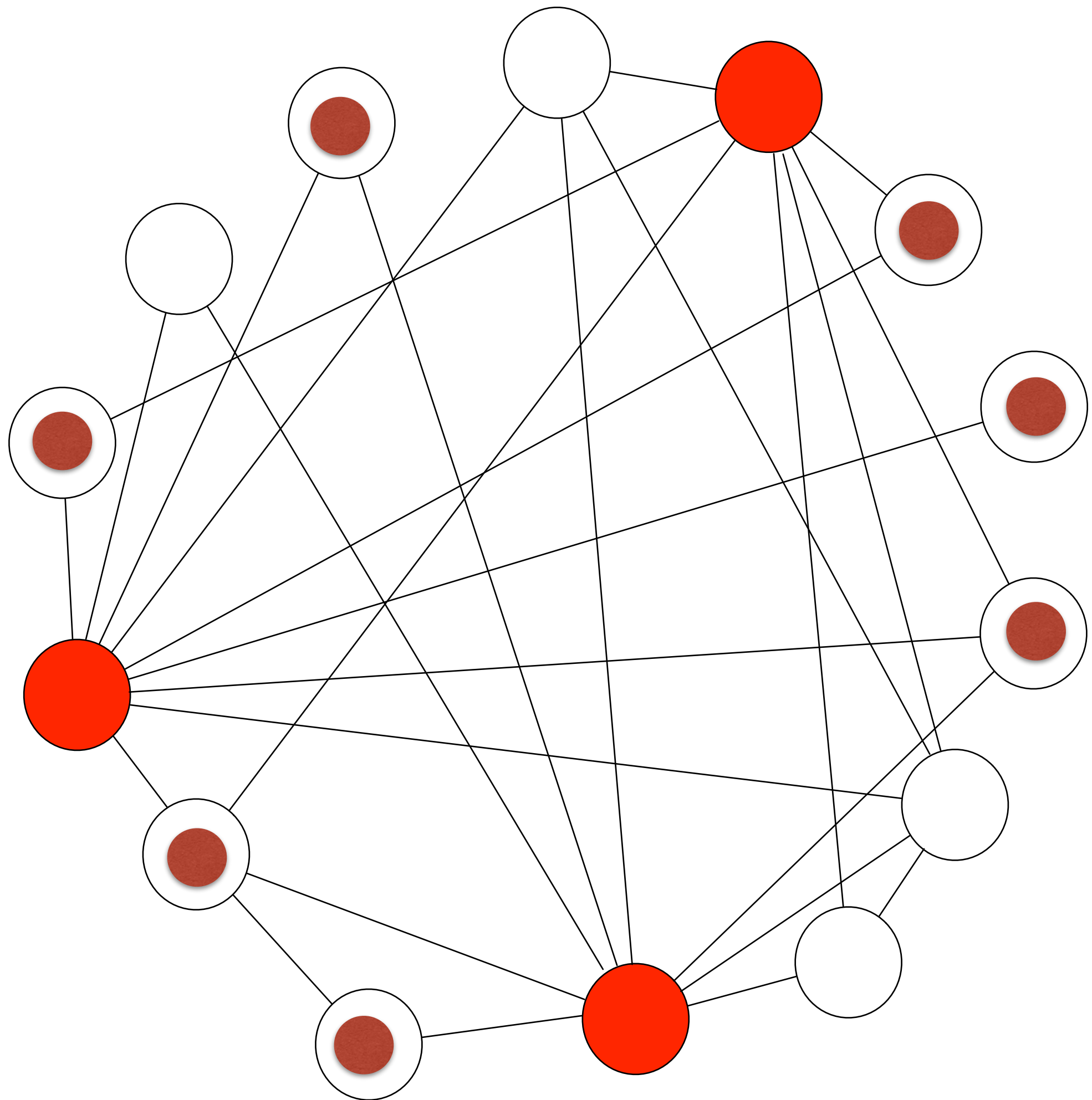




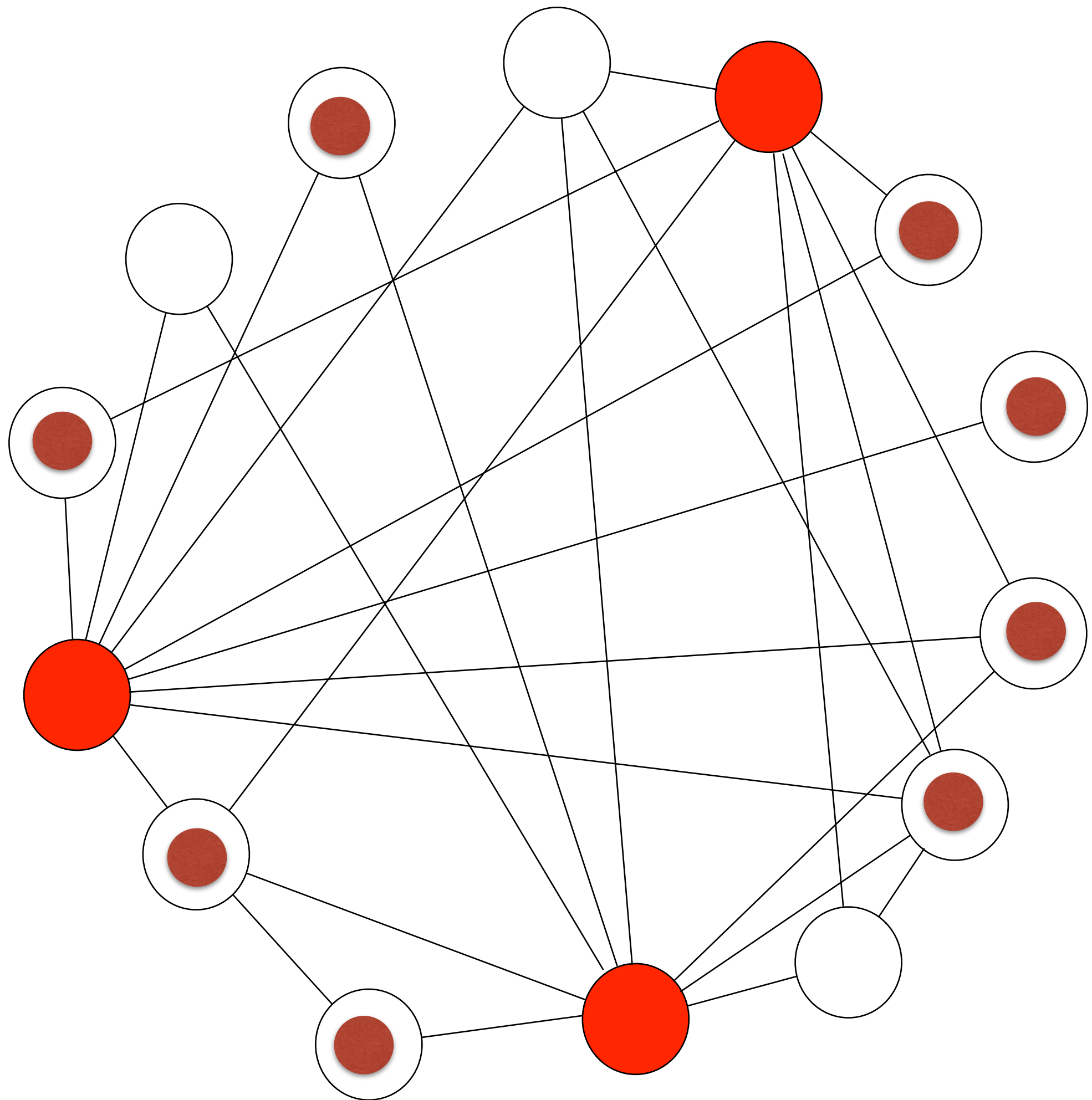


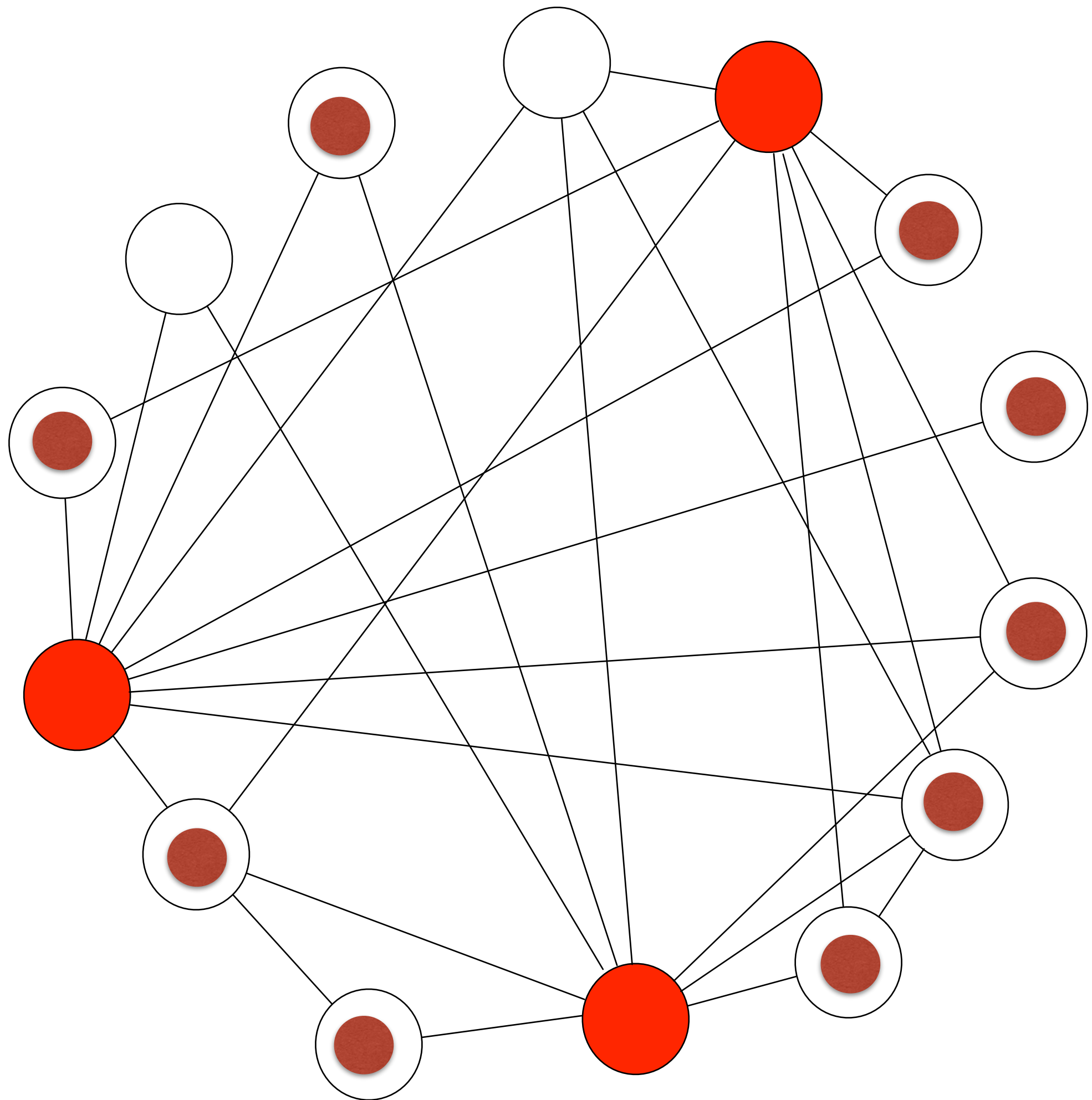


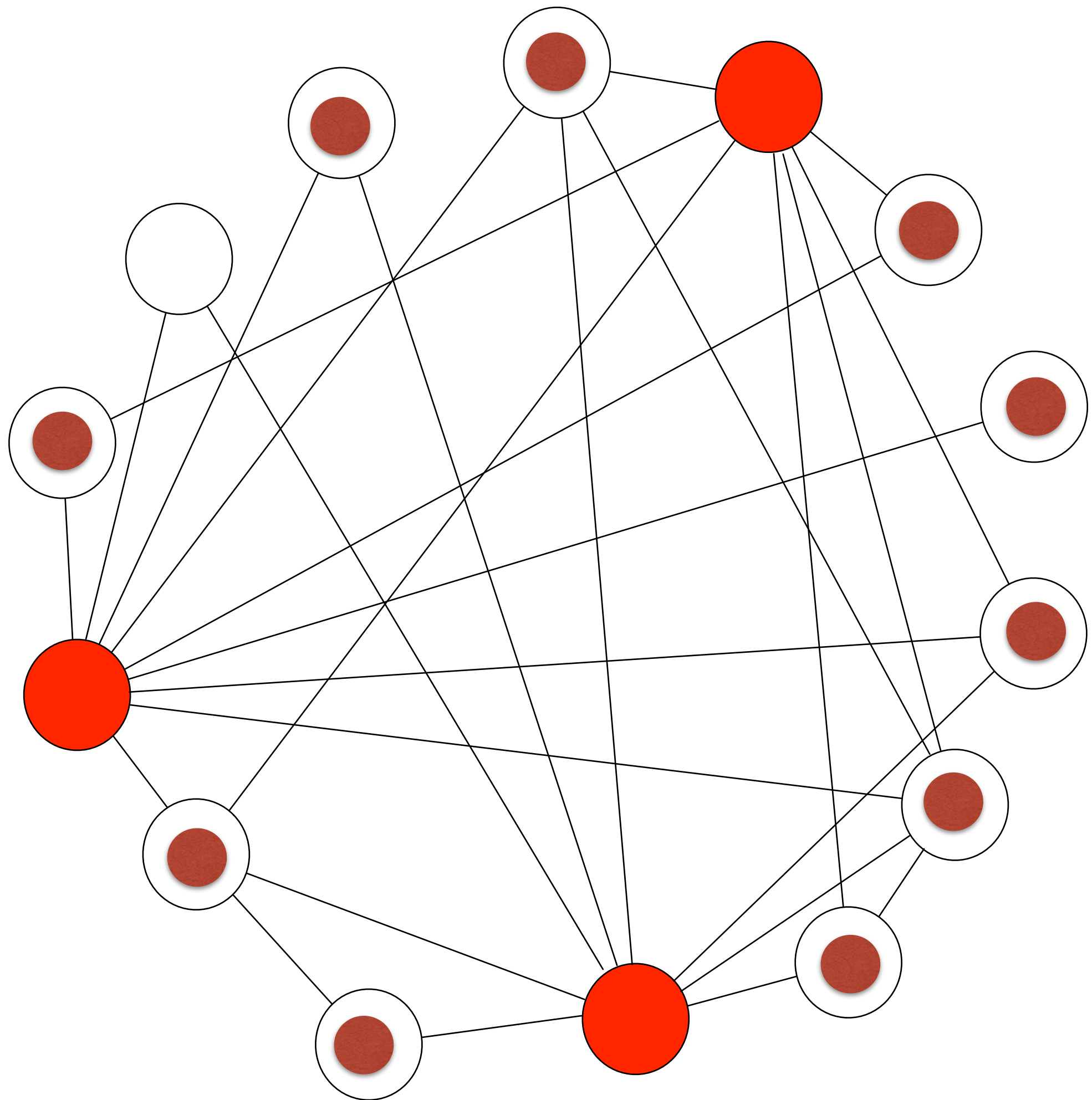


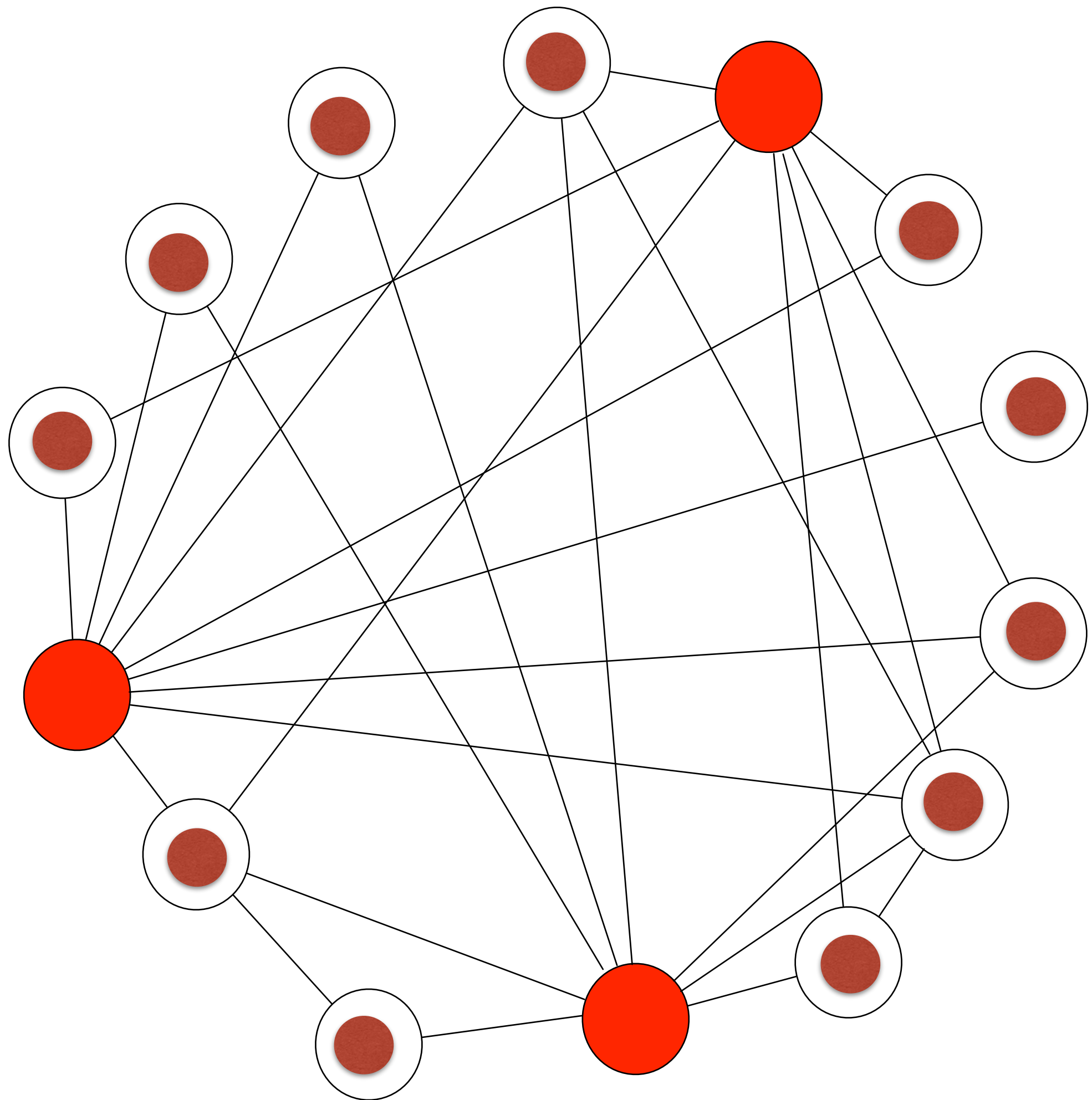












# Pluralistic ignorance



Article |  Full Access

## The discovery of pluralistic ignorance: An ironic lesson

Hubert J. O'Gorman

First published: October 1986

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# Pluralistic ignorance

# Pluralistic ignorance



"All those in favour say 'Aye'."

"Aye."

"Aye."

"Aye."

"Aye."

"Aye."

# Pluralistic ignorance



"All those in favour say 'Aye'."

"Aye."

"Aye."

"Aye."

"Aye."

"Aye."





# So far...

only symptoms of influence... but no models of social influence