The Complexity of Elections with Rational Actors

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Abstract. Voting and elections are among the most common methods of making collective decisions. The voters express their preferences regarding the candidates, and a voting rule aggregates them to provide the final election winner. We believe that to better understand elections, it is important to consider also the rationales behind the voters' individual preferences when aggregating them. To do this, we need to model and execute elections in such a way that the "rationality" is not optional or subjective. In this paper we propose to extend the traditional election model with information about the reasons for voters' choices.

1 The Model

We first briefly recall the approval-based election model and then extend it.

An approval election $E = (C, V)$ consists of a set of candidates $C = \{c_1, \dots, c_m\}$ and a collection of voters $V = (v_1, \dots, v_n)$. Each voter v_i has an approval set A_i that consists of those candidates from C that v_i approves of. The approval score of candidate c_j, denoted $\mathrm{score}_E(c_j)$, is defined as the number of voters that approve c_j. Formally, $\mathrm{score}_E(c_j) = |\{v_i \in V \mid c_j \in A_i\}|$. The set of approval winners, denoted $\mathcal{R}(E)$, consists of those candidates that receive the highest approval score in a given election. Typically, we expect to have only a single winner, but we have to take into account the possibility of ties. In practice, tie-breaking mechanisms are used when this happens, but in this paper we disregard this issue.

In an *active candidate* model, we assume that the candidates take the action of announcing the issues that they intend to address when in the office, and the voters judge if these agendas are sufficiently convincing for them to grant their approvals.

We are given a set of candidates $C = \{c_1, \dots, c_m\}$, a set of voters $V = (v_1, \dots, v_n)$, and a set $\mathcal{P}$ of political positions. Each candidate c_i is associated with a position $p(c_i) \in \mathcal{P}$, and each voter v_i has an evaluation function $f_i \colon \mathcal{P} \to \{\mathit{True}, \mathit{False}\}$ that specifies if the candidate approves a given position or not. The set A_i of the candidates approved by voter v_i is:

$$A_i = \{c_j \mid f_i(p(c_j)) = \mathit{True}\}.$$

This model allows us to formulate variants of the classic POSSIBLE WINNER problems, see *e.g.,* the works of [Konczak and Lang, 2005; Xia and Conitzer, 2011], but perhaps in a somewhat more realistic format.

Definition 1. *In the* POSSIBLE WINNER WITH ACTIVE CANDIDATES (PWAC) *problem, we are given an election $E = (C, V, \mathcal{P})$, where C is a set of candidates, V is a collection of voters (with their functions for evaluating positions), and $\mathcal{P}$ is a set of* *possible positions; the input also contains the preferred candidate c_p. Each candidate $c \in C$ is associated with a set P_c of positions that he or she may assume. We ask if it is possible to associate each candidate $c \in C$ with a position from P_c, so that c_p is a winner of the resulting election.*

The active candidate model is appealing because it seems to be capturing the natural dynamics present in political elections: The candidates announce the platforms on which they run, and each voter individually evaluates each of them.

Single-Peaked Elections. Consider an election where taxation level is the main issue. The set of possible positions of the candidates is $\mathcal{P} = [0, 1]$. Each candidate c_j announces his or her ideal taxation level $p(c_j) \in [0, 1]$. Each voter also has his or her interval $[a_i, b_i] \subseteq [0, 1]$ of acceptable taxation levels. Voter v_i approves candidate c_j if $p(c_j) \in [a_i, b_i]$. Formally, each evaluation function f_i is defined as follows (for each $x \in [0, 1]$):

$$f_i(x) = \begin{cases} True, & \text{if } x \in [a_i, b_i], \\ False, & \text{otherwise.} \end{cases}$$

We choose to model the sets of possible candidate positions in the PWAC problem so that for each candidate c, P_c is an interval $[x_c, y_c]$. In this case, our problem is polynomial-time computable.

Theorem 1. *For the active candidate model, the* PWAC *problem is in* P.

Proof. For the possible winner problem it suffices to choose the position of the preferred candidate so that it is approved by as many candidates as possible, and the positions of the remaining candidates to be approved by as few voters as possible. If in consequence c_p has at least as high approval score as every other candidate, then we accept. Otherwise we reject. Computing positions of the candidates is easy: Indeed, it boils down to finding a point from a given interval that intersect either as many as possible or as few as possible given intervals; this is a classic problem that can be solved by a simple greedy algorithm.

2 Summary

In addition to active candidates, we also intend to propose an *active voter model*, in which it is the voters that are represented with a set of issues they care about. A voter then approves of a candidate if that candidate satisfies (most) of the issues the voter cares about. In addition to the possible winners, we can also study necessary winners. It is also natural to consider both rational election models and explore further issues of strategic behavior.

Bibliography

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